# Cyber-Physical Systems

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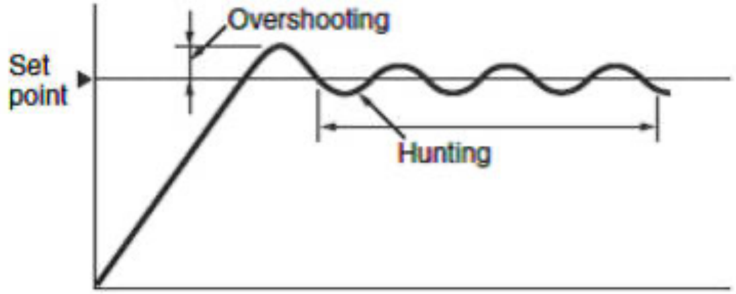
II Semestre 2022

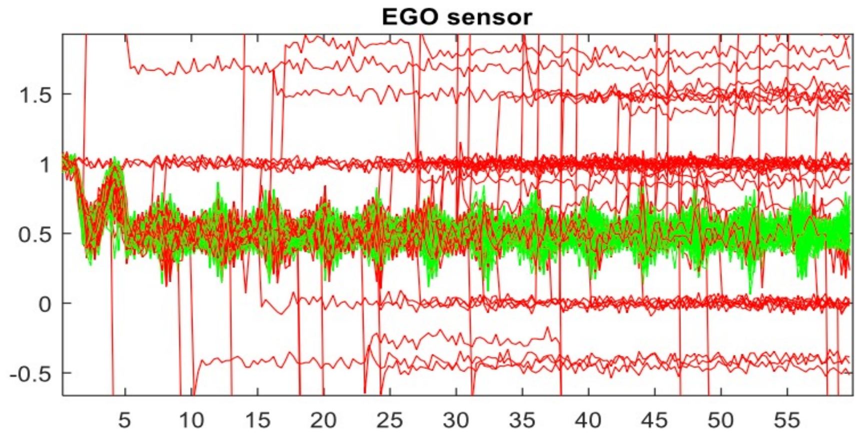
## Lecture 16: STL learning

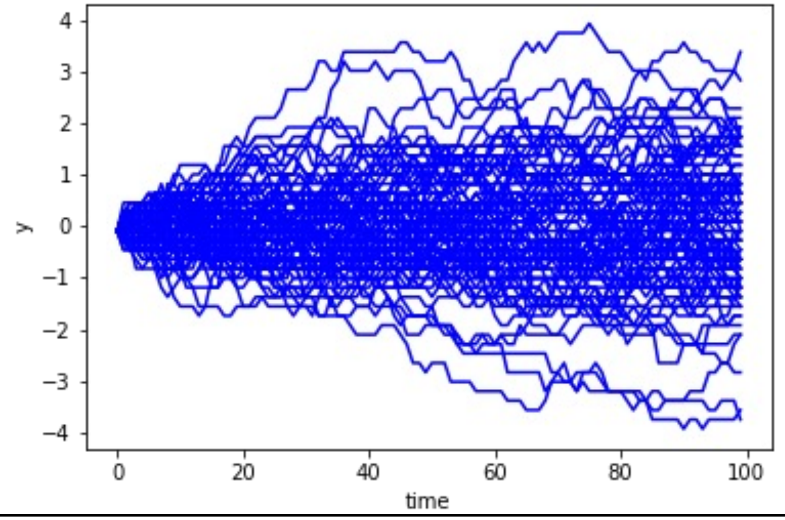
[Many Slides due to J. Deshmukh, S. Silvetti]

