Cyber-Physical Systems

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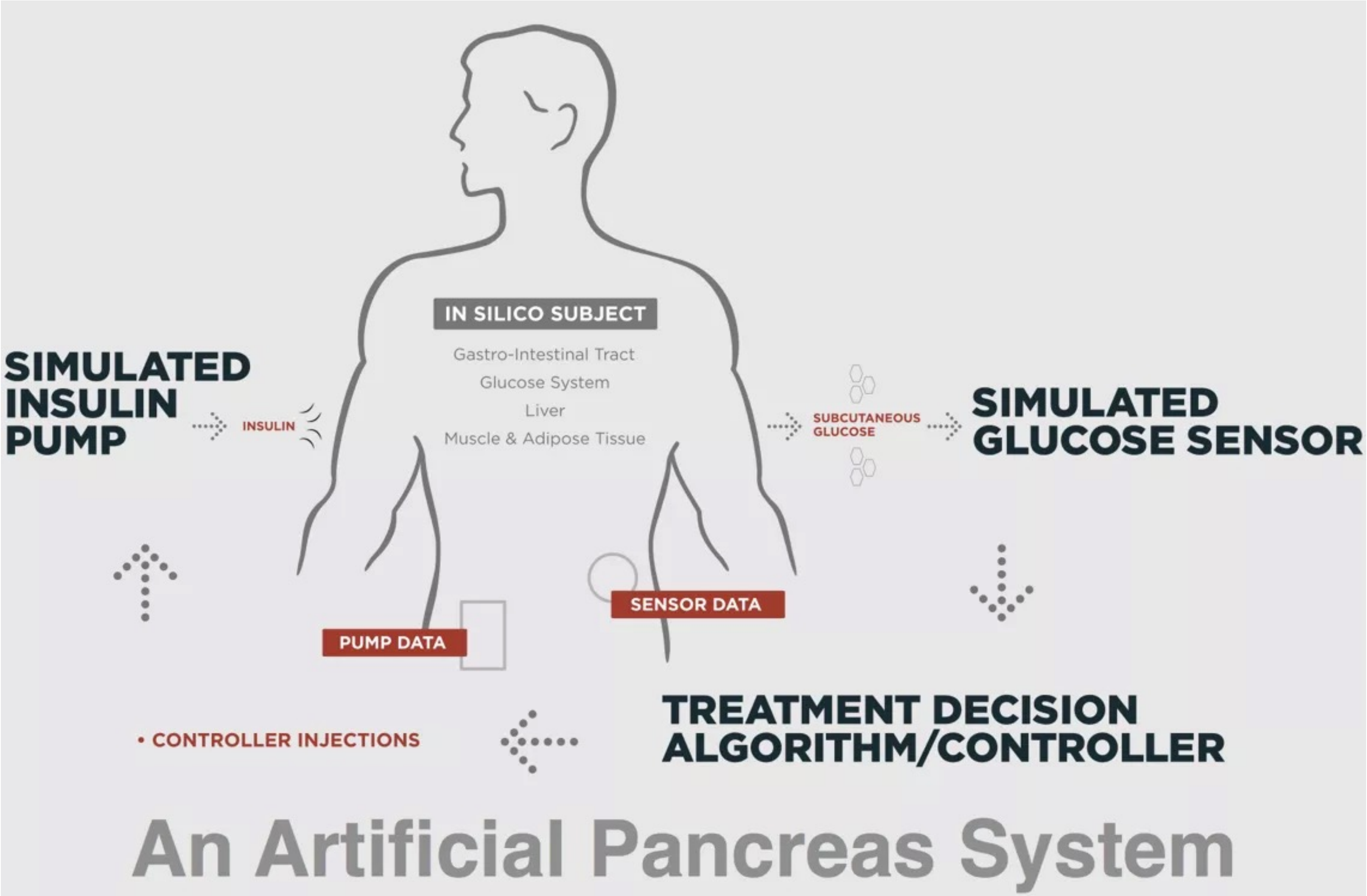
Lecture: Examples

Artificial Pancreas

Type 1 diabetes occurs when the pancreas produces little or none of the insulin needed to regulate blood glucose

They rely on external ad- ministration of insulin to manage their blood glucose levels.

Artificial Pancreas



Stochastic Hybrid Systems Of Glucose

$$\frac{d}{dt}\boldsymbol{x}(t) = F(\boldsymbol{x}(t);\ u(t);\Theta); \qquad y(t) = x_1(t)$$

glucose concentration

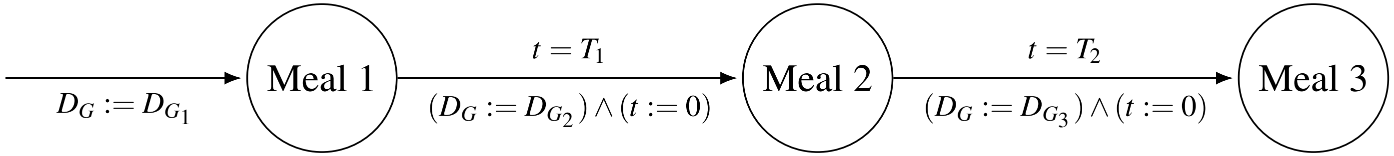
the control parameters

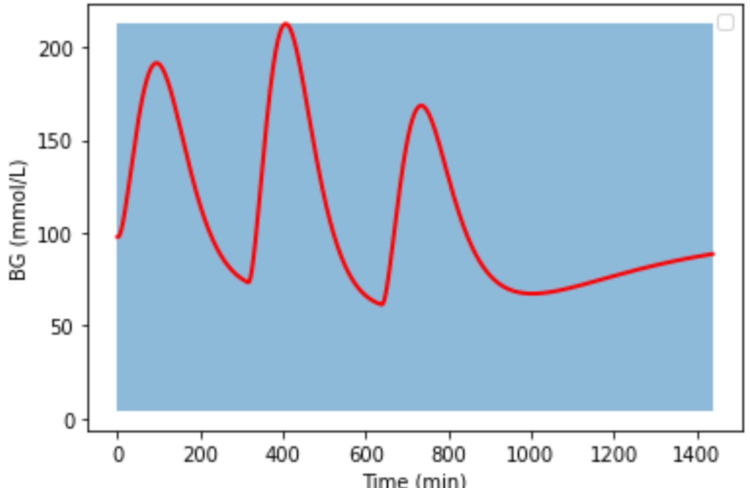
Infusion rate of bolus insulin

$\Theta = (D_{G_1}; D_{G_2}; D_{G_3};\ T_1;\ T_2)$ are the control parameter

$(D_{G_1}; D_{G_2}; D_{G_3}) \in (N(40;\ 10); N(90;\ 10); N(60;\ 10))$ are the three daily meals

$(T_1;\ T_2) \in \sim N$ (300, 10) and $T2 \sim N$ (300, 10) are the inter-times between each of them





Stochastic Hybrid Systems Of Glucose

$$\frac{d}{dt}Q_1(t) = -F_{01} - x_1Q_1 + k_{12}Q_2 - F_R + EGP_0(1-x_3) + \frac{D_G A_G}{t_{maxG}^2} t e^{-\frac{t}{t_{maxG}}}$$

$$\frac{d}{dt}Q_2(t) = x_1Q_1 - (k_{12} + x_2)Q_2;$$

$$\frac{d}{dt}S_1(t) = u(t) + u_b - \frac{S_1}{t_{maxI}};$$

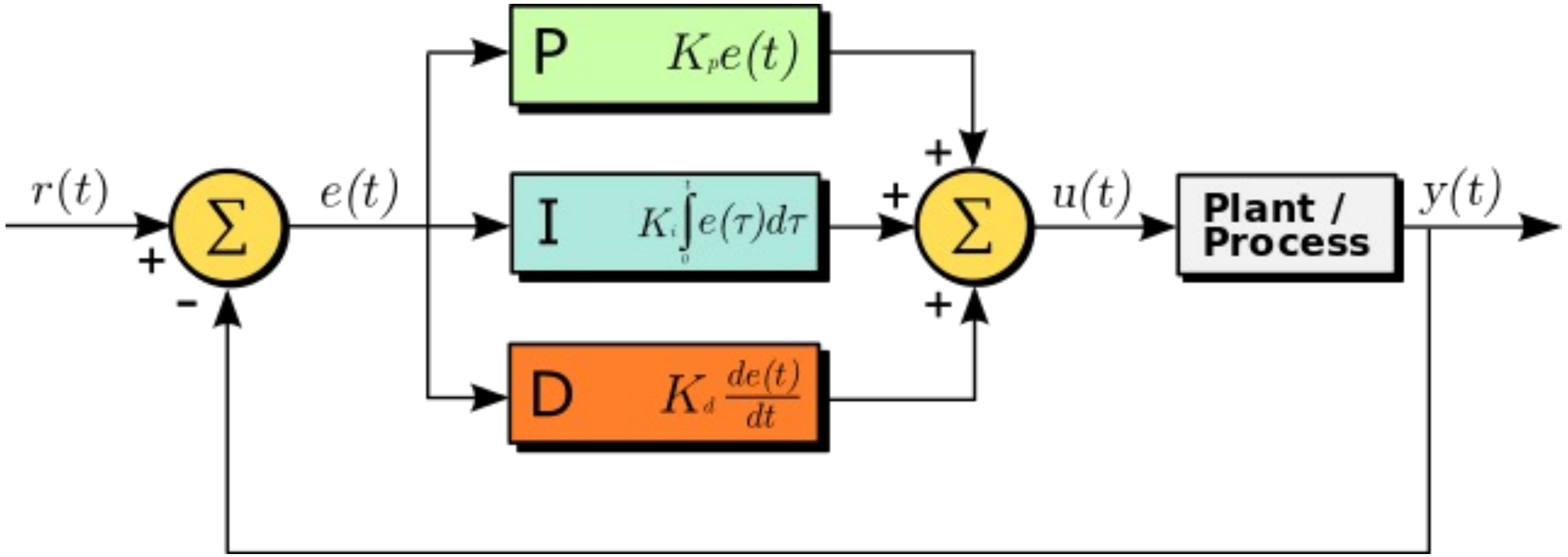
$$\frac{d}{dt}S_2(t) = S_1 - \frac{S_2}{t_{maxI}};$$

$$\frac{d}{dt}I(t) = \frac{S_2}{t_{maxI}V_I} - keI;$$

$$\frac{d}{dt}x_i(t) = -k_{a_i}x_i + k_{b_i}I; \qquad (i = 1,2,3)$$

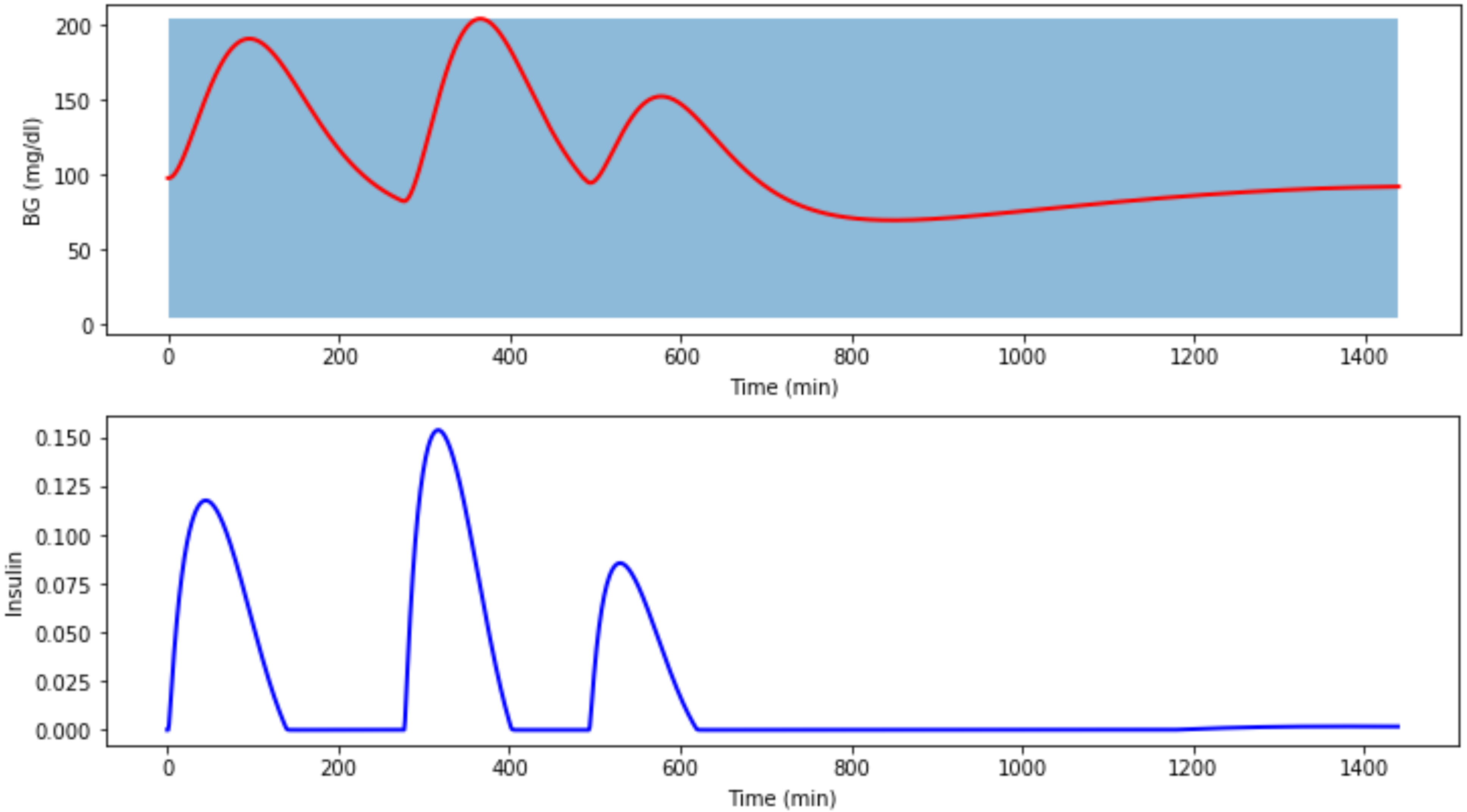
$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{d}{dt}e(t), \qquad e(t) = r(t) - y(t)$$

