Condensed Matter Physics I
II written test
academic year 2016/2017
January 16, 2017

(*Time: 2 hours)*

Exercise 1: *Weak potential*

1. Consider a weak potential $V(x) = V_0 cos(2\pi x)$ in one dimension.
2. Calculate E(k) for the lowest energy band in the nearly free electron approximation.
3. Determine the size of the energy gap between first and second band at k $= \pm\frac{\pi}{a}$.
4. What is the Fermi energy if the first band is fully occupied with electrons?

Exercise 2: *Tight-binding model and Semiclassical model of Electron Dynamics*

1. Calculate the expression for the energy band of a BCC crystal with lattice parameter a, using the tight-binding model assuming one s orbital per atom and nearest neighbor interaction of strength γ, neglecting overlaps and higher order hopping contributions.
2. Using the band calculated above and the semiclassical model of electron dynamics, write the explicit expression for the velocity of an electron of this band, $\mathbf{v}(\mathbf{k}(t))$ (i.e., v_x, v_y, v_z as a function of $\mathbf{k}$(t)).
3. Consider an electron in a uniform electric field E constant in time. Calculate explicitly $\mathbf{k}$(t)) considering that $\mathbf{k}(0)$=0.
4. Consider the electron initially at rest and at the origin. Derive the explicit expression of $\mathbf{r}$(t), i.e. of $x(t)$, $y(t)$ and $z(t)$, using the expression of $\mathbf{k}$(t)) previously derived.
5. Specify the results for $x(t)$, $y(t)$ and $z(t)$ in the case of $\mathbf{E}$ in [110] direction.
6. You should find an oscillatory motion. Sketch the trajectory in the real space. Estimate Δr, the amplitude of oscillation, for a realistic electric field and bandwidth.

NOTE:

- Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.