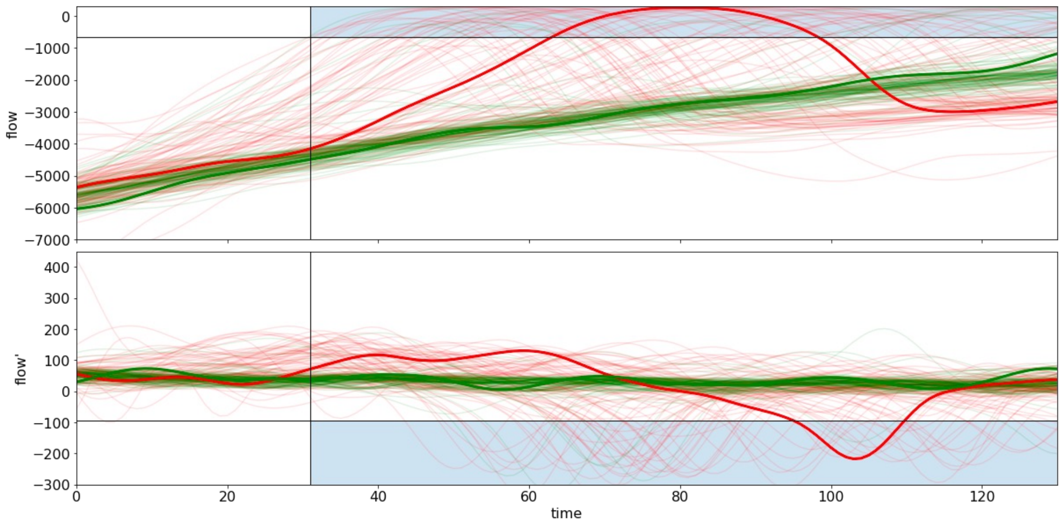
# Context and Problem

## Data are trajectories



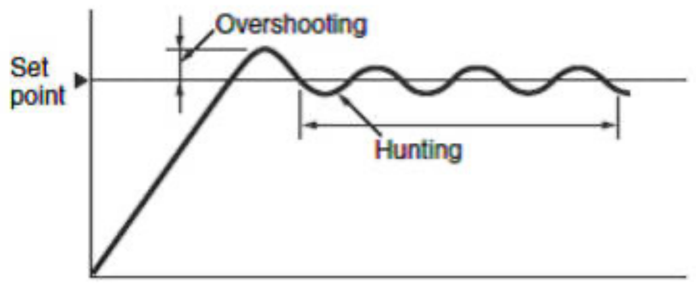



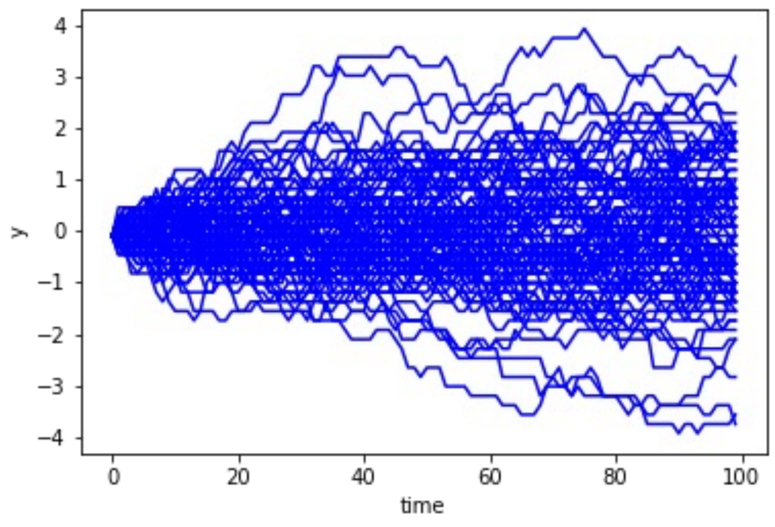



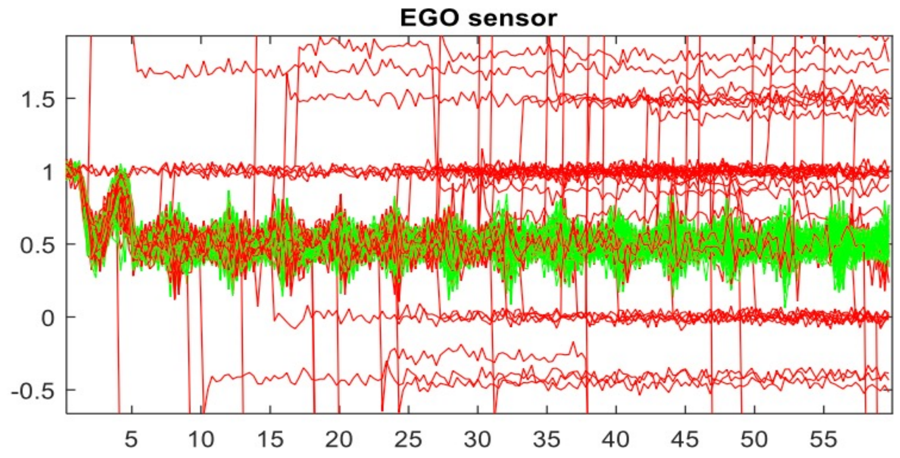
# Context and Problem

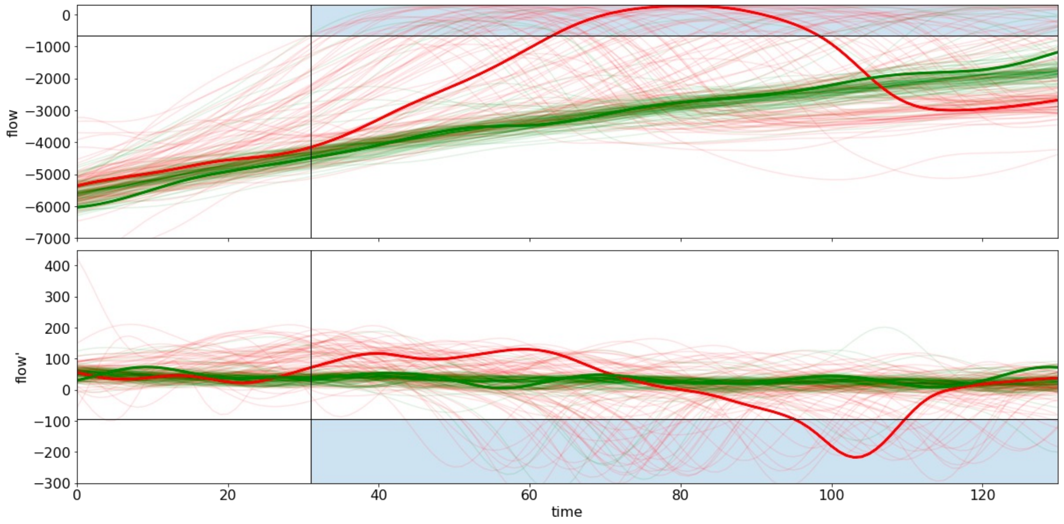
## Mining tight STL



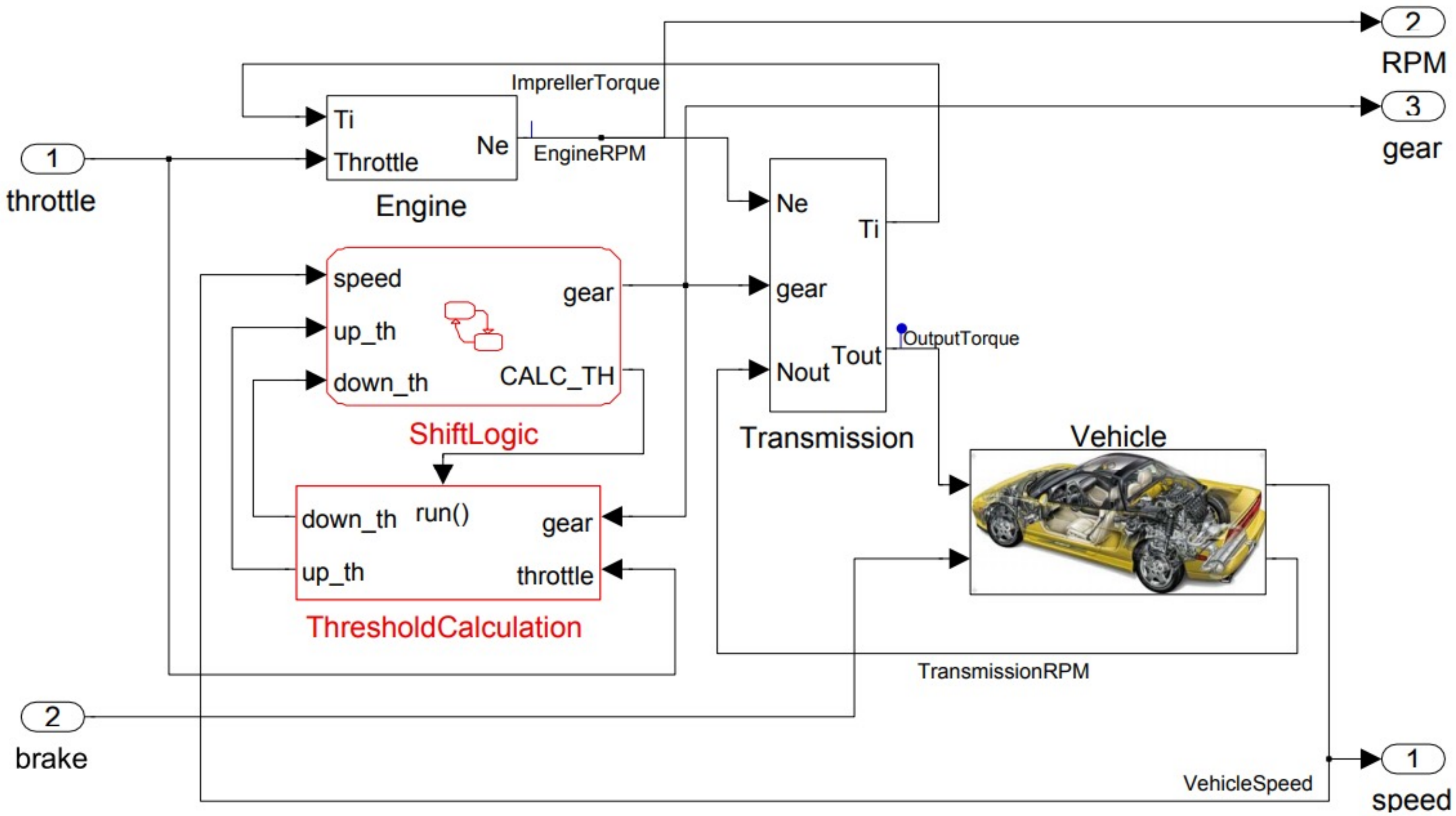




## Mining STL classifier



# Specification Mining




- What is the maximum speed that the vehicle can reach ?
- What is the minimum d well time in a given gear ?