PID Control



$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{d}{dt} e(t), \qquad e(t) = r(t) - y(t)$$

Artificial Pancreas Simulation



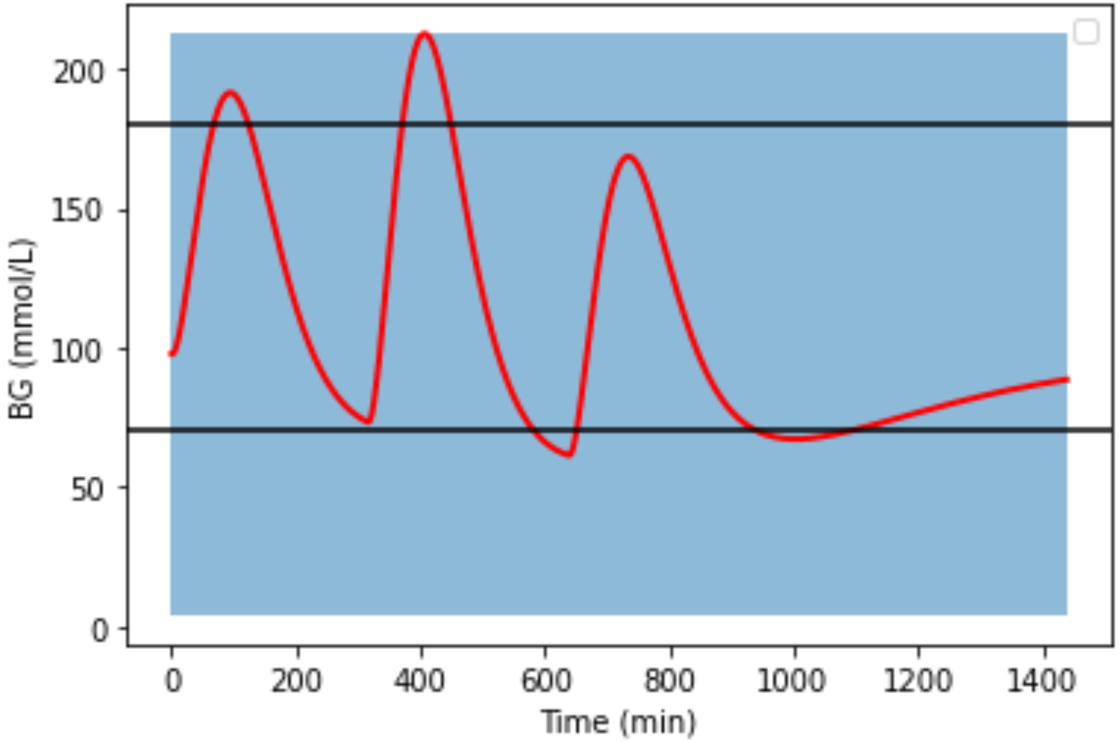
STL Properties for the Artificial Pancreas

- Hyperglycemia
  - "during the day the level of glucose goes above 180mg/dl"

$$\neg G_{[0,24h]}(BG(t) < 180)$$

- Hypoglycemia
  - "during the day the level of glucose goes below 70mg/dl"

$$\neg G_{[0,24h]}(BG(t) > 70)$$



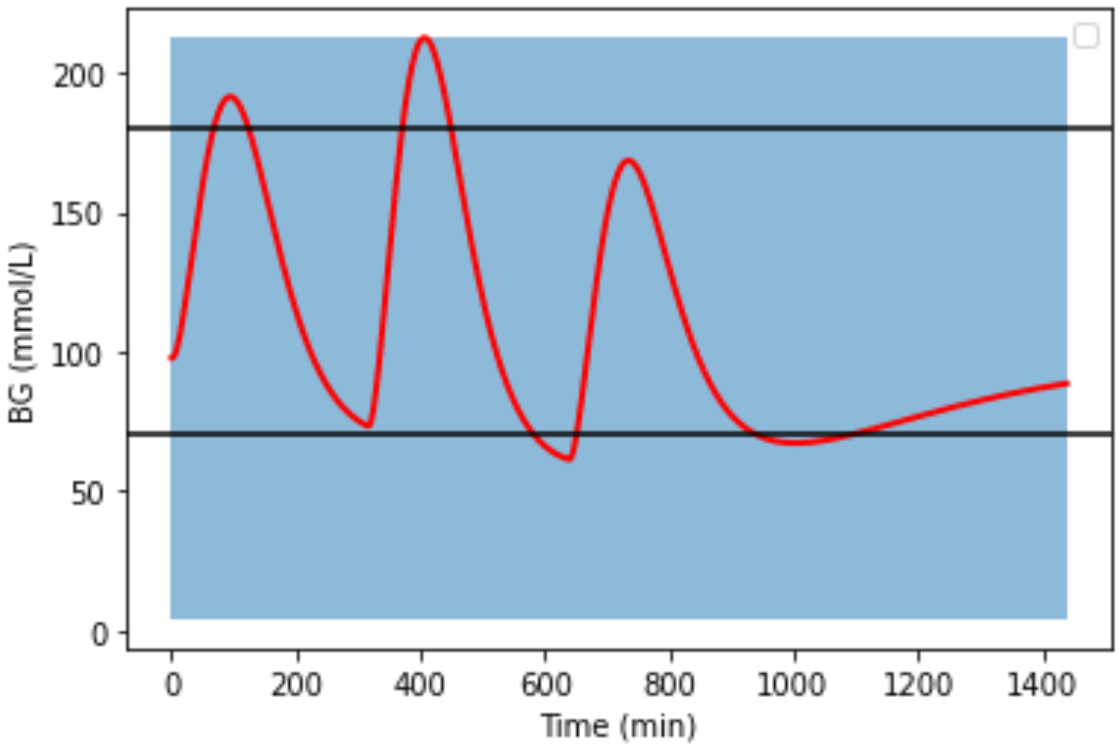
STL Properties for the Artificial Pancreas

- Prolonged Hyperglycemia
  - “during the day the level of glucose goes above 180mg/dl for 3 hours”

$$F_{[0,21h]}(G_{[0,3]}(BG(t) \geq 180))$$

- Prolonged Hypoglycemia
  - “during the day the level of glucose goes below 70mg/dl for 30 minutes”

$$F_{[0,21h]}(G_{[0,0.5]}(BG(t) < 70)$$



Falsification

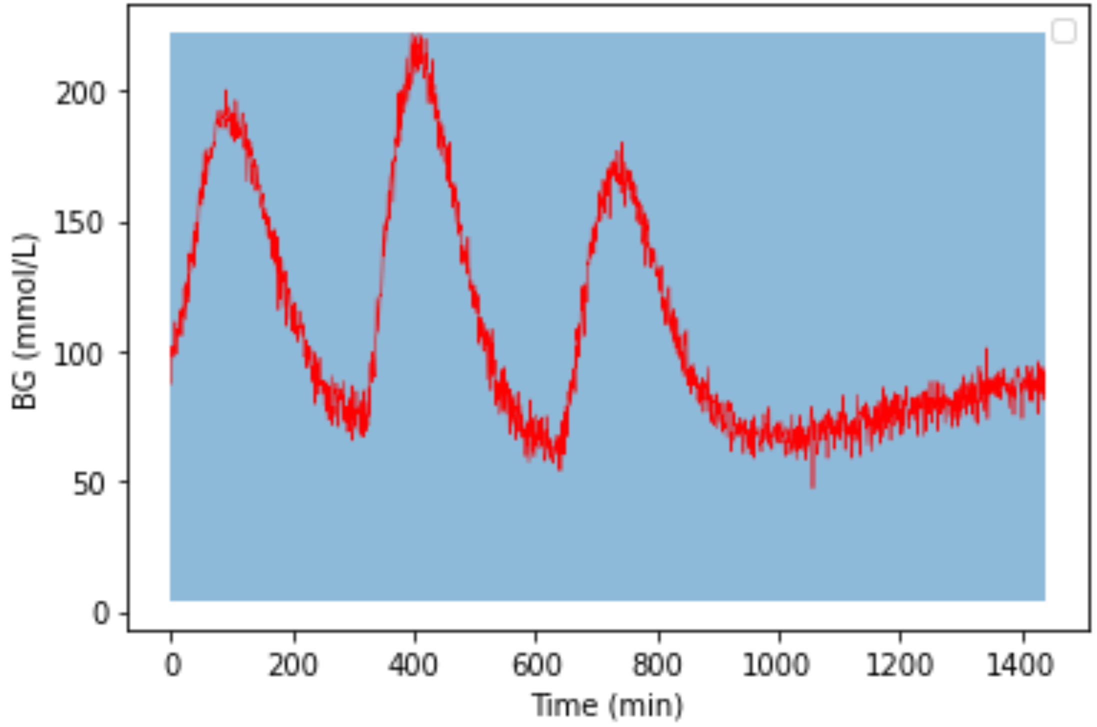
The most simple way to do falsification with respect a property ϕ is minimizing the robustness over N iterations considering random samples on control parameters, i.e:

```
minSTL = 'inf'
For i = 1,..,N:
    Θ  = sampling (D_G1, D_G2, D_G3, T1, T2)
    t ,y =simulation(Θ)
    stl= computeRobustness(y, ϕ)
    if (stl < minSTL):
        minSTL = stl
        vSTL = [D_G1, D_G2, D_G3, T1, T2]
```

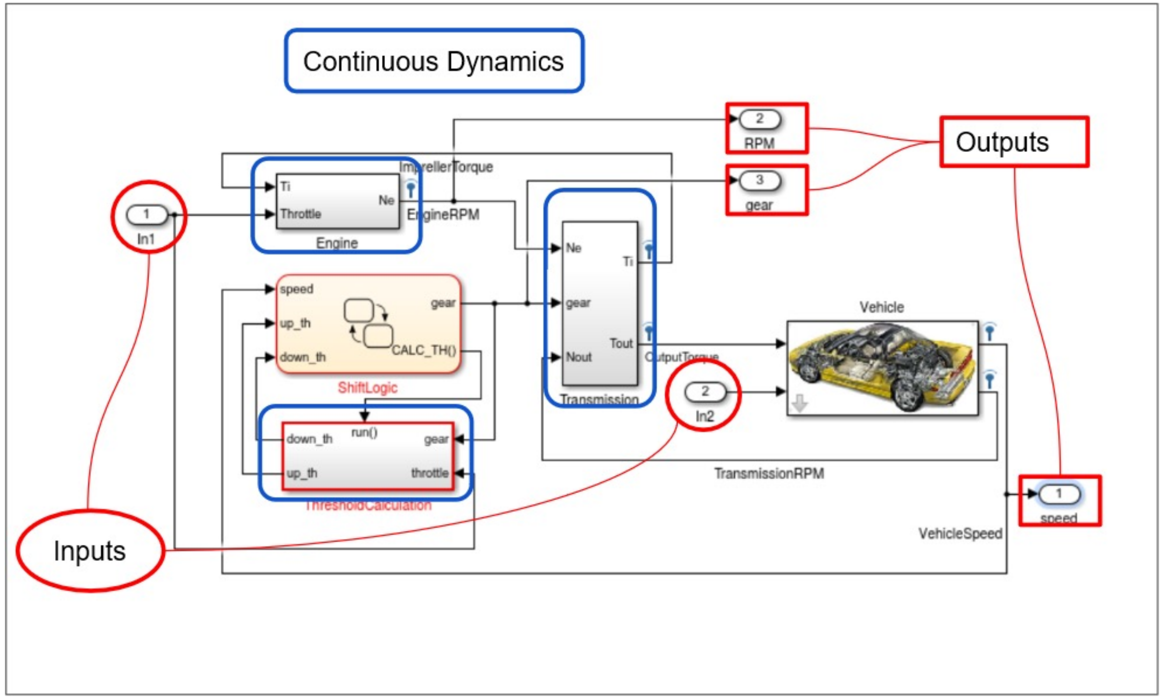
For fixed control parameter spaces you can consider to sample with respect to the grids over it.

