Electrons in crystals – final written test
Academic year 2006/2007 – April 11, 2007

(*Time: 3 hours*)

— Solve all the exercises, corresponding to a total maximum score of 36. If the score is between 33 and 36 it is considered equal to 30/30 *cum laude*, if it is between 30 and 32 it is considered equal to 30/30.
— Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.
— When required, numerical evaluations should be given exactly with 3 significant figures, if not otherwise indicated.

Exercise 1: *Free-electron model*

Consider a system with two types of charge carriers in the Drude model. The two carriers have the same density n and opposite charges $q_1 = e$ and $q_2 = -e$. Let m_1, m_2, τ_1, τ_2 be their masses and relaxation times, respectively (and consequently $\mu_1 = \tau_1/m_1$ and $\mu_2 = \tau_2/m_2$ their mobilities).

1. Calculate the total transverse magnetoresistance $\Delta\rho(H) = \rho(H) - \rho(H = 0)$.
2. Calculate the total Hall coefficient R_H.
3. Which is R_H if the mobilities of the two types of charge carriers are equal?

Hint: remember that the resistivity and the conductivity in presence of an external magnetic field are tensors . . . ; remember also that:

$$\hat{\rho}^{-1} = \begin{pmatrix} \rho & -R_H H \\ R_H H & \rho \end{pmatrix}^{-1} = \frac{\begin{pmatrix} \rho & R_H H \\ -R_H H & \rho \end{pmatrix}}{\begin{vmatrix} \rho & -R_H H \\ R_H H & \rho \end{vmatrix}}$$

Exercise 2: *Electrons in periodic solids*

A cubic crystal has a band whose dispersion is described as:

$$E(k) = E_0 - \beta k^2 + \alpha k^4 + \gamma(k_x^2 k_y^2 + k_x^2 k_z^2 + k_y^2 k_z^2)$$

where $k^2 = k_x^2 + k_y^2 + k_z^2$ and the parameters α, β, γ are all positive.

1. Show that the bandstructure has the full cubic symmetry.
2. Show that there are six degenerate minima of the energy located along the six Cartesian directions in k-space, and find the position of these minima.
3. Find the inverse effective mass tensor $(1/m)_{ij}$ at each minimum.

Exercise 3: *Crystalline structures*

Consider the two-dimensional lattice shown in the figure with a= 2.5 Å.

1. How you will classify it? Find its primitive translation vectors and draw its Wigner-Seitz cell. What is the area of this unit cell?
2. Draw the reciprocal lattice, give its primitive vectors specifying their k_x and k_y coordinates.
3. Draw the first Brillouin zone. What is its area?

Exercise 4: *Semiclassical model of electron dynamics*

Consider a solid with band dispersion $E(\mathbf{K}) = 2\gamma[cos(k_x a) + cos(k_y a) + cos(k_z a)]$, with $\gamma < 0$, a the lattice parameter, and a proper choice of the zero of energy, under a uniform static magnetic field $\mathbf{H} = H\hat{x}$.

1. Write the expression of the Bloch electron velocity in the (y,z) plane.
2. Write explicitely the equation of the orbit (in $\mathbf{k}$ space) for $k_x = 0$ on the constant energy surface $E(\mathbf{k}) = E^* = 2\gamma$ and draw it.
3. Specify the expression of the velocity (a) on this orbit in terms of k_y only. Indicate the direction of the motion along the orbit.