Metodo di Lax-Wendroff

Esempio

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \quad , \quad c = \text{cost.}$$

$$u(t+\Delta t, x) \simeq u(t,x) + \left.\frac{\partial u}{\partial t}\right|_t \Delta t + \frac{1}{2}\left.\frac{\partial^2 u}{\partial t^2}\right|_t (\Delta t)^2$$

$$\left.\frac{\partial u}{\partial t}\right|_t = -c\left.\frac{\partial u}{\partial x}\right|_t$$

$$\left.\frac{\partial^2 u}{\partial t^2}\right|_t = \left[\frac{\partial}{\partial t}\left(-c\frac{\partial u}{\partial x}\right)\right]_t =$$

$$= \left[-c\frac{\partial}{\partial x}\frac{\partial u}{\partial t}\right]_t = -c\left[-c\frac{\partial^2 u}{\partial x^2}\right]_t$$

$$= c^2\left.\frac{\partial^2 u}{\partial x^2}\right|_t$$

$$\Rightarrow u(t+\Delta t, x) \simeq u(t,x) - c \left.\frac{\partial u}{\partial x}\right|_t \Delta t$$
$$+ c^2 \left.\frac{\partial^2 u}{\partial x^2}\right|_t \frac{(\Delta t)^2}{2}$$

$$\left.\frac{\partial u}{\partial x}\right|_t \simeq \frac{u_{i+1}^t - u_{i-1}^t}{2h}$$

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_t \simeq \frac{u_{i+1}^t - 2u_i^t + u_{i-1}^t}{h^2}$$

$$\Rightarrow \quad u_i^{n+1} = u_i^n + \Delta t \, \frac{u_{i+1}^n - u_{i-1}^n}{2h}$$
$$+ \frac{(\Delta t)^2}{2} \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{h^2}$$