Noise Robustness

- To consider noisy sensor we can add a Gaussian noise to the generated glucose trajectory, i.e. $GB(t) + \gamma$ with $\gamma \in N(0;\ 5)$



Automatic Transmission

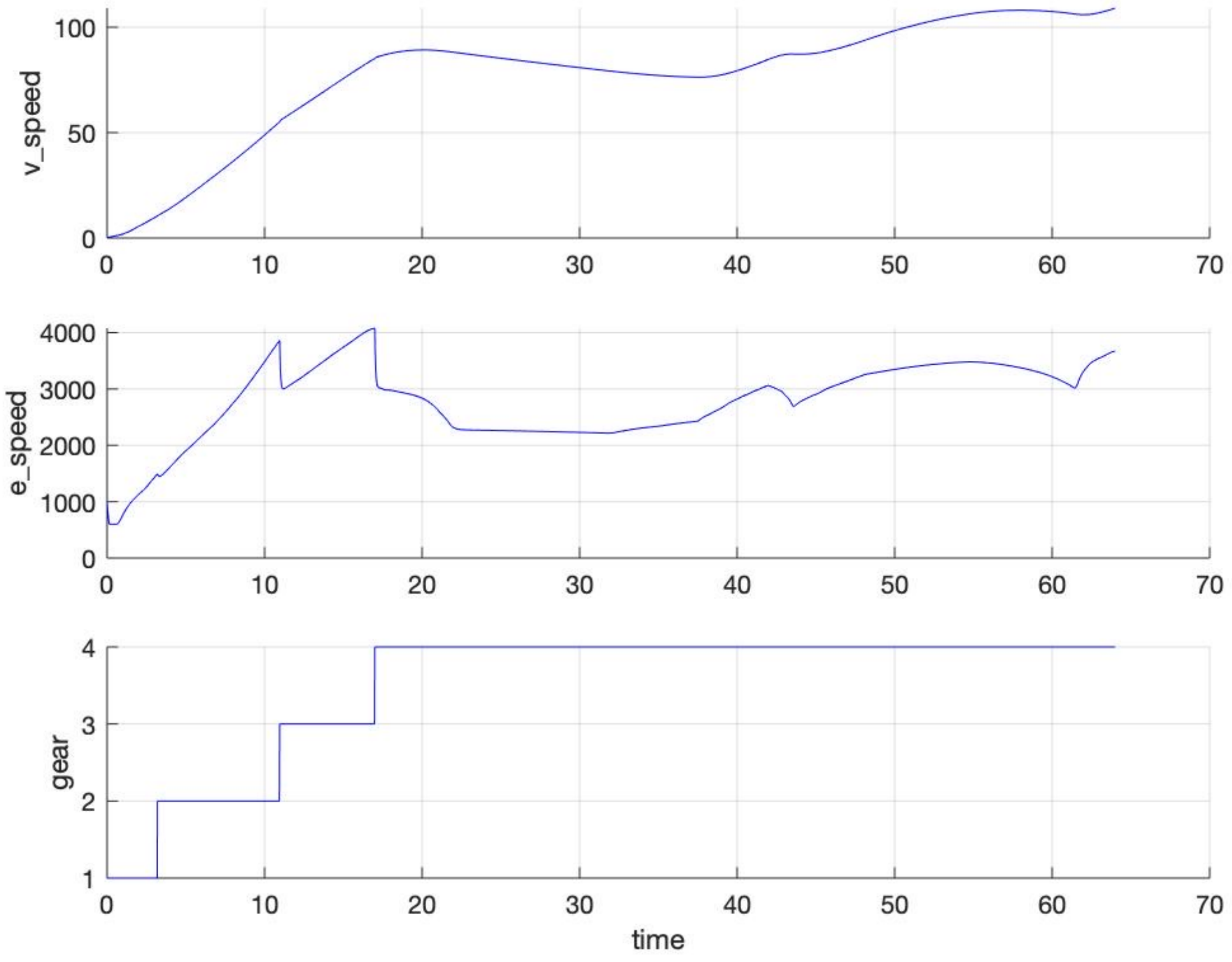


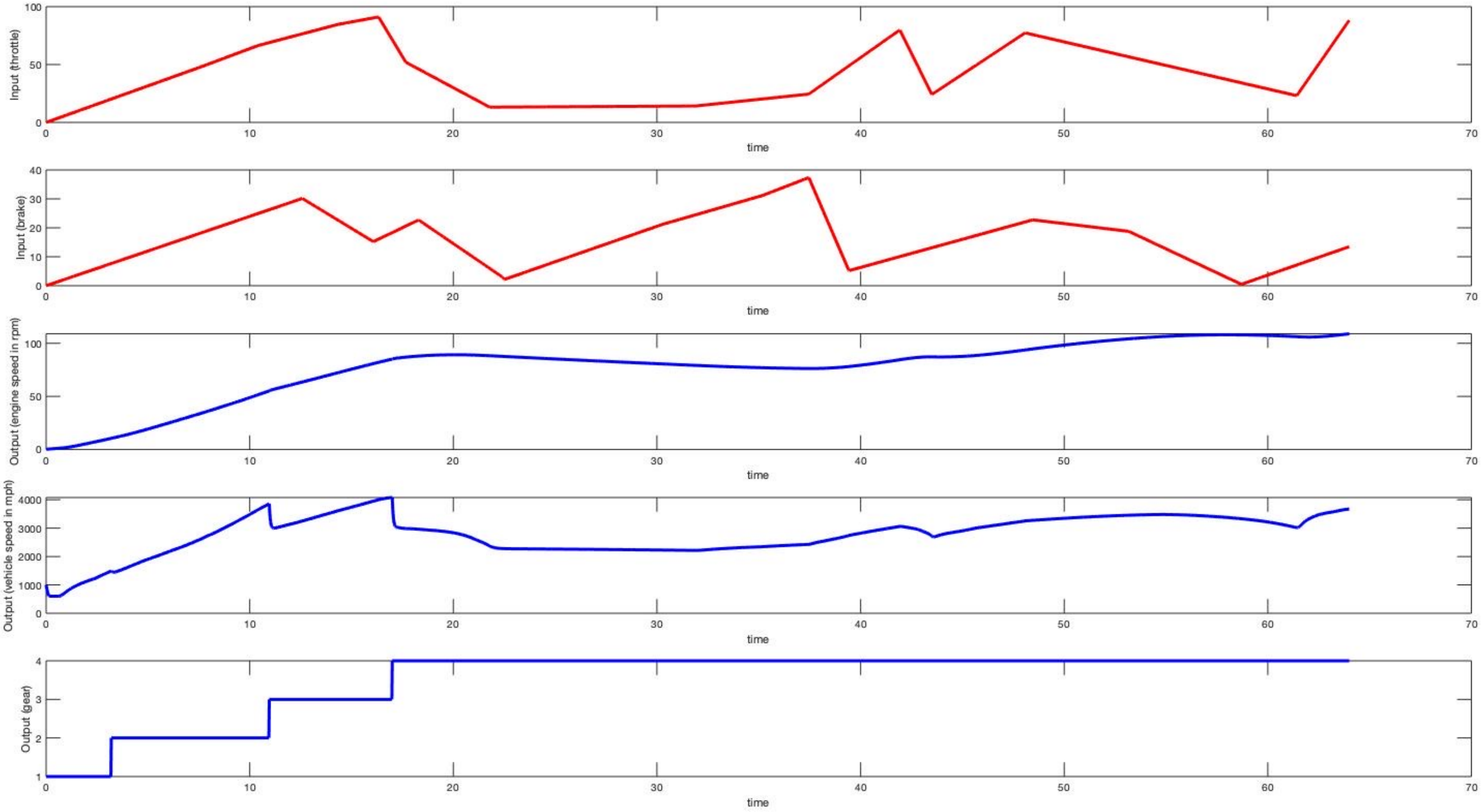
Most material that follows is from this paper:

- Bardh Hoxha, Houssam Abbas, Georgios E. Fainekos: Benchmarks for Temporal Logic Requirements for Automotive Systems. ARCH@CPSWeek 2014: 25-30

Automatic Transmission

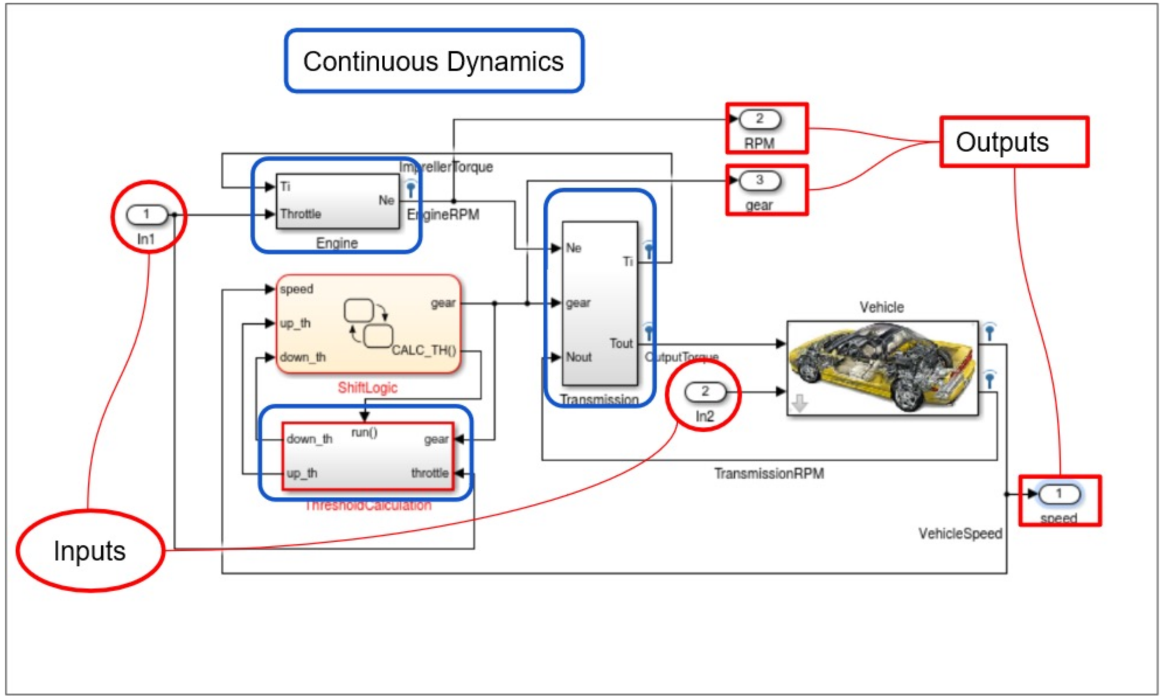
- Inputs: the throttle and break
- Outputs: the speed of the engine ω (RPM), the speed of the vehicle v (mph) and the gear.
- Initially, the vehicle is at rest at time 0, i.e. the speed v = 0 and engine speed ω = 0
- Therefore, the output trajectories depend only on the input signals ut and ub which model the throttle and break inputs.
- The throttle and break, at each point in time, can take any value between 0 (fully closed) to 100 (fully open).