# Parametric Signal Temporal Logic

**Definition (PSTL syntax)**

$$\phi := (x_i \bowtie \pi) \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \,\mathcal{U}_{[\tau_1,\tau_2]}\, \varphi_2$$

with $\bowtie \in \{>, \leq\}$

- $\pi$ is **threshold** parameter
- $\tau_1$, $\tau_2$ are **temporal** parameters

- $\mathbb{K} = (\mathcal{T} \times \mathcal{C})$ be the **parameter space**
- $\boldsymbol{\theta} \in \mathbb{K}$ is a **parameter configuration**

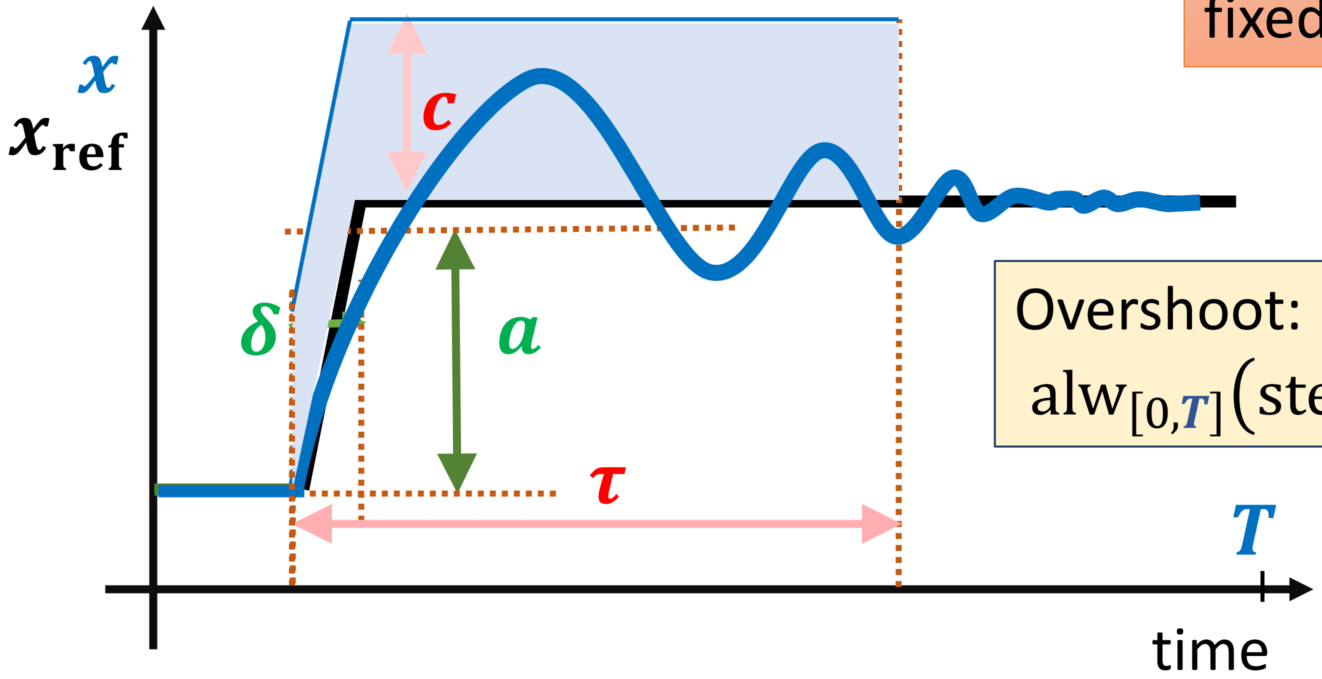
e.g., $\phi = \mathcal{F}_{[a,b]}(x_i > k), \boldsymbol{\theta} = (0, 2, 3.5)$ then $\phi_{\boldsymbol{\theta}} = \mathcal{F}_{[0,2]}(x_i > 3.5)$.

# Specification Mining

- Specification Mining: Try to find values of parameters of a PSTL formula from a given model

- Why?
  - Good to know “as-is” properties of the model
  - Finds worst-case behaviors of the model
  - Discriminates between regular and anomalous behaviours

# Specification Templates using PSTL




In previous lecture, $a, c, T, \tau, \delta$ were some fixed values, here they represent parameters

Step:

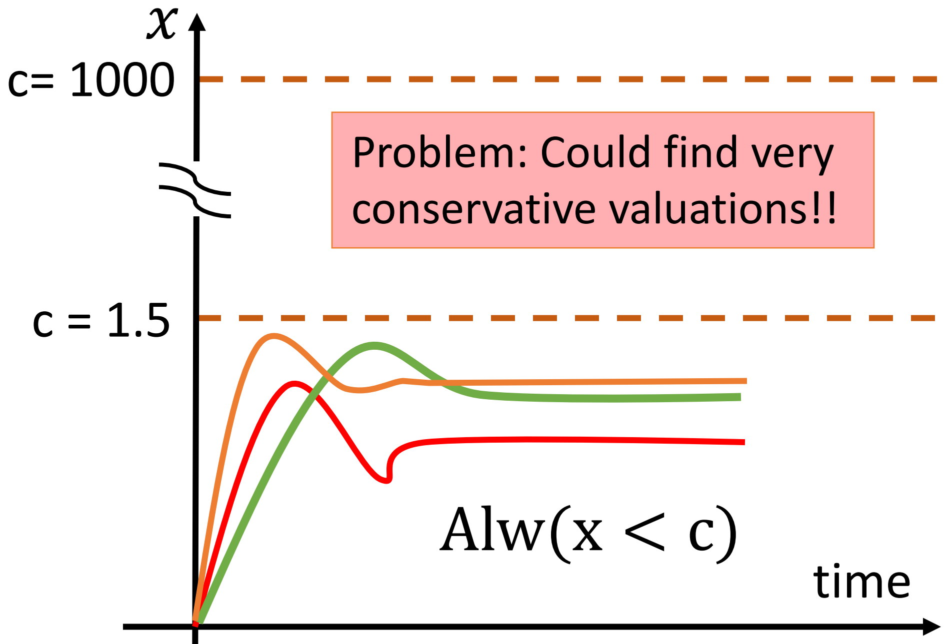
$\text{step}(y) \coloneqq y(t + \delta) - y(t) > a$

Overshoot:

$\text{alw}_{[0,T]}\left(\text{step}(x_{\text{ref}}) \Rightarrow \text{alw}_{[0,\tau]}(x(t) - x_{\text{ref}}(t) < c)\right)$

# Parameter inference for PSTL

- Given:
  - PSTL formula $\varphi(\mathbf{p})$, $[\mathbf{p} = (p_1, p_2, \ldots, p_m)]$
  - Traces $x_1, \ldots, x_n$
- Find:
  - ~~Valuation~~ $\nu(\mathbf{p})$ such that: $\forall i : x_i \vDash \varphi(\nu(\mathbf{p}))$
    *δ-tight valuation*
  - and $\exists i: x_i \nvDash \varphi(\nu(\mathbf{p}) \pm \delta)$ :
    i.e. small perturbation in $\nu(\mathbf{p})$ makes some trace not satisfy formula



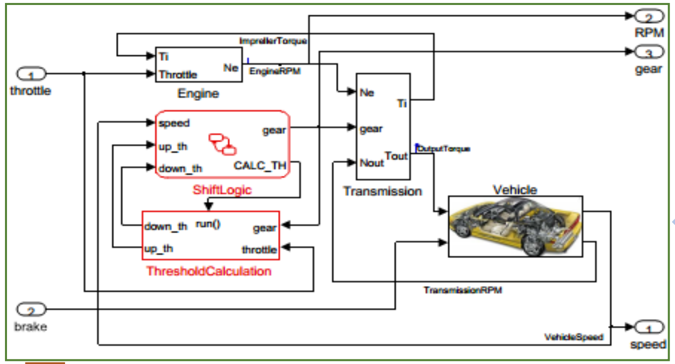

Problem: Could find very conservative valuations!!

formula sat for given valuation ⇒ ∀ greater (or lesser) valuations sat

Finding $\delta$-tight valuations hard in general, but **efficient** for **Monotonic PSTL**

Binary search on parameter space

# Specification Mining




Falsification:
∃ **trace** ⊭ Property?

