Advanced Mathematical Physics Mod. A.
Academic year 2022/2023
Detailed program

1) Generalities and the transport equation.
   A- Linear, quasi-linear and semi-linear equations. Concept of well-posedness.
   B- The transport equation: general solutions. Characteristics. Non-homogeneous transport equation. D'Alembert's formula.
2) The Laplace Equation.
   A - Definitions and the Poisson equation. The "baby example" D=1. Rotational invariance. Radial fundamental solution. Green's identities.
   B - Uniqueness for Dirichlet and Neumann boundary conditions. Neumann compatibility condition. Mean values theorems.
   C- Solution of the Poisson equation by convolution with the fundamental solution.
   D- Maximum principles. Newton's theorem.
   E- Poisson formula for D=2. Case of the circle and the rectangle. Green representation for solutions in bounded domains.
   F- Applications: Green function for the half-space and Green function for the ball in R^D. Energy methods and L^2 uniqueness.
   G- D=2 and analytic functions. Conformal transformations and harmonic functions. Ill-posedness for the Cauchy problem for the Laplace equation.
3) The Heat (diffusion) Equation.
   A- Thermodynamical derivation. Random-walk derivation. The case D=1. Fundamental solution and Dirac's δ-function.
   B- D>1. Symmetries and scalings. Fundamental solution. Non-homogeneous heat equation.
   C- Boundary conditions and parabolic boundary.
   D- Application: Heat diffusion in a finite length bar.
   E- Problems in unbounded domains. Inhomogeneous heat equation and the Duhamel method.
   F- Fourier series. Parseval identity. Fourier series of real functions. Fourier transform and Dirac's δ-function. Fourier transform of the Gaussian function and the fundamental solutions for the heat equation in D=1.

4) The Wave (D’Alembert) Equation.
   A- The wave equation for the vibrating string. The case of an infinite sequence of oscillators. Sound waves. Maxwell’s equation. IVP and Boundary conditions.
   B- D=1. L^2 Uniqueness and stability of the solution by energy methods. D’Alembert formula and causality. The D’Alembert equation on the half-line. Plane wave elementary solutions and the dispersion relation.
   C- D=2 and D=3. Spherical averages and the Euler-Poisson-Darboux equation. D=3 and the Kirchhoff formula. Reduction method and the case D=2. The Huygens phenomenon. Inhomogeneous Wave equations and the Duhamel principle. Examples in D=1 and D=3.
5) Topics in Fluid Dynamics.
   A – Kinematics. Axiomatic setting. The Lagrange and Euler description of the fluid’s motion. Reynolds’ transport theorem. Mass conservation. Material derivative.
   B – Dynamics. The linear momentum balance equation and the Stress Tensor. Cauchy Equations. The angular momentum balance equations: symmetry of the Stress Tensor. Euler fluids. The concept of incompressibility.
   C – Homogeneous Euler fluids, Lamb’s form of the Euler equations and the Bernoulli Theorem. Helmholtz’s equations for the vorticity field. Newtonian fluids and the Navier-Stokes equations for a homogeneous fluid.
   D – Applications: 1) Flow past a circular obstacle in 2D. The lift.
   2) Water waves: derivation of the equations. Linearization and the dispersion relation.