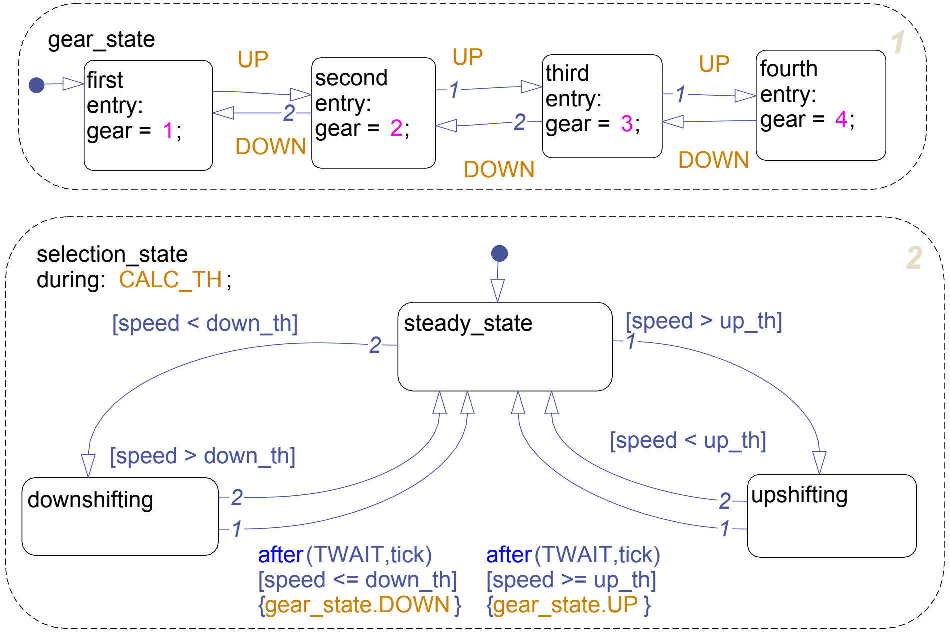



Input (throttle)
Input (brake)
Output (engine speed in rpm)
Output (vehicle speed in mph)
Output (gear)
time
0
10
20
30
40
50
60
70
0
50
100
0
10
20
30
40
0
50
100
0
1000
2000
3000
4000
1
2
3
4

Automatic Transmission

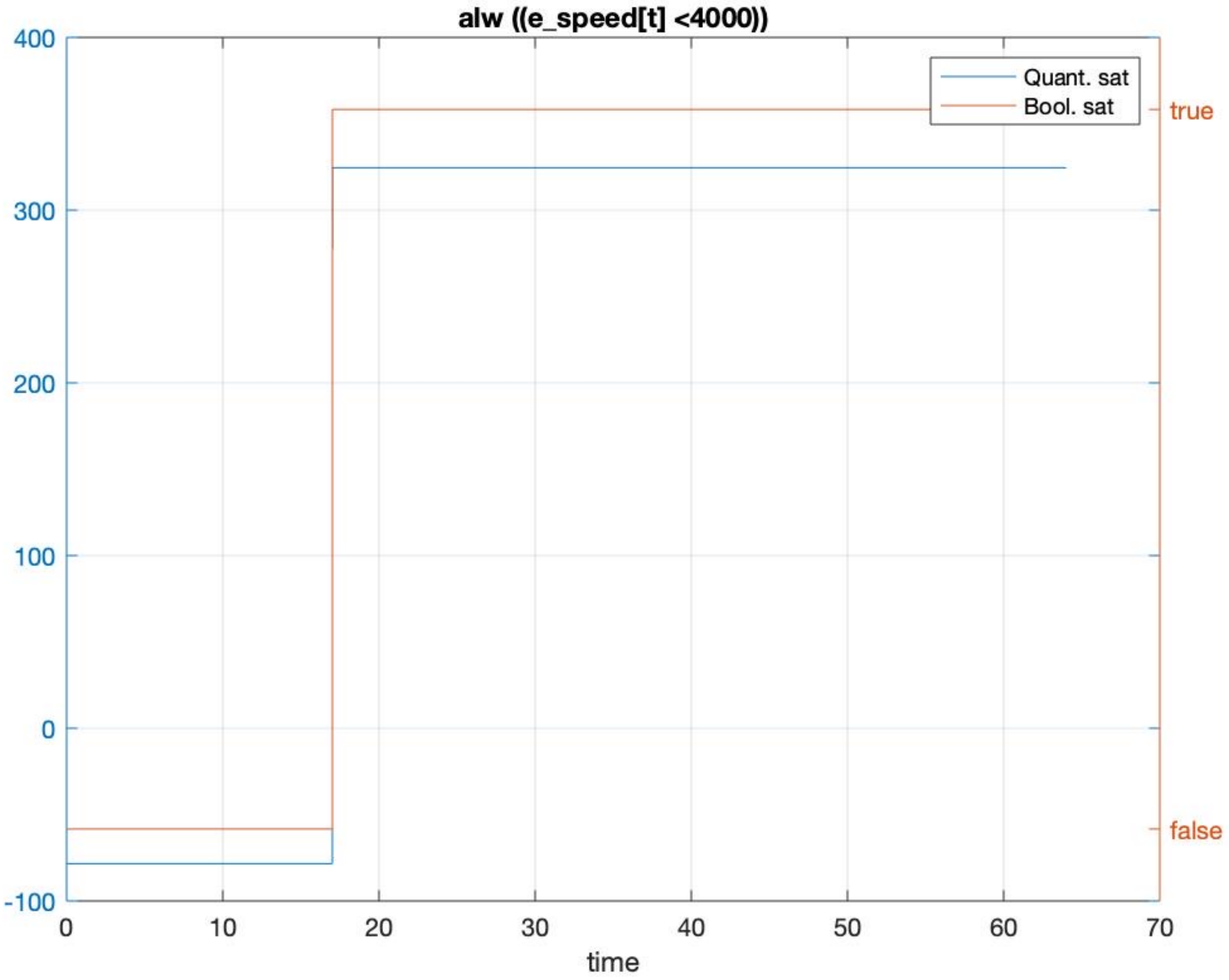
- The model contains 69 blocks among which there are 2 integrators (i.e., 2 continuous state variables), and a Stateflow chart. The Stateflow chart contains two concurrently executing Finite State Machines with 4 and 3 states, respectively.





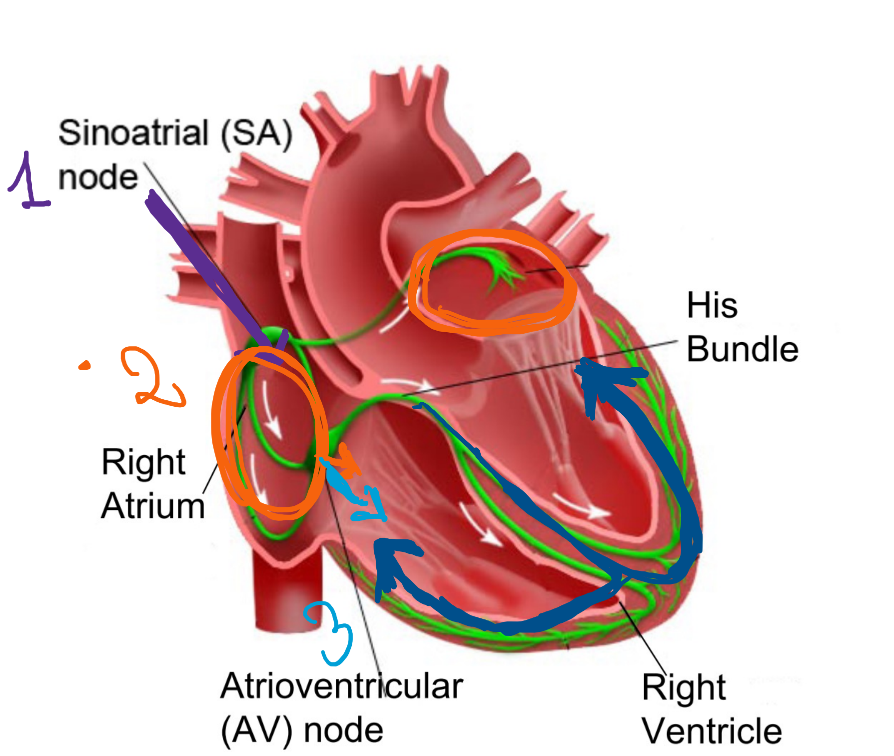
Properties

Automatic Transmission		
	Natural Language	**MTL**
ϕ_1^{AT}	The engine speed never reaches $\bar{\omega}$.	$\Box(\omega < \bar{\omega})$
ϕ_2^{AT}	The engine and the vehicle speed never reach $\bar{\omega}$ and $\bar{v}$, resp.	$\Box((\omega < \bar{\omega}) \wedge (v < \bar{v}))$
ϕ_3^{AT}	There should be no transition from gear two to gear one and back to gear two in less than 2.5 sec.	$\Box((g_2 \wedge X g_1) \rightarrow \Box_{(0,2.5]} \neg g_2)$
ϕ_4^{AT}	After shifting into gear one, there should be no shift from gear one to any other gear within 2.5 sec.	$\Box((\neg g_1 \wedge X g_1) \rightarrow \Box_{(0,2.5]} g_1)$
ϕ_5^{AT}	When shifting into any gear, there should be no shift from that gear to any other gear within 2.5sec.	$\wedge_{i=1}^{4} \Box((\neg g_i \wedge X g_i) \rightarrow \Box_{(0,2.5]} g_i)$
ϕ_6^{AT}	If engine speed is always less than $\bar{\omega}$, then vehicle speed can not exceed $\bar{v}$ in less than T sec.	$\neg(\Diamond_{[0,T]}(v > \bar{v}) \wedge \Box(\omega < \bar{\omega}))$
ϕ_7^{AT}	Within T sec the vehicle speed is above $\bar{v}$ and from that point on the engine speed is always less than $\bar{\omega}$.	$\Diamond_{[0,T]}((v \geq \bar{v}) \wedge \Box(\omega < \bar{\omega}))$
ϕ_8^{AT}	A gear increase from first to fourth in under 10secs, ending in an RPM above $\bar{\omega}$ within 2 seconds of that, should result in a vehicle speed above $\bar{v}$.	$((g_1 \,\mathcal{U}\, g_2 \,\mathcal{U}\, g_3 \,\mathcal{U}\, g_4) \wedge \Diamond_{[0,10]}(g_4 \wedge \Diamond_{[0,2]}(\omega \geq \bar{\omega}))) \rightarrow \Diamond_{[0,10]}(g_4 \rightarrow X(g_4 \,\mathcal{U}_{[0,1]}\, (v \geq \bar{v})))$



alw ((e_speed[t] <4000))
400
300
200
100
0
-100
0
10
20
30
40
50
60
70
time
Quant. sat
Bool. sat
true
false

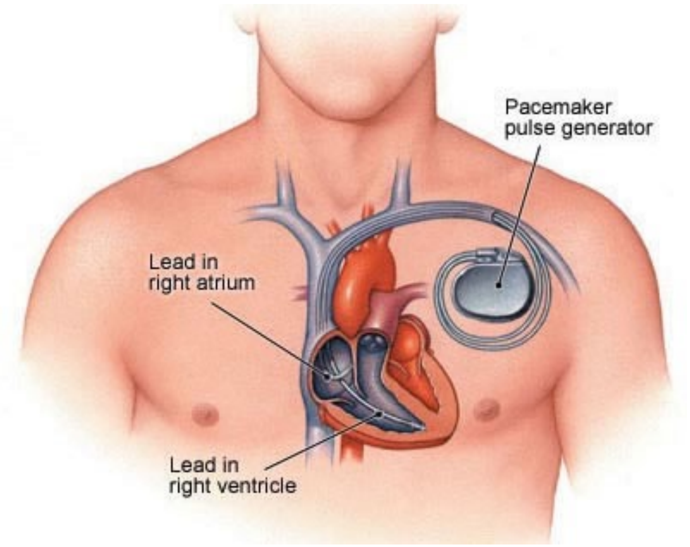
Pacemaker



Electrical Conduction System of the Heart

- 1 SA node (controlled by nervous system) periodically generates an electric pulse
- 2 This pulse causes both atria to contract pushing blood into the ventricles
- 3 Conduction is delayed at the AV node allowing ventricles to full fill
- 4 Finally the His-Purkinje system spreads electric activation through ventricles causing them both to contract, pumping blood out of the heart

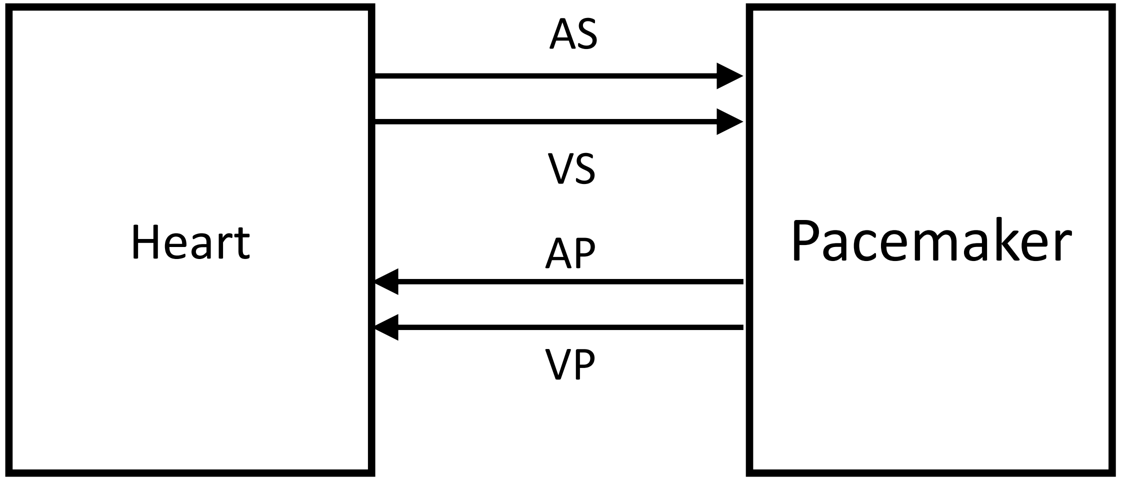
What do pacemakers do?