YES

NO

Secret Sauce:

- Infer parameters for a given PSTL formula from traces
- Falsify given STL formula

Param…
Find "Tightest"
Answers

Overshoot is 5.5 KPa
Upper Bound on x is 5

Settling Time is **??**
Overshoot is **??**
Bounds on *x* are **??**

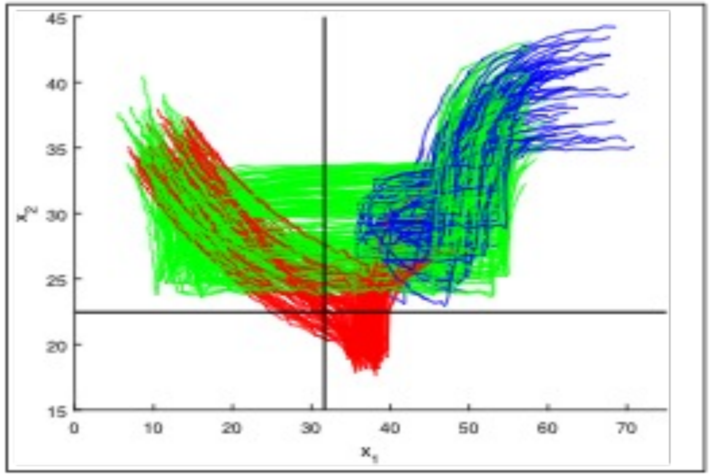
Settling Time is 6 ms
Overshoot is 5.5 KPa
Upper Bound on x is 5

# Learning STL classifiers

**Goal**: Given sets of good and bad trajectories (or generative models), learn STL properties that can separate the two behaviours (a STL classifier)

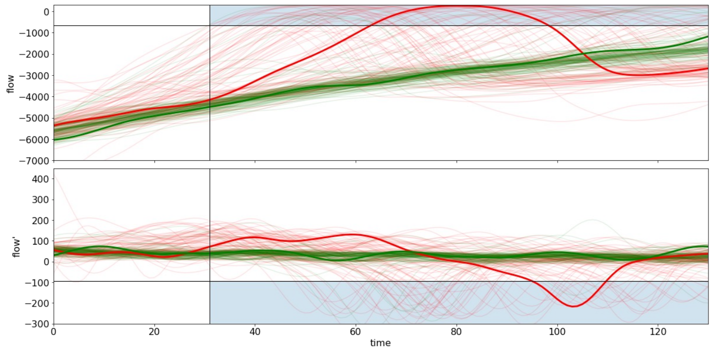
**Idea**: for a fixed template formula, learn optimally separating parameters by **Bayesian Optimisation**.

$$\varphi = ((x_2 > 22.46)\,\mathcal{U}_{[49,287]}\,(x_1 \leq 31.65))$$



Maritime surveillance

$$\phi = \mathcal{F}_{[31,130]}((flow \geq -670) \vee (flow' \leq -94))$$



Light entrainment of biological oscillator

**Idea**: explore formula structure by **genetic programming** on syntactic trees

# Problem Formulation

A supervised two-class classification problem

Given a training set of $D_p$(good) and $D_n$(bad) find the best $\phi$ that better separates the two sets.

**Discrimination Function**

$$G(\phi) = \frac{\mathbb{E}(R_\phi|\vec{X}_p) - \mathbb{E}(R_\phi|\vec{X}_n)}{\sigma(R_\phi|\vec{X}_p) + \sigma(R_\phi|\vec{X}_n)}.$$

**Observation**: only statistical and noisy evaluations of $G(\phi)$

**Goal**: maximize $G(\phi)$

# ROGE – RObustness GEnetic Algorithm

It is a bi-level optimization algorithm. A GEnetic algorithm to learn the structure and a Bayesian optimization algorithm to learn the parameters.

**Require:** $\mathcal{D}_p, \mathcal{D}_n, \mathbb{K}, Ne, Ng, \alpha, s$

$gen \leftarrow \textsc{GenerateInitialFormulae}(Ne, s)$

$gen_\Theta \leftarrow \textsc{LearningParameters}(gen, G, \mathbb{K})$

**for** $i = 1 \dots Ng$ **do**

  $subg_\Theta \leftarrow \textsc{Sample}(gen_\Theta, F)$

  $newg \leftarrow \textsc{Evolve}(subg_\Theta, \alpha)$

  $newg_\Theta \leftarrow \textsc{LearningParameters}(newg, G, \mathbb{K})$

  $gen_\Theta \leftarrow \textsc{Sample}(newg_\Theta \cup gen_\Theta, F)$

**end for**

**return** $gen_\Theta$

$$\phi_{best} = \mathrm{argmax}_{\phi_\theta \in gen_\Theta}(G(\phi_\theta))$$

# Learning the Parameters

Problem

Given a PSTL formula $\phi$, a parameter space $\mathbb{K}$, find $\Theta^*$ that maximises the discrimination function $G(\phi_\Theta)$.

Methodology

1. Sample $\{(\theta_{(i)}, y_{(i)}), i = 1,\ldots,n\}$
2. Emulate (**GP Regression**): $G[R_\phi] \sim GP(\mu,k)$
3. Optimize the emulation via **GP-UCB algorithm**, new $\theta_{(n+1)}$

$\exists\delta$ s.t. $\mathbb{E}(R_{\phi_{\Theta^*}}|\vec{X}_p) > \delta$ and $\mathbb{E}(R_{\phi_{\Theta^*}}|\vec{X}_n) \leq \delta$

**Translation**. $(\vec{x} - \delta) \Rightarrow \mathbb{E}(R_{\phi_{\Theta^*}}|\vec{X}_p) > 0$ and $\mathbb{E}(R_{\phi_{\Theta^*}}|\vec{X}_n) \leq 0$

# Learning the Structure

Problem

Given a set of PSTL formulas *gen*, find the best $\phi$ such that $\phi_\Theta$ maximises the discrimination function $G(\phi_\Theta)$.

Methodology

1. GenerateInitialFormulae: $gen=\{\phi_1,\cdots,\phi_{N_e}\}$
2. Sample$(gen_\Theta,F)=subg_\Theta$, $N_e/2$ formulae, $F(\phi)=G(\phi)-S(\phi)$
3. Evolve$(subg_\Theta, \alpha) = newg_\Theta$, based on two genetic operators, a **recombination** and a **mutation** operator.

Regularization

Formula size penalty $S(\phi)$ and complexity of initial population.

