$$y_t + y\,y_x = 0$$

$$y^{n+1} = y^n + \left.\frac{dy}{dt}\right|^n \Delta t + \frac{1}{2}\left.\frac{d^2y}{dt^2}\right|^n (\Delta t)^2$$

$$= y^n - y^n y_x^n \Delta t + \frac{1}{2}(\Delta t)^2 \cdot \frac{d}{dt}\left[-yy_x\right]^n$$

$$\frac{d}{dt}\left[-yy_x\right] = -y_t^n y_x^n - y^n y_{xt}^n = (yy_x)^n y_x^n - y^n y_{xt}^n$$

$$y_{xt} = \frac{\partial}{\partial x}\left(-yy_x\right) = -(y_x)^2 - y\,y_{xx}$$

$$\Rightarrow \frac{d}{dt}\left[-yy_x\right] = y^n (y_x^n)^2 + y^n (y_x^n)^2 + (y^n)^2 y_{xx}^n$$

$$= 2y^n (y_x^n)^2 + (y^n)^2 y_{xx}^n$$

$$y^{n+1} = y^n - y^n y_x^n \Delta t + \frac{1}{2}(\Delta t)^2 \left[2y^n (y_x^n)^2 + (y^n)^2 y_{xx}^n\right]$$

$$\left(y_x^n\right)_j \simeq \frac{y_{j+1}^n - y_{j-1}^n}{2h}$$

$$\left(y_{xx}^n\right)_j \simeq \frac{y_{j+1}^n - 2y_j^n + y_{j-1}^n}{h^2}$$