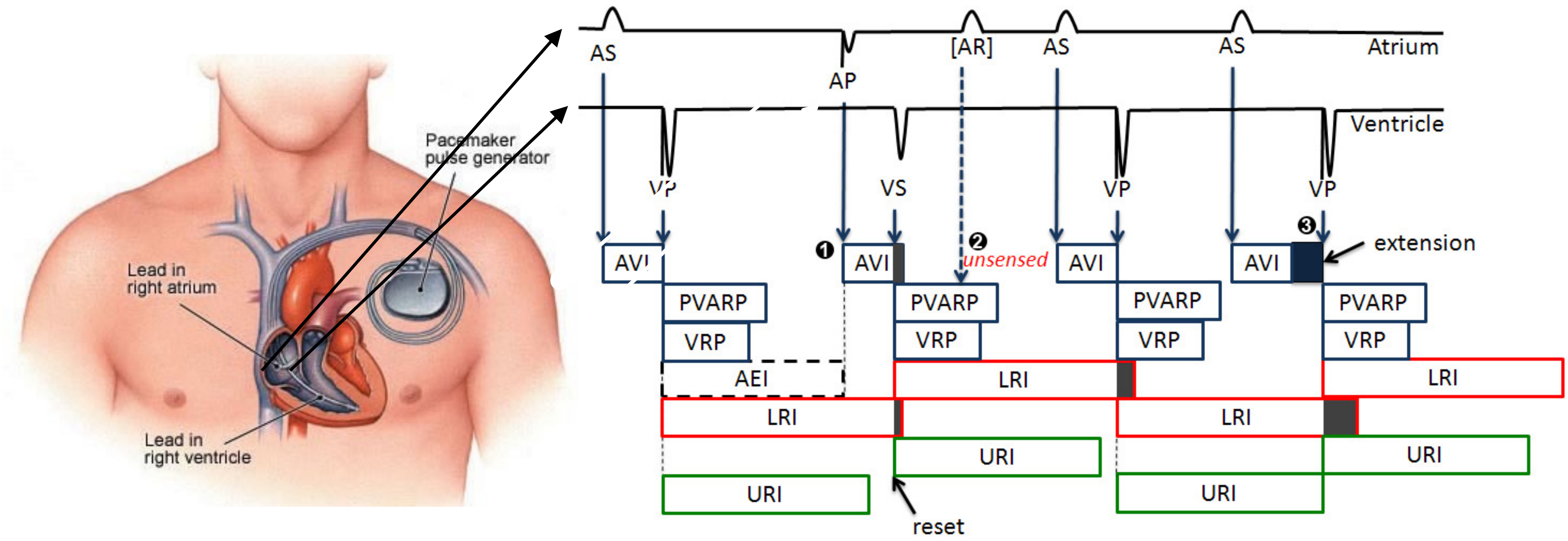
- Aging and/or diseases cause conduction properties of heart tissue to change leading to timing anomalies in heart rhythm (arrhythmias)
- Tachycardia: faster than desirable heart rate impairing hemo-dynamics (blood flow dynamics)
- Bradycardia: slower heart rate leading to insufficient blood supply
- Pacemakers can be used to treat bradycardia by providing pulses when heart rate is low

How dual-chamber pacemakers work

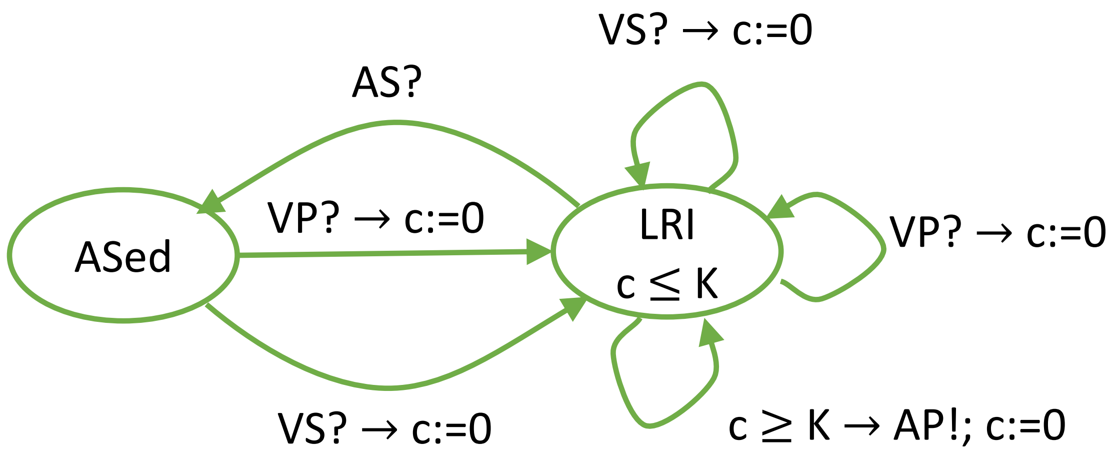
- Two fixed leads on wall of right atrium and ventricle respectively
- Activation of local tissue sensed by the leads (giving rise to events Atrial Sense (AS) and Ventricular Sense (VS))
- Atrial Pacing (AP) or Ventricular Pacing (VP) are delivered if no sensed events occur within deadlines



Implantable Pacemaker modeling



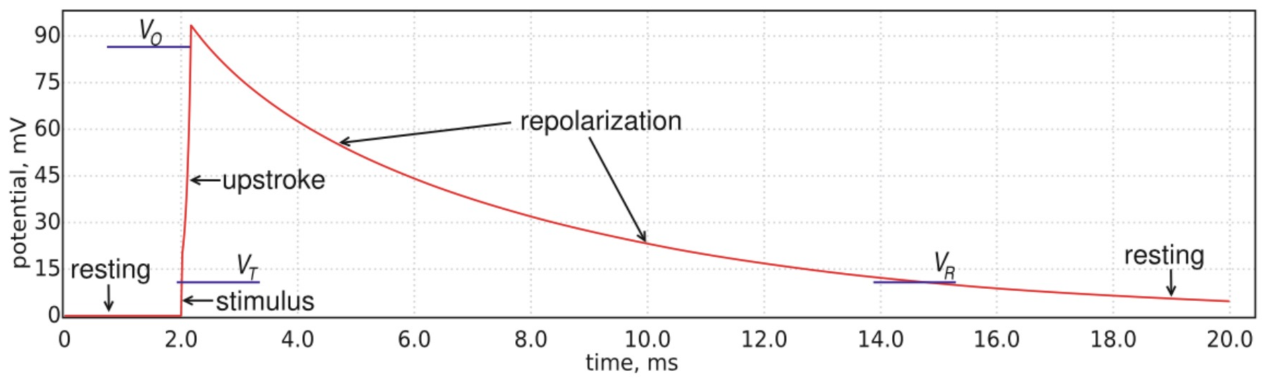
The LRI mode of operation explained

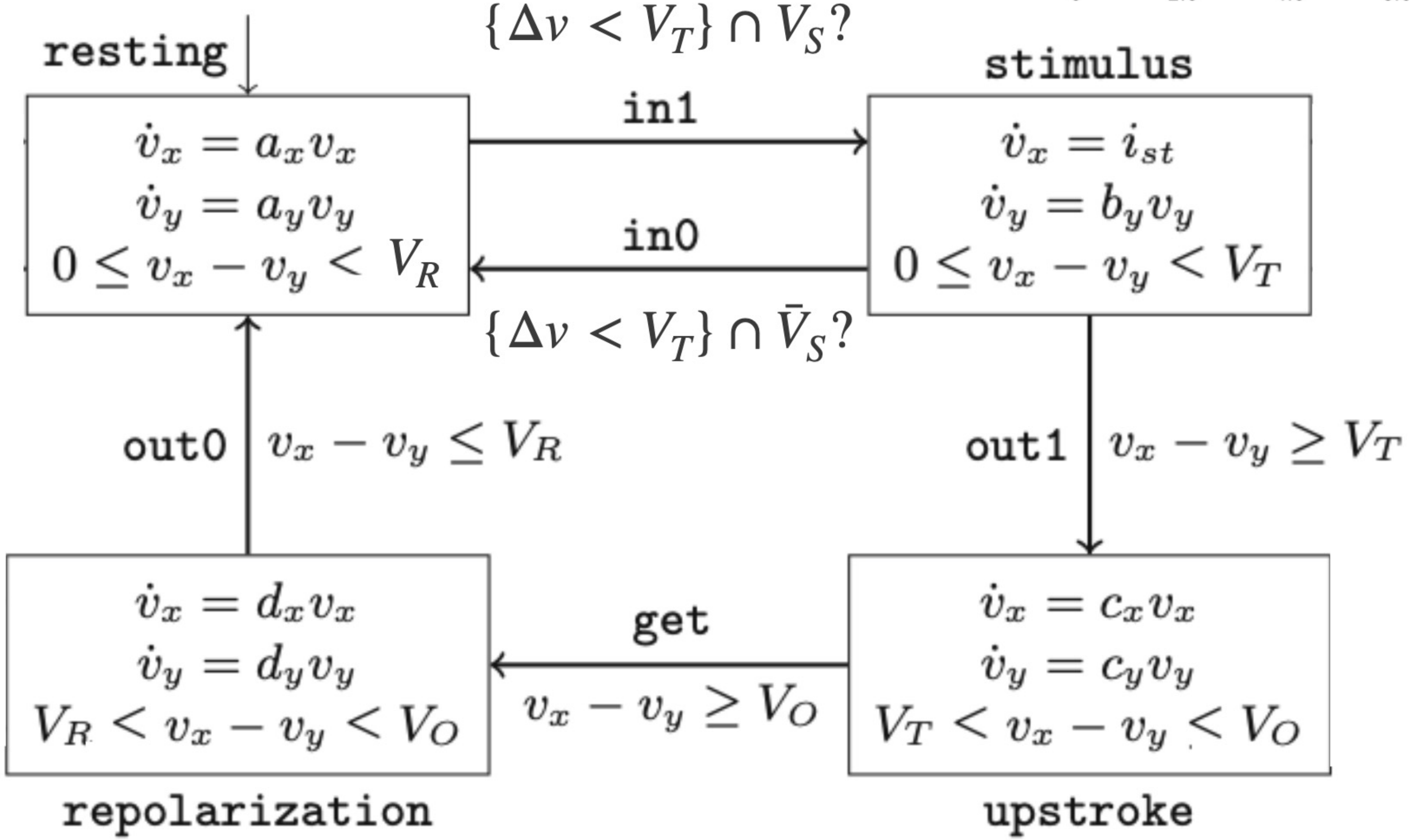


K= 850ms

- LRI (Low Rate Interval) component keeps heart rate above minimum level
- One of the pacemaker modes of operation that models the basic timing cycle
- Measures the longest interval between ventricular events
- Clock reset when VS or VP received
- No AS received ⇒ LRI outputs AP after K (TLRI-TAVI) time units

