

$$\begin{cases} y_{,xx} - f(x) = 0 & x \in (0,1) \\ y(0) = 0 \\ y_x(1) = 1 \end{cases}$$

$$\int_0^1 (y_{,xx} - f) N_i dx = 0 \quad \forall i$$

$$\bullet \quad y_{,x} N_i \Big|_0^1 - \int_0^1 y_{,x} N_{i,x} dx - \int_0^1 f N_i dx = 0 \quad \forall i$$

$$N_i = 0 \text{ at } x=0 \quad \forall i \text{ (Dirichlet)}$$

$$y_{,x}(1) = 1 \Rightarrow y_{,x} N_i \Big|_0^1 = N_i(x=1)$$

$$\bullet \quad \int_0^1 y_{,x} N_{i,x} dx = \int_0^1 N_{j,x} N_{i,x} dx y_j \quad (\text{Euler's summation conv.})$$

$$\int_0^1 = \sum_e \int_{\Omega^e}$$

$$\int_{\Omega^e} N_{j,x} N_{i,x} dx = \begin{cases} 1/h^{(e)} & \text{se } i=j \\ -1/h^{(e)} & \text{se } i \neq j \end{cases}$$

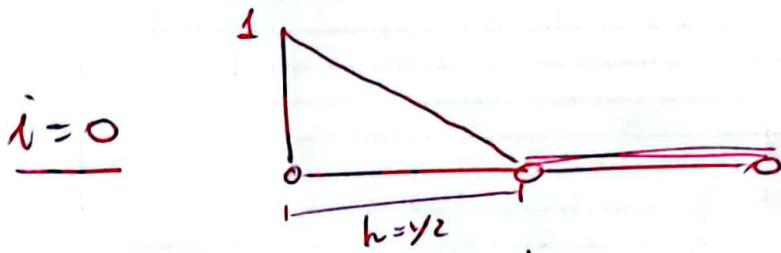
$$[K] = \begin{bmatrix} \boxed{K^{(1)}} & & \\ & \boxed{K^{(2)}} & \\ & & \end{bmatrix}$$

global
pre-consol. cond.

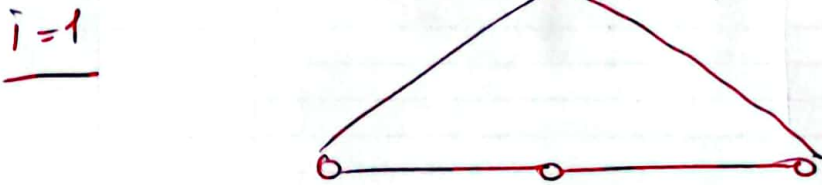
$$= \begin{bmatrix} +y/h & -y/h & 0 \\ -y/h & y/h + 1/h & -1/h \\ 0 & -y/h & y/h \end{bmatrix}$$

$$= \frac{1}{h} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\int_0^1 N_i f(x) dx :$$



$$\int_0^1 N_0 \sec x dx = \int_0^h \left(1 - \frac{x}{h}\right) \sec x dx = 1 - \frac{\sec h}{h}$$



$$\int_0^1 N_1(x) \sec x dx = \left(\frac{\sec h}{h} - \cosh h \right) + \left[\cosh h + \frac{\sec h - 2 \sec h \cosh h}{h} \right]$$

$$= - \frac{\sec(2h) - 2 \sec h}{h}$$

$i=2$

$$\int_0^1 N_2(x) dx = \int_{0.5}^1 \frac{x-h}{h} \sec x dx = - \frac{\sec h - 2 \cosh \sec h + h(2 \cosh^2 - 1)}{h}$$

$$b_i \equiv y_{i,x} U_i(x=1) - \int_0^1 f N_i dx \equiv U_i(x=1) - \int_0^1 f U_i dx$$

$$K_{ij} y_j = b_i \quad i=1,2,3$$

Sostituendo le espressioni di Dirichlet:

$$\begin{bmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} - \int_0^1 \sin(x) U_1 dx \\ 1 - \int_0^1 \sin(x) U_2 dx \end{bmatrix}$$

Per $h=1/2$:

$$\begin{bmatrix} 4 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -2.3476 \dots \times 10^{-1} \\ 8.1621 \dots \times 10^{-1} \end{bmatrix}$$

etc.