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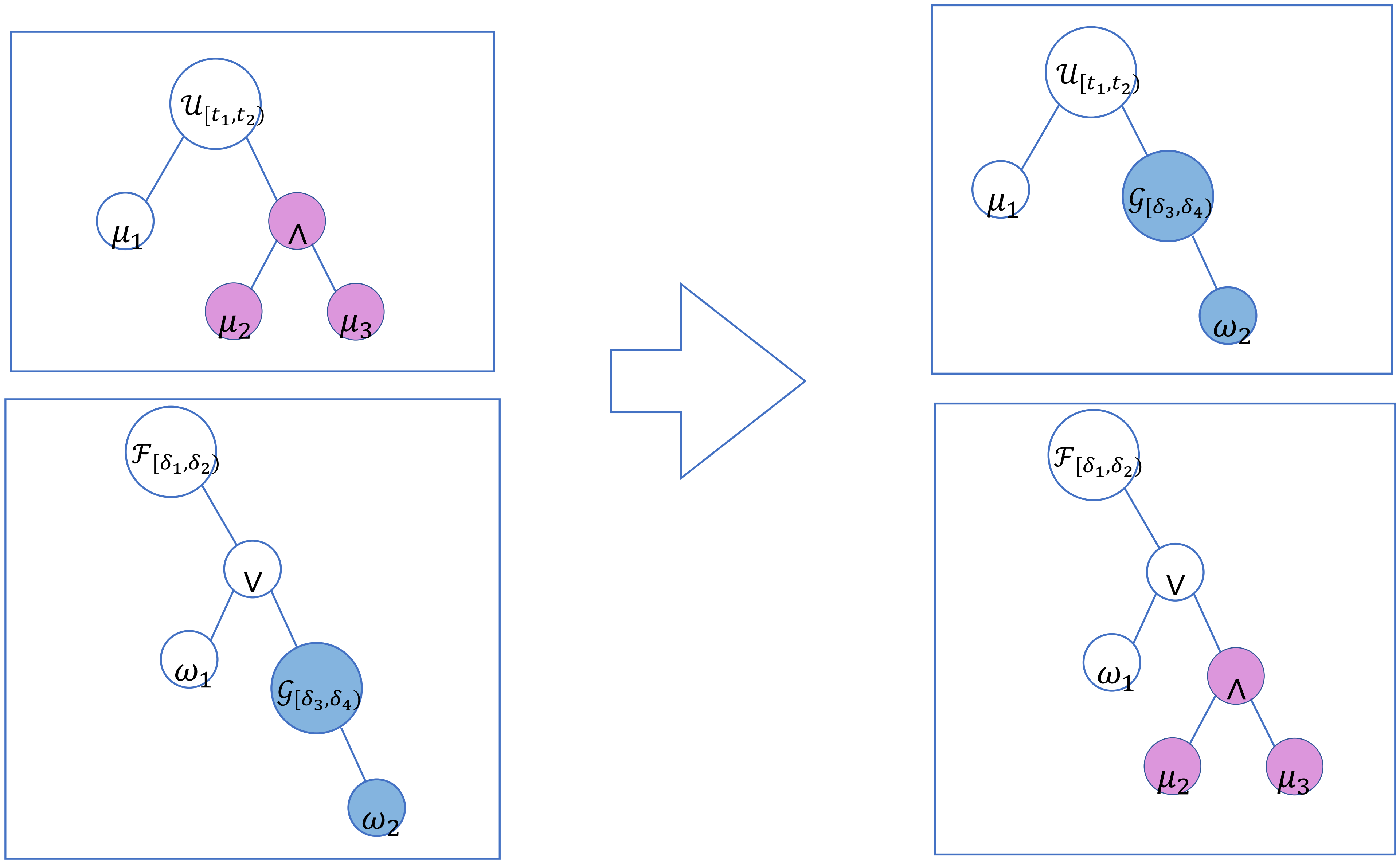
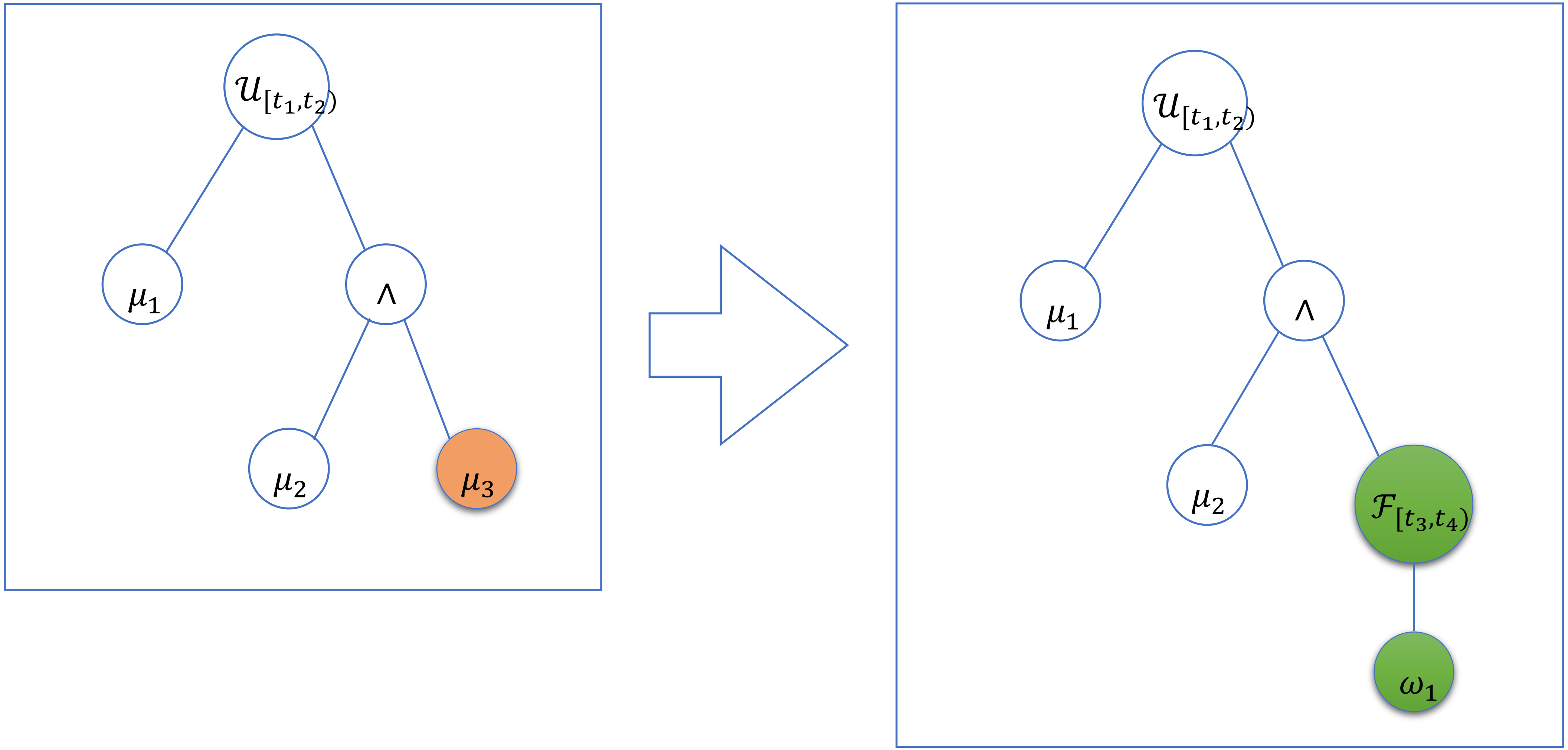
# Recombination operator

