NAME and SURNAME:

Condensed Matter Physics I
a.y. 2023/24
15 January 2024

(*Time available: 2h 30'*)

Hand in the assignment giving also this sheet of paper (solve here the Exercise n. 3)

Exercise 1: *Electronic specific heat in the Sommerfeld model*

1. Show that the electronic density of states varies as $g(E) \sim E^{-1+d/2}$ in spatial dimension d.

2. Show that the electronic heat capacity varies as $C_{el} \sim T^p$, where the power p is independent of dimension d. Briefly explain the value of p and why C_{el} does not depend on d.

Exercise 2: *Semiclassical model of the electron dynamics and Bloch oscillations*

Consider an electron moving in a one-dimensional periodic potential $V(x+a) = V(x)$. Let the dispersion relation be $E(k) = -A\cos(bk)$ within the band where the electron is confined.

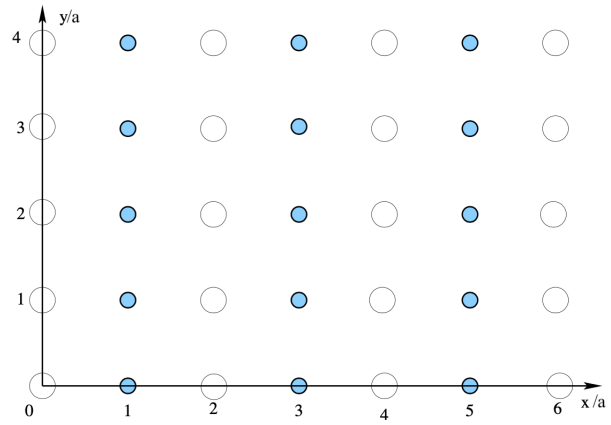
1. What is the value of b? What is the group velocity of the electron with wavenumber k?

2. A constant applied electric field $E = -E\hat{x}$ accelerates the electron. What is dk/dt? Neglect any scattering by impurities, defects or phonons, i.e. take the mean free time $\tau \to \infty$.

3. Assume the electron is initially localized around the origin $x = 0$ in a wave packet with $k = 0$. Qualitatively describe the motion of the electron in response to the applied field, and comment on how the motion differs from that of a free particle.

Exercise 3: *Crystalline structures*

1. In the hypothetical 2D crystal structure shown below, the large and small circles are atoms of different type, spaced by a. Sketch one unit cell. Write the expression for the primitive translation vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ chosen parallel to the axes and draw them on the figure. Write also the vectors of the unit cell internal basis $\{\mathbf{d}_i\}$.



$\mathbf{a}_1$=...

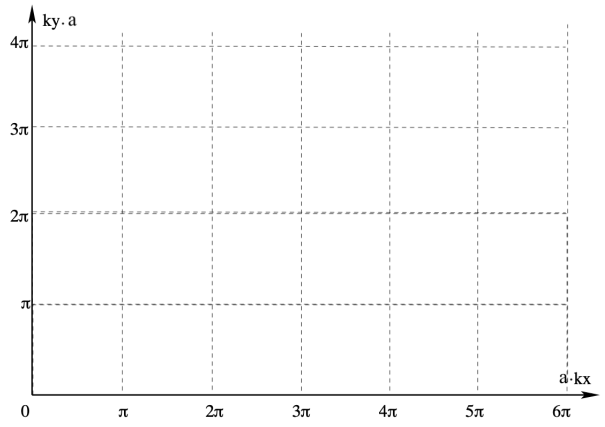
$\mathbf{a}_2$=...

$\mathbf{d}_1$=...

$\mathbf{d}_2$=...

...

2. Write the primitive reciprocal lattice vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ and draw them in the plane (k_x, k_y) shown below.



$\mathbf{b}_1$=...

$\mathbf{b}_2$=...

Miller indices (1,0): $\mathbf{K}$=...

Miller indices (0,1): $\mathbf{K}$=...

Miller indices (1,1): $\mathbf{K}$=...

Miller indices (1,2): $\mathbf{K}$=...

Miller indices (2,1): $\mathbf{K}$=...

3. Remember that the Miller indices (h, k) of a lattice plane are the coefficients of the vector normal to the plane in terms of the primitive reciprocal lattice vectors. Consider the planes corresponding to the Miller indices (1,0), (0,1), (1,1), (1,2), (2,1); for each plane write the corresponding reciprocal lattice vector $\mathbf{K}$ normal to it and place and label the points in the plane (k_x, k_y).

4. Draw and label in the figure (x,y) the lattice planes corresponding to the Miller indices above (one plane for each Miller index).

5. Consider the diffraction corresponding to the family of planes of the Miller indices above. For each diffraction peak calculate the value of the structure factor $S_{(h,k)}$.

6. If the form factors of the two atoms are equal, for which values of the Miller indices the structure factor vanishes?