Hodgkin - Huxley model

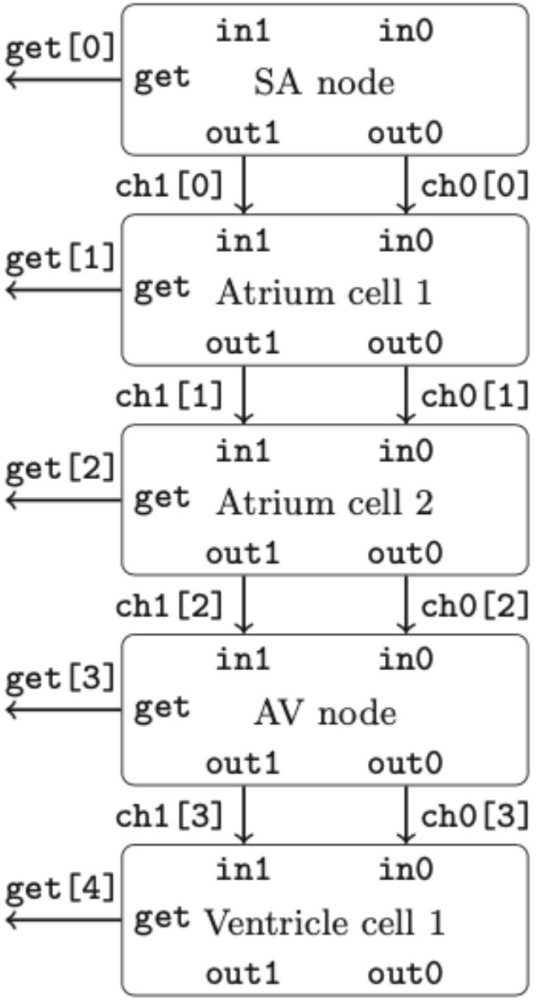




Hodgkin - Huxley model

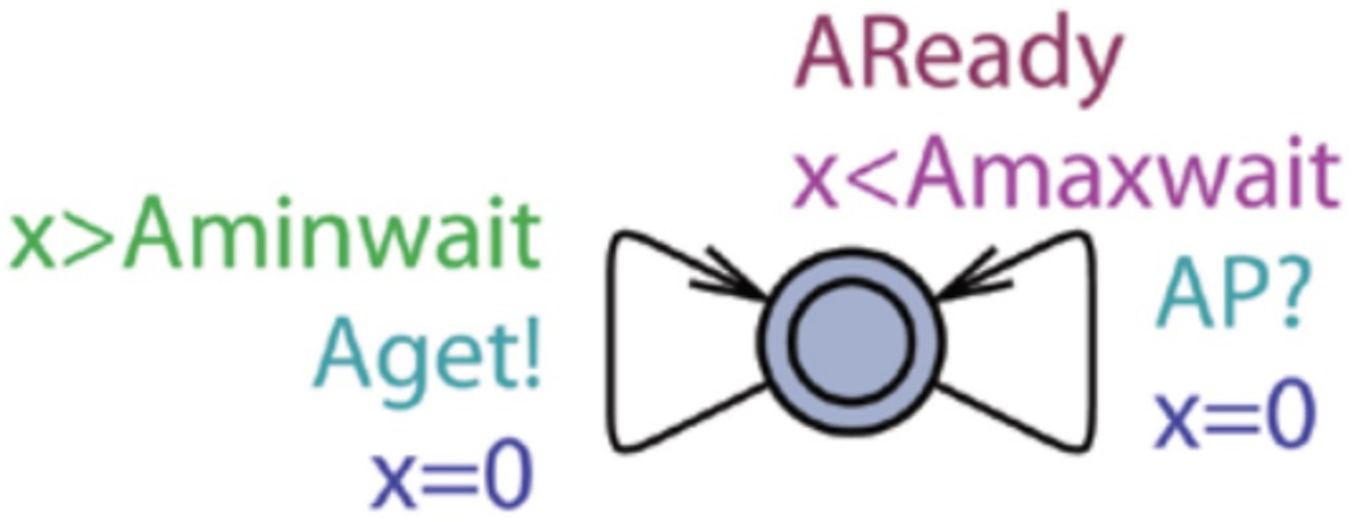
The whole heart model consists of a linear composition of cell models, which synchronize according to their output and input stimuli

At the top of the network, we have the sinoatrial (SA) node: it’s input stimulus can come from the natural pacing of the heart or from pacemaker’s actuator.



Random Heart Model (RHM)

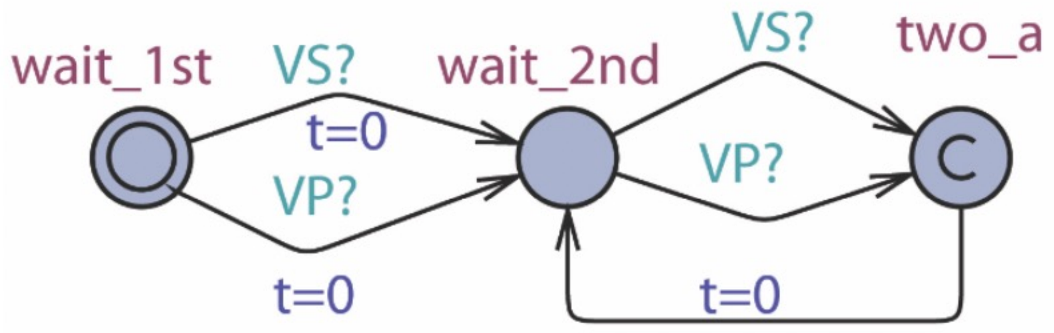
- RHM is designed to cover open-loop heart behaviors For the atrial region for instance, the interval between each action (*Aget!*) is a random value from the interval (*Aminwait, Amaxwait*).



Property

TCTL formula : $A\square((VS! \vee VP!) \rightarrow A\,\square_{\leq TLRI}(VS! \vee VP!))$

The interval between two ventricular events should be less than TLRI



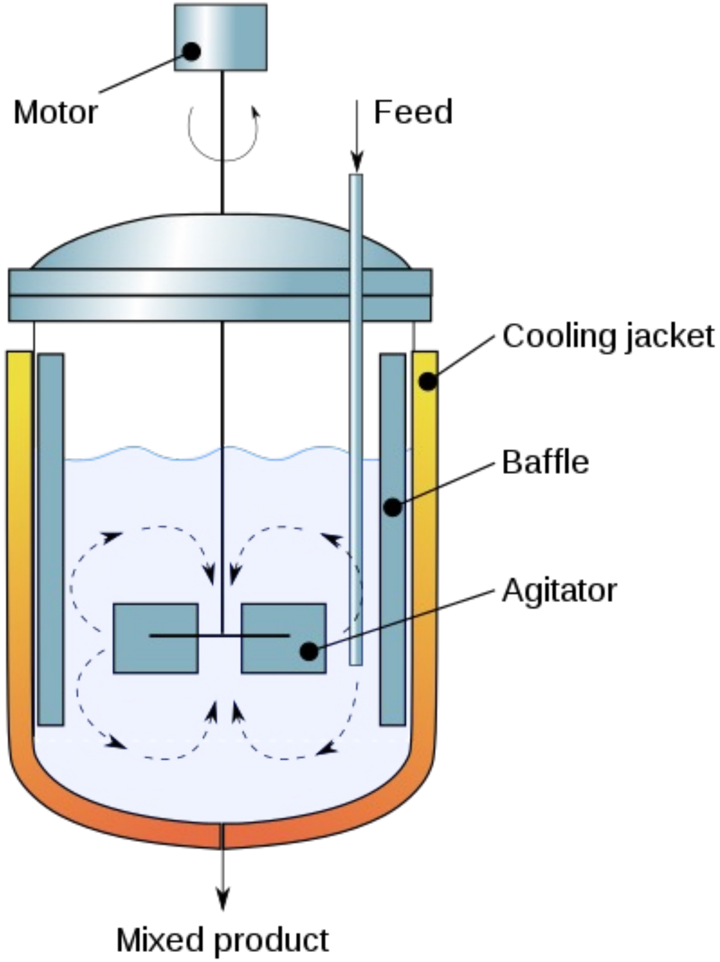
Property

TCTL formula : $A\Box(\, ch1[0]\,! \rightarrow A\Diamond\, ch1[N]\,!)$

Given an initial input, the signal should propagate all the way from SA node to atrium and then ventricle, and eventually be visible at the end of the N cells chain described in the previous section.

Temperature Control of a Continuous Stirred Tank Reactor

- Control (PID and MPC) the temperature of an exothermic CSTR so that it follows a constant set point;
- Requirement specification and checking using STL;
- Falsification of the requirements;



https://github.com/GaiaSaveri/CPS-Project

Plant Model

First Order Reaction: $A \rightarrow_{k \cdot C_A} B$

Reaction rate per unit volume (Arrhenius law):

$$k(T) = k_0\, e^{-E_a/_{R \cdot T}}$$

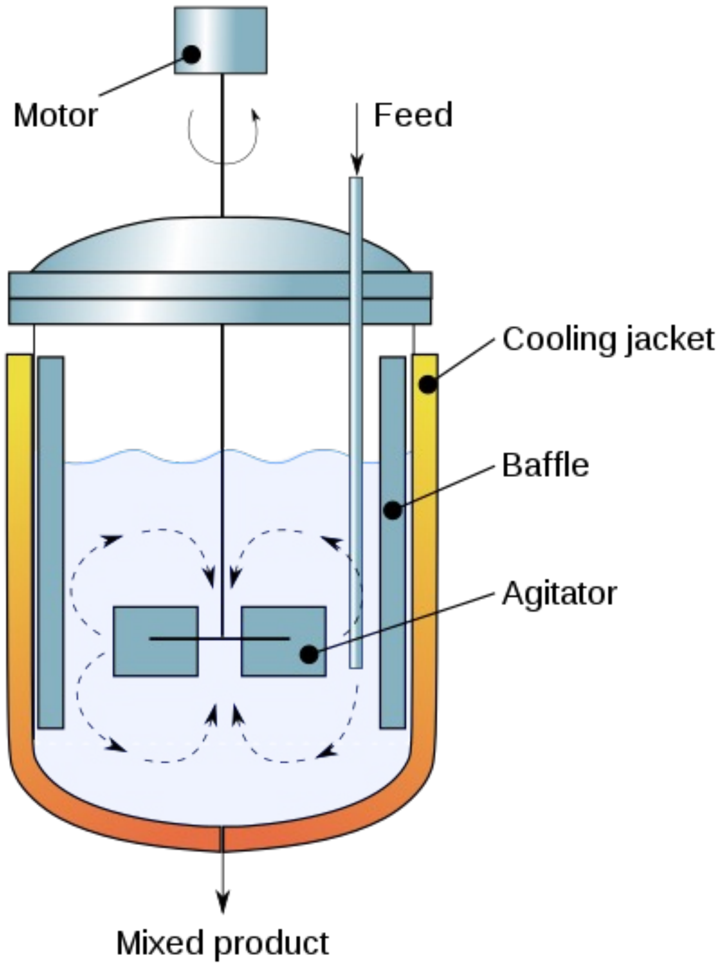
Mole balance equation:

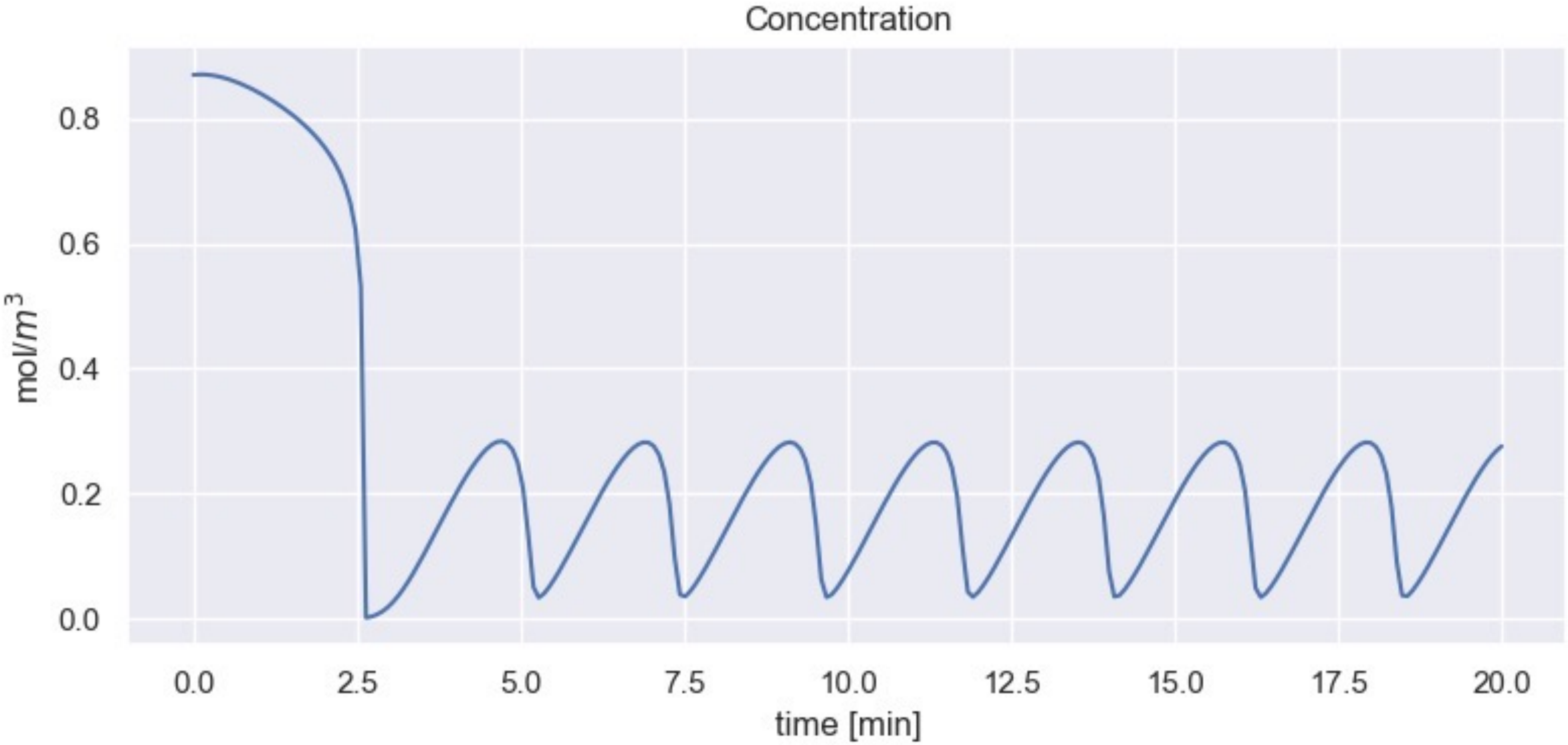
$$\frac{dC_A}{dt} = \frac{q}{V}(C_{Af} - C_A) - k(T)C_A$$