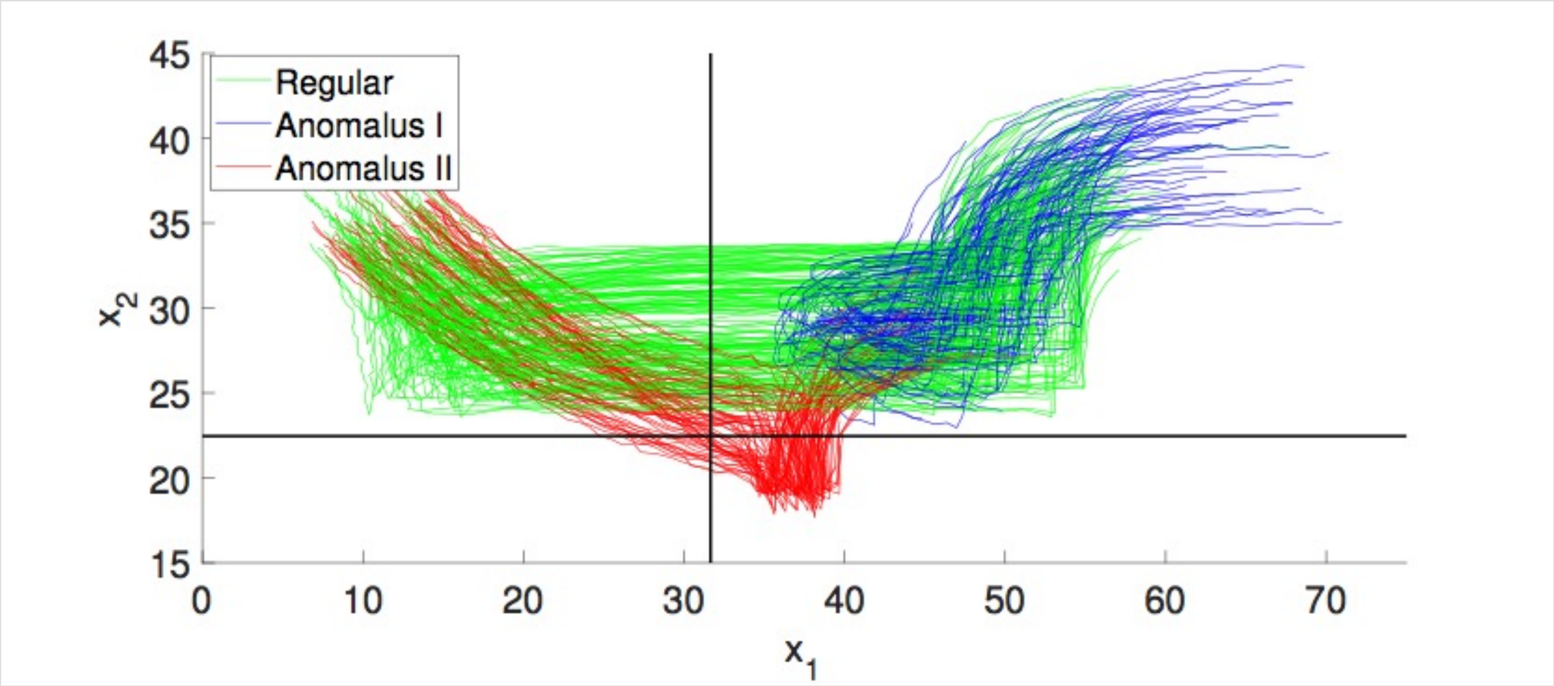


# Mutation operator



# Maritime Surveillance

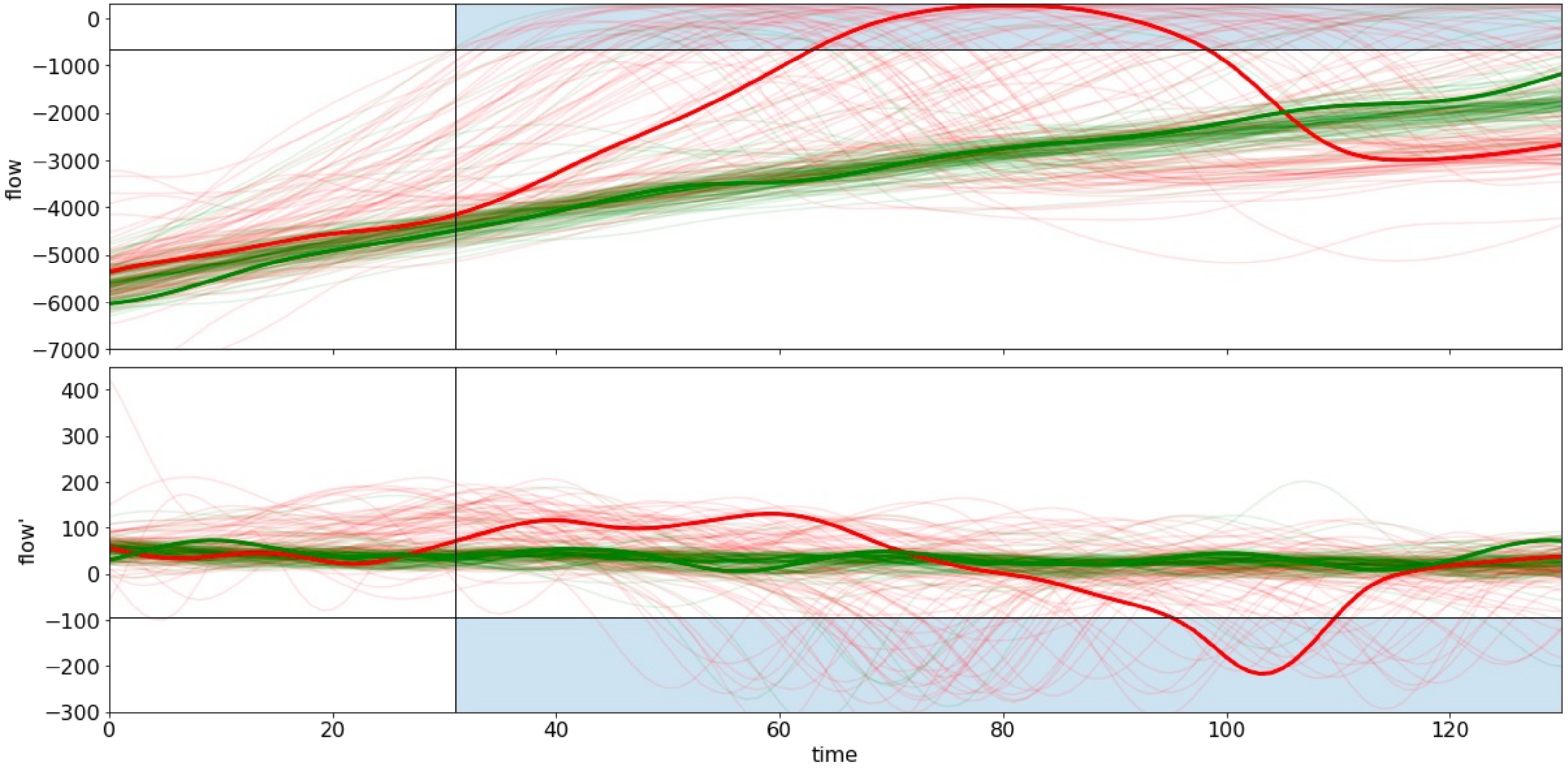
Synthetic dataset of naval surveillance of 2-dimensional coordinates traces of vessels behaviours.




$$\phi_{ROGE} = ((x_2 > 22.46)\,\mathcal{U}_{[49,287]}\,(x_1 \leq 31.65))$$

# Ineffective Inspiratory Effort (IIE)

The dataset consists of 2-dim traces of flow and its derivative, flow’.




$$\phi = \mathcal{F}_{[31,130]}((flow \geq -670) \vee (flow' \leq -94))$$

# Bibliography

**Mining Requirements:**


- Ezio Bartocci, Luca Bortolussi, Laura Nenzi, Guido Sanguinetti, System design of stochastic models using robustness of temporal properties. Theor. Comput. Sci. 587: 3-25 (2015)
- Jyo Deshmukh et al. Mining Requirements from Closed-loop Control Models (HSCC ’13, IEEE Trans. On Computer Aided Design ’15)
- Bartocci, E., Bortolussi, L., Sanguinetti, G.: Data-driven statistical learning of temporal logic properties, FORMATS, 2014
- Bufo, S., Bartocci, E., Sanguinetti, G., Borelli, M., Lucangelo, U., Bortolussi, L.i,Temporal logic based monitoring of assisted ventilation in intensive care patients, ISoLA, 2014.
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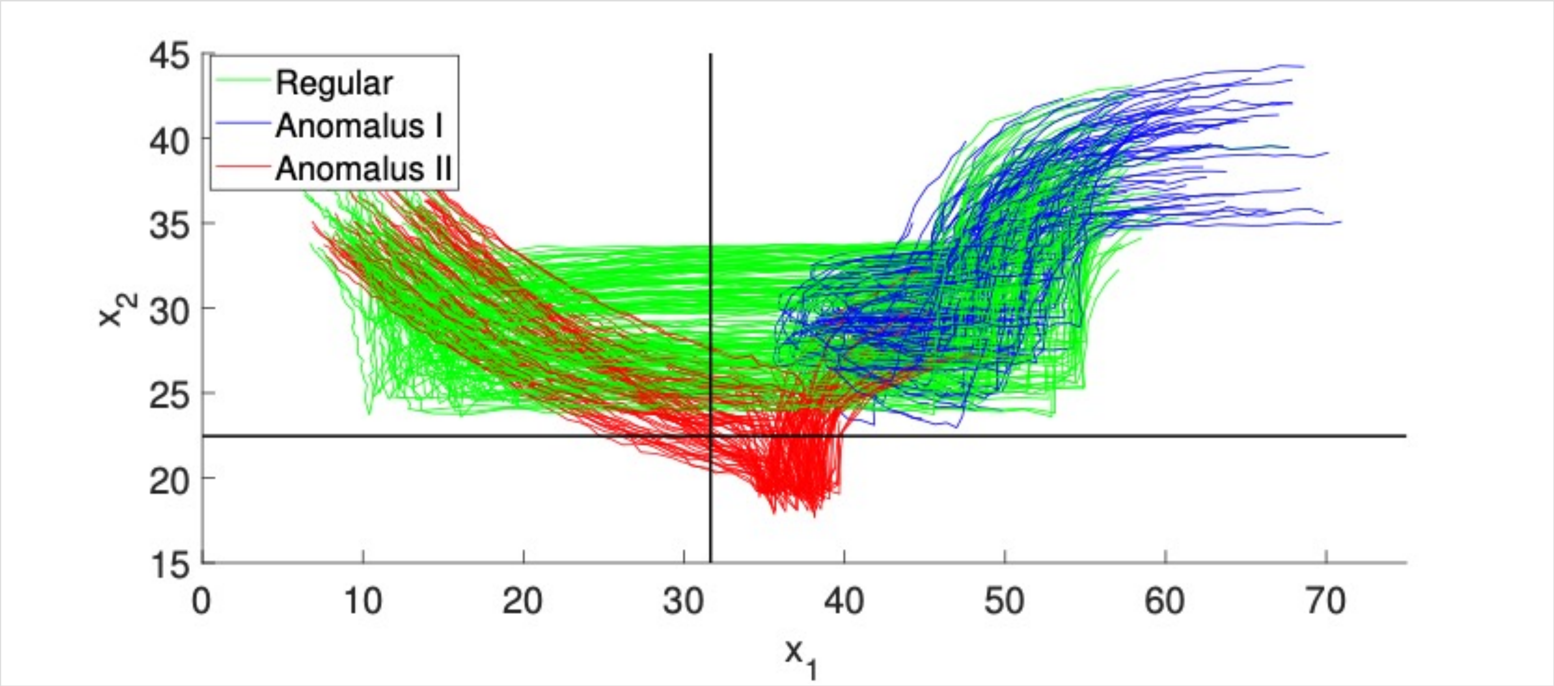
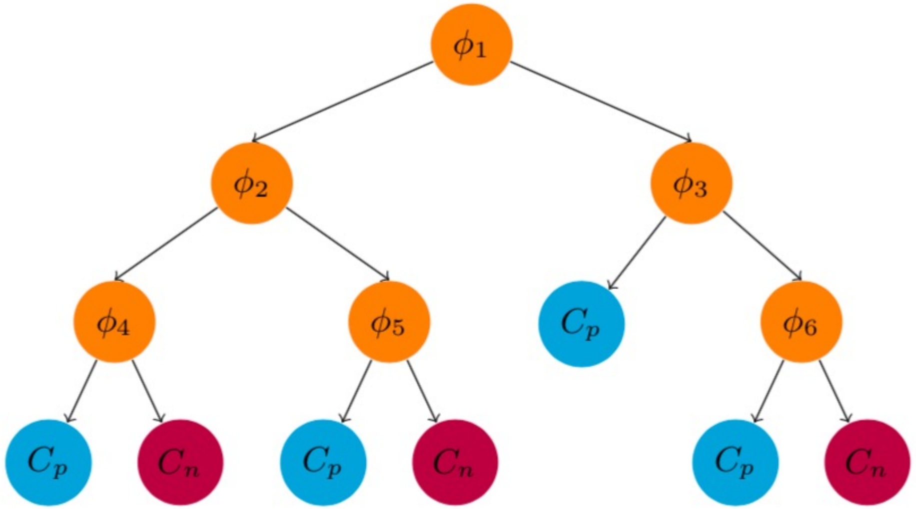



$$\phi_{ROGE} = ((x_2 > 22.46)\,\mathcal{U}_{[49,287]}\,(x_1 \leq 31.65))$$

# A Decision Tree Approach to Data Classification

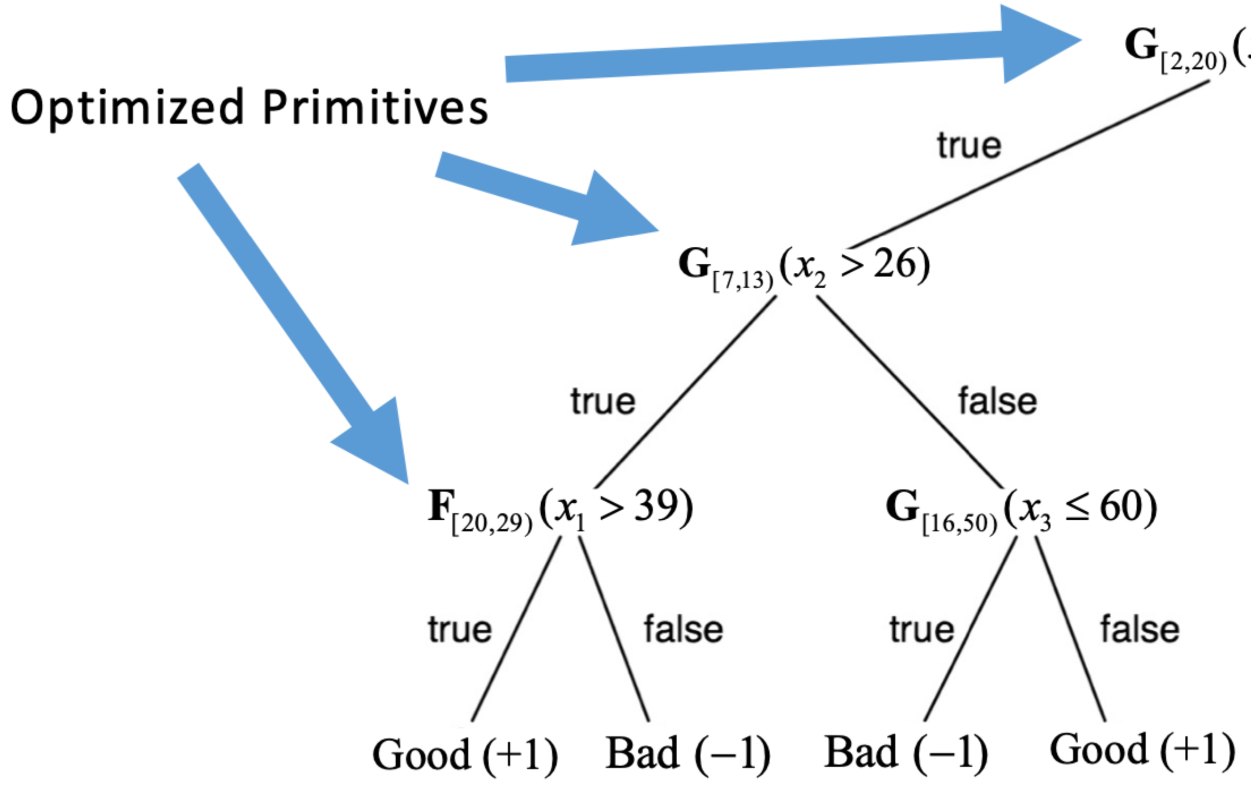
DTL4STL [2], that uses a decision tree algorithm for the structure of the formula and an heuristic impurity measure for parameter synthesis




$$\phi_{Tree} = (\phi_1 \wedge ((\phi_2 \wedge \phi_4) \vee (\neg\phi_2 \wedge \phi_5))) \vee (\neg\phi_1 \wedge (\phi_3 \vee (\neg\phi_3 \wedge \phi_6)))$$


[2] Bombara, G et all, A Decision Tree Approach to Data Classification Using Signal Temporal Logic. In: Proc. of HSCC, 2016.