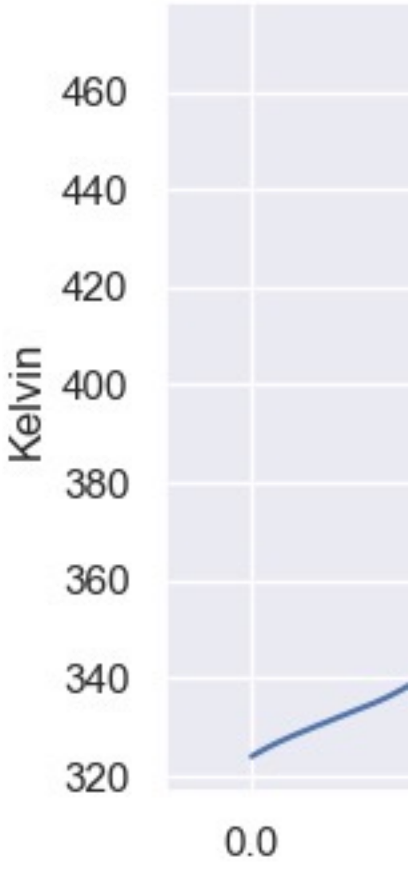
Energy balance equation:

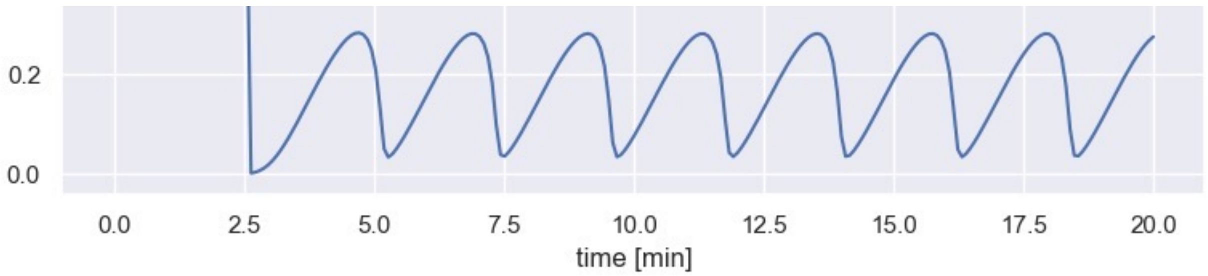
$$\frac{dT}{dt} = \frac{q}{V}(T_f - T) + \frac{-\Delta H_R}{\rho C_p} k(T)C_A + \frac{UA}{\rho C_p V}(T - T_c)$$

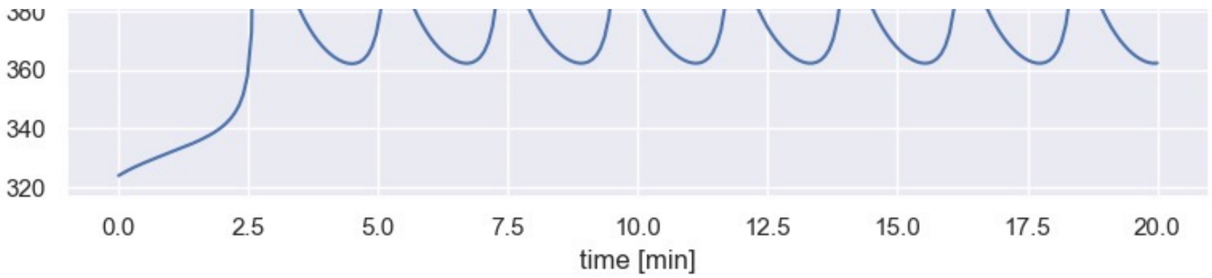
Input constraint: $T_C \in [250, 350]$

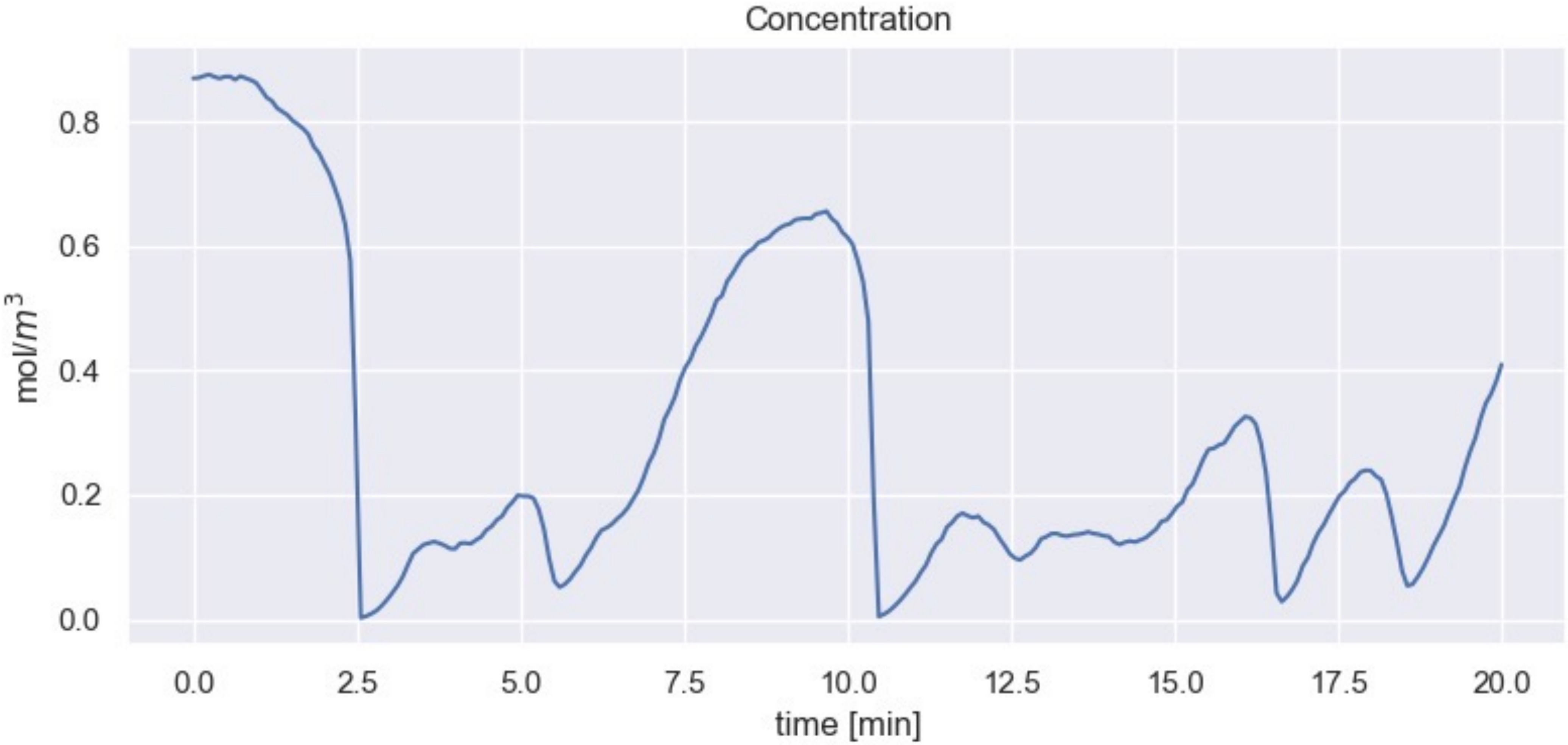


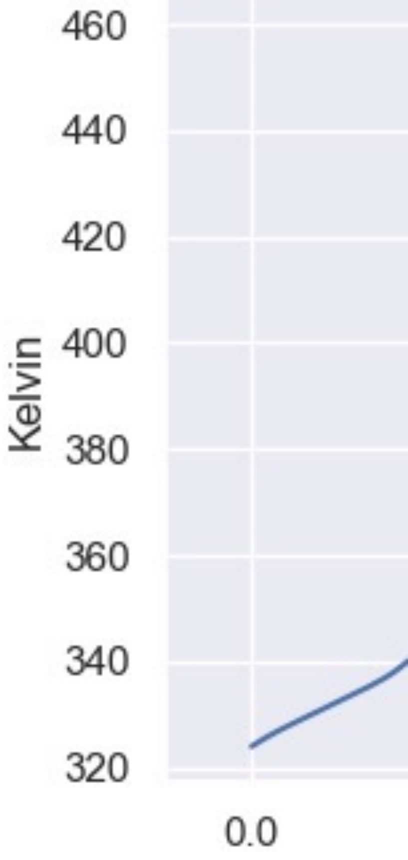

Concentration
mol/m^3
0.0
0.2
0.4
0.6
0.8
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
time [min]


Kelvin
320
340
360
380
400
420
440
460
0.0


0.0
0.2
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
time [min]

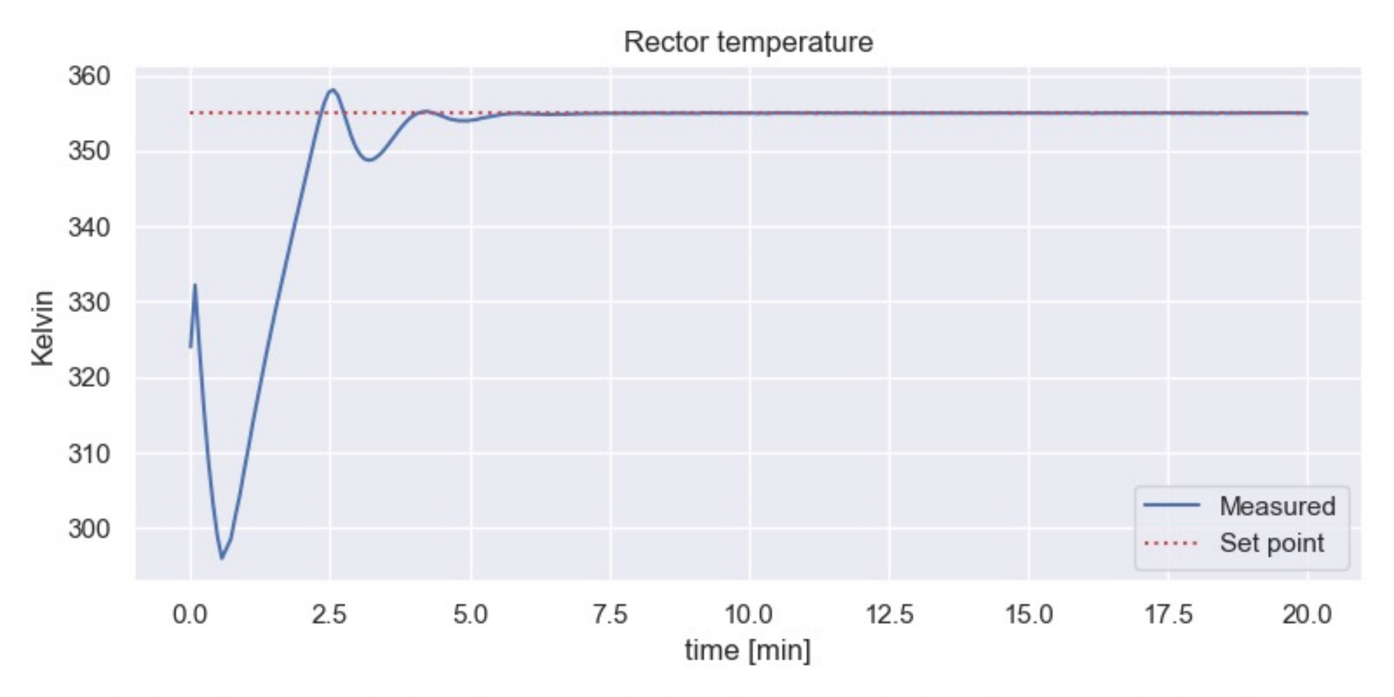

320
340
360
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
time [min]


Concentration
0.8
0.6
0.4
0.2
0.0
mol/m^3
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
time [min]


460
440
420
400
380
360
340
320
Kelvin
0.0

PID Control Cont'd

Performance of PID controller when tracking a temperature of $355K$



PID parameters:

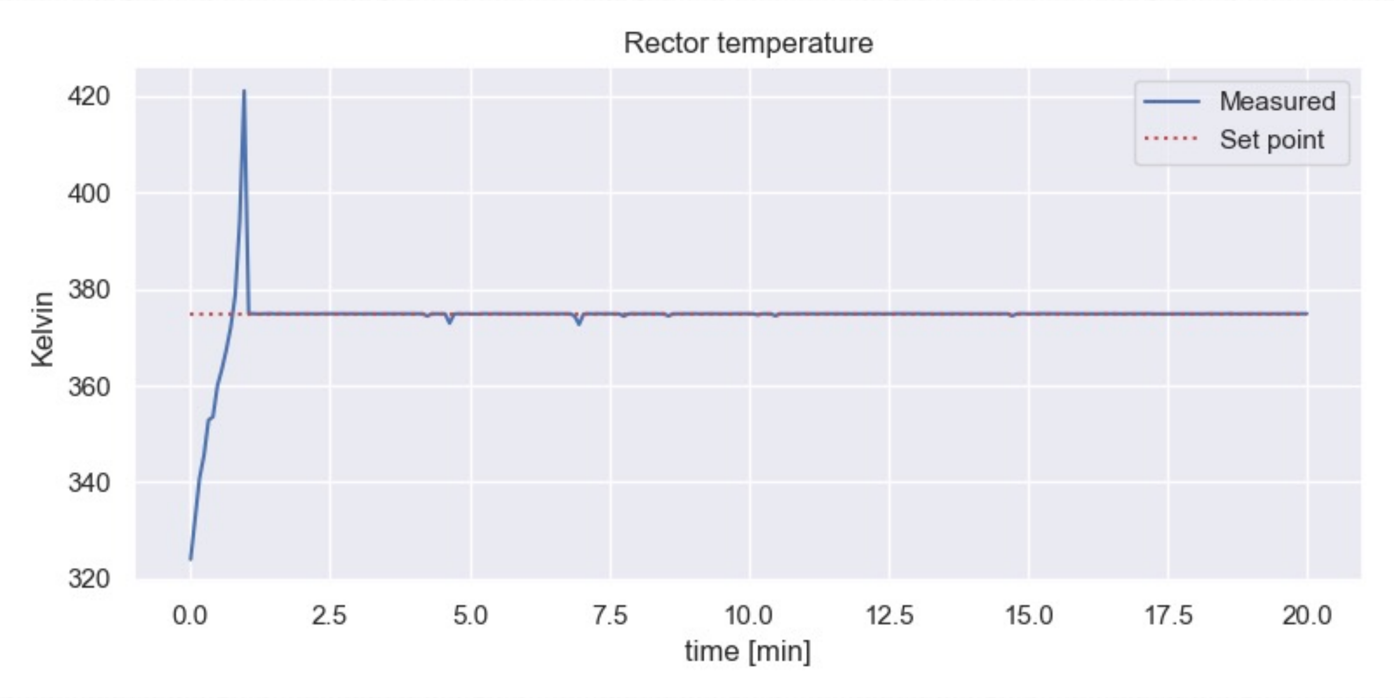
$K_p = 1.7$

$\tau_i = 0.8$

$\tau_d = 0.2$

Non-linear MPC Cont’d

Performance of MPC when tracking a temperature of 375K



MPC parameters:

$Q = 2.0$

$R = 0.01$

$H = 10.0$

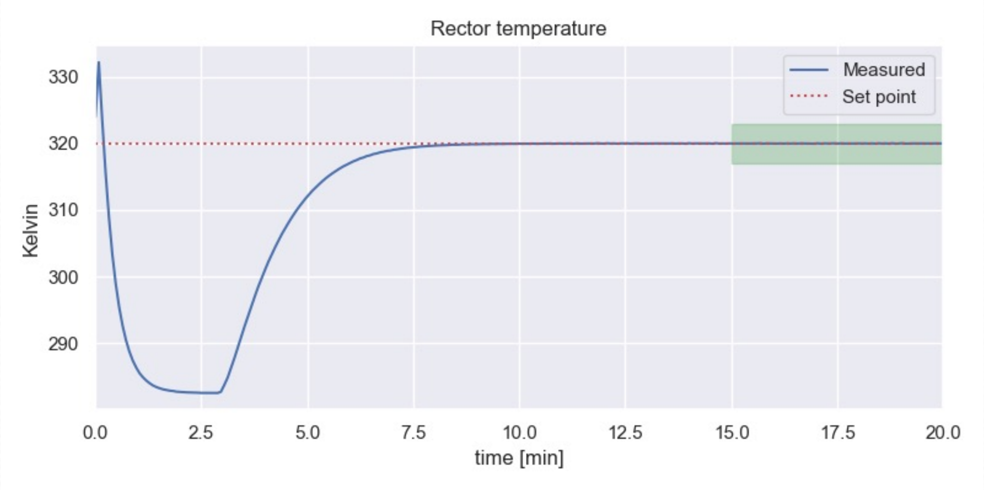
STL Requirements 3

Goal: CSTR should closely follow reference temperature

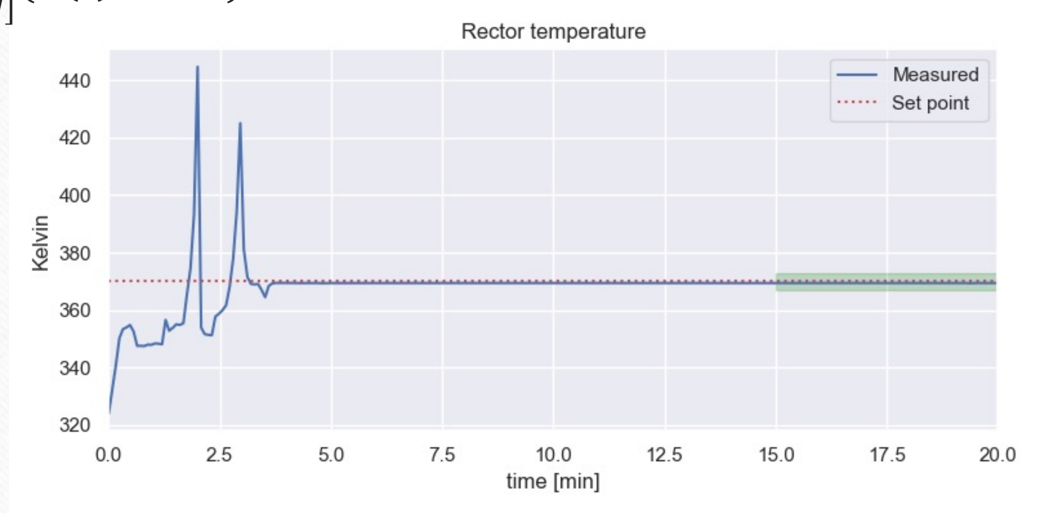
Difference from reference: $\mathrm{d}(t) = |T(t) - ref(t)|\ \forall t$

1. In the last part of the simulation, difference from reference should not exceed $3K$

$$\phi_3 = \boldsymbol{G}_{[\frac{2N}{3}, N]}(d(t) < 3.0)$$



PID

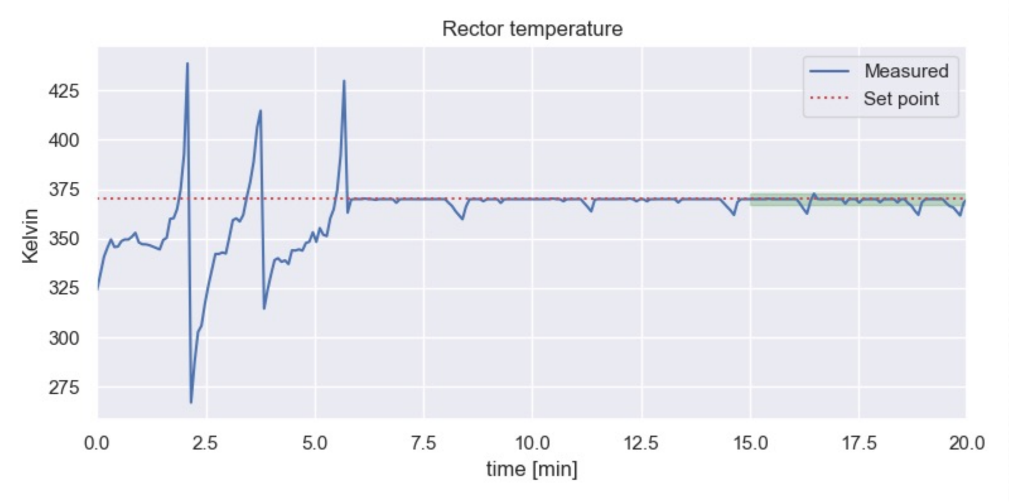


MPC

Falsification Cont'd

MPC parameters found in falsification analysis

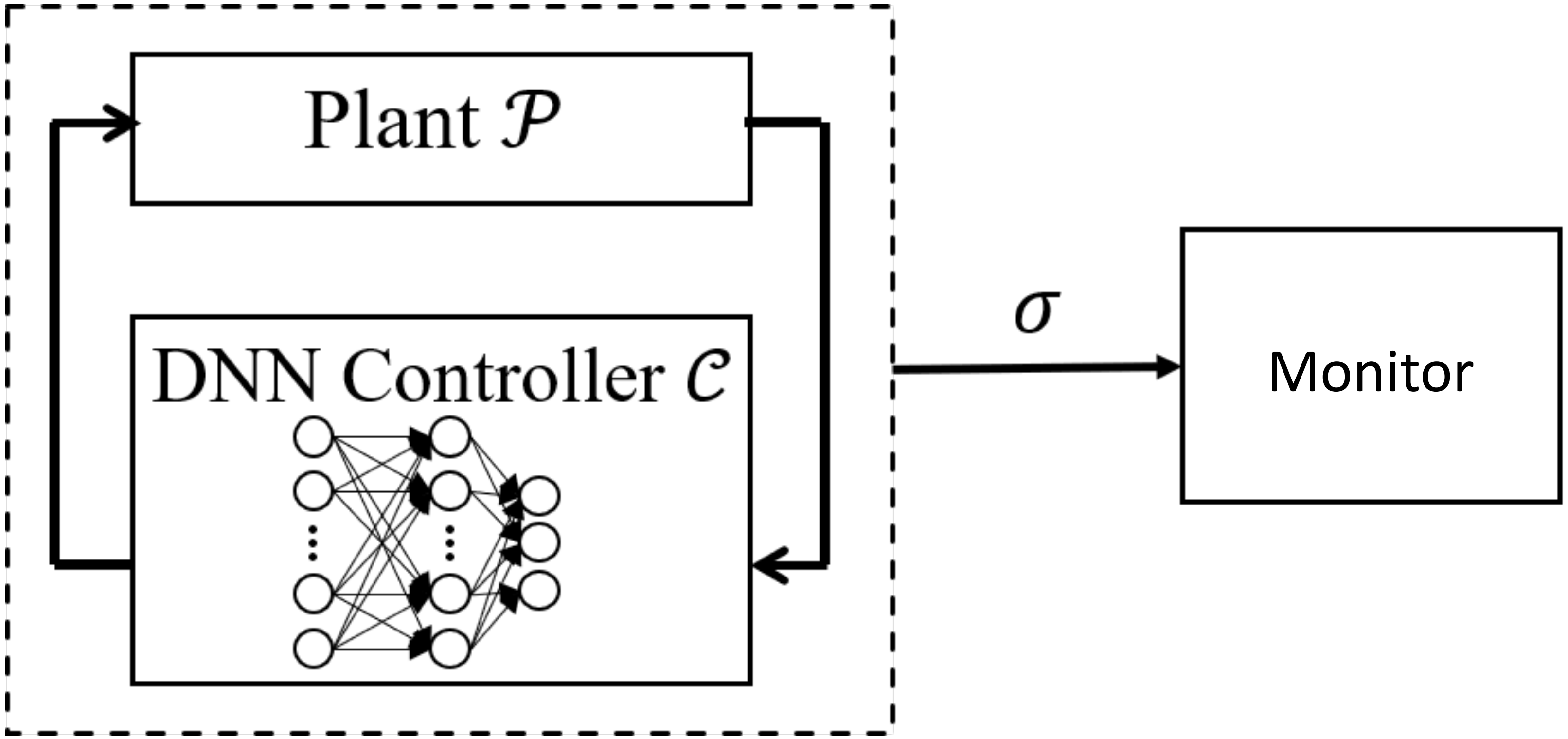
Reference	Q	R	Robustness
320	1.33663	0.020416	-1.28741
325	0.376885	0.024445	-20.7467
330	0.210134	0.0191757	-31.3692
335	1.60591	0.0040131	-0.423672
340	1.63118	0.00144311	-0.0405561
345	2.45878	0.0092065	-0.10704
350	2.67307	0.0045229	-2.28309
355	1.57202	0.00161315	-3.23189
360	1.67468	0.0150942	-8.42903
365	2.70502	0.00979266	-58.8853
370	2.59014	0.00121103	-5.30934
375	1.73716	0.0150608	-0.0113018
380	1.48972	0.018182	-2.94381
385	0.226472	0.0107259	-8.14433



Falsification for reference temperature $370K$

A Deep Neural Network controller

$S = P||C$



Mojtaba Zarei, Yu Wang, Miroslav Pajic: [Statistical verification of learning-based cyber-physical systems.](#) HSCC 2020: 12:1-12:7

Bibliography

Nice survey on Specification-Based Monitoring of CPSs: http://www-verimag.imag.fr/PEOPLE/maler/Papers/monitor-RV-chapter.pdf

Artificial Pancreas:

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