# A Decision Tree Approach to Data Classification




**Construction Process**

$S$ = set of traces reaching this node

$\mathrm{P}$ = set of primitives

$IR$ = Impurity Reduction Measure

$$\phi^* = \arg\max_{\psi \in \mathrm{P},\, \theta \in \Theta} IR(S, \psi(\theta))$$

[2] Bombara, G et all, A Decision Tree Approach to Data Classification Using Signal Temporal Logic. In: Proc. of HSCC, 2016.

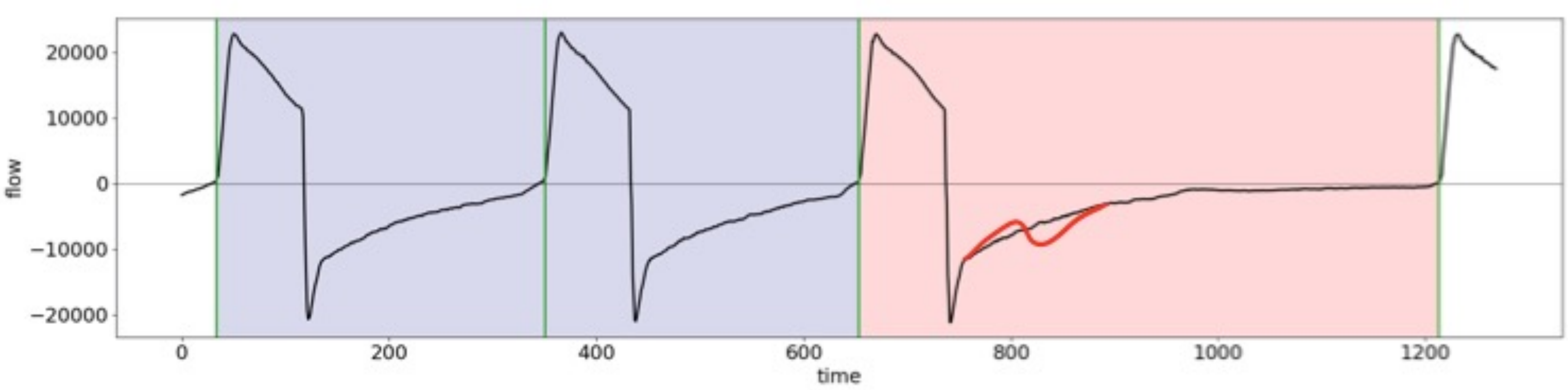
# Maritime Surveillance

$$\phi_{ROGE} = ((x_2 > 22.46)\,\mathcal{U}_{[49,287]}\,(x_1 \leq 31.65))$$

$$\psi_{DTL4STL} = (((\mathcal{G}_{[187,196)}x_1 < 19.8) \wedge (\mathcal{F}_{[55.3,298)}x_1 > 40.8)) \vee ((\mathcal{F}_{[187,196)}x_1 > 19.8) \wedge ((\mathcal{G}_{[94.9,296)}x_2 < 32.2) \vee ((\mathcal{F}_{[94.9,296)}x_2 > 32.2) \wedge (((\mathcal{G}_{[50.2,274)}x_2 > 29.6) \wedge (\mathcal{G}_{[125,222)}x_1 < 47)) \vee ((\mathcal{F}_{[50.2,274)}x_2 < 29.6) \wedge (\mathcal{G}_{[206,233)}x_1 < 16.7))))))$$

| | ROGE | DTL4STL | DTL4STL$_p$ |
|---|---|---|---|
| Mis. Rate | 0 | $0.01 \pm 0.013$ | $0.007 \pm 0.008$ |
| Comp. Time (sec.) | $73 \pm 18$ | $144 \pm 24$ | - |

# Ineffective Inspiratory Effort (IIE)




- Regular breaths
- Irregular breaths (IIE)

# Ineffective Inspiratory Effort (IIE)

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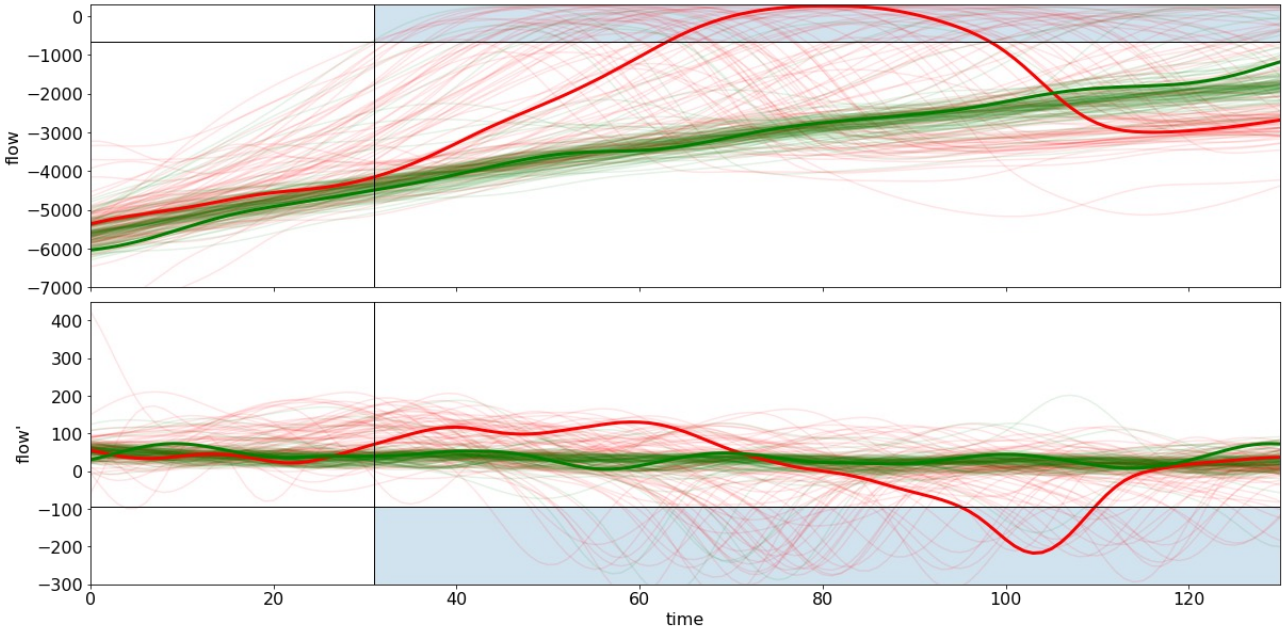


| | ROGE | DTL4STL |
|---|---|---|
| Mis. Rate | $0.17 \pm 0.01$ | $0.23 \pm 0.07$ |
| False. Pos | $0.20 \pm 0.02$ | $0.23 \pm 0.07$ |
| False. Neg | $0.14 \pm 0.02$ | $0.20 \pm 0.15$ |
| Comp. Time (sec.) | $65 \pm 7$ | $201 \pm 7$ |

$$\phi = \mathcal{F}_{[31,130]}((flow \geq -670) \vee (flow' \leq -94))$$

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