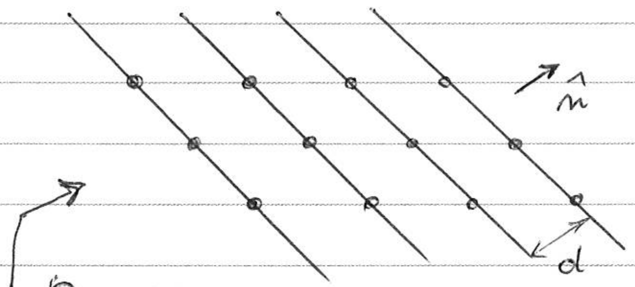
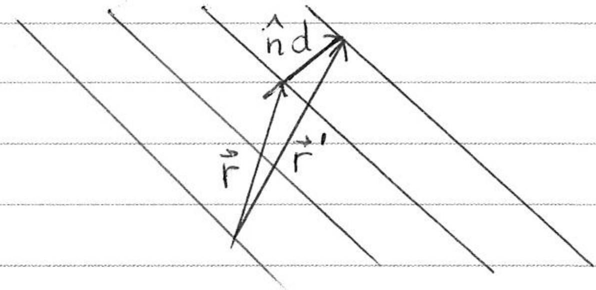
FAMILY OF LATTICE PLANES and $\vec{K}$





- Consider a FAMILY OF LATTICE PLANES $\perp \hat{n}$ and equispaced by d.
  The eq. of a plane (in general, in the continuum space) is $\boxed{\vec{r}\cdot\hat{n} = \text{constant}}$ (a different constant for $\neq$ planes)

- Consider a specific wave vector: $\boxed{\vec{k} = \frac{2\pi}{d}\hat{n}}$

$\Rightarrow$ the corresponding planewave $e^{i\vec{k}\cdot\vec{r}}$ is constant on those planes: $\vec{r}\cdot\hat{n} = \text{const.} \Rightarrow e^{i\vec{k}\cdot\hat{n}} = e^{i\frac{2\pi}{d}\hat{n}\cdot\hat{n}} = \text{const}'$

More precisely, it has the same value on all the planes:

$$e^{i\vec{k}\cdot\vec{r}'} = e^{i\vec{k}\cdot(\vec{r}+m\hat{n}d)} = e^{i\vec{k}\cdot\vec{r}}$$

Without any loss of generality one of these planes can contain the origin and therefore $e^{i\vec{k}\cdot\vec{r}} = 1$, on the whole family of planes. Such planes are lattice planes, contain all the Bravais lattice points, therefore $\vec{k} = \frac{2\pi}{d}\hat{n}$ satisfy $\boxed{e^{i\vec{k}\cdot\vec{R}} = 1 \quad \forall \vec{R} \in \text{Bravais lattice}}$

$\Rightarrow$ $\vec{k}$ is a reciprocal lattice vector $\vec{K}$

$\Rightarrow$ (a)

A family of lattice planes $\perp \hat{n}$ and equispaced by d	$\Longrightarrow$	are defined by: a vector of the reciprocal lattice: $\vec{K} = \frac{2\pi}{d}\hat{n}$

(b)

Each vector $\vec{K}$ $\in$ reciprocal lattice	$\Longrightarrow$	defines a family of lattice planes $\perp \vec{K}$ and with interplanar distance $d = 2\pi / K^*$ where $\vec{K}^*$ is the shortest rec. lattice vector $\vec{K}^* /\!/ \vec{K}$

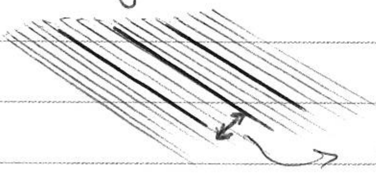
- Consider $\vec{K} \in$ reciprocal lattice
- Consider the planes in real space with $\vec{r}$ such that: $\boxed{e^{i\vec{K}\cdot\vec{r}} = 1}$ (✱)

For sure, one of them contains the origin, since $e^{i\vec{K}\cdot(000)} = 1$

But the eq. (✱) defines <u>many planes</u> (continuous space — in the continuous space) (a set of planes) which are $\perp \vec{K}$
(all the planes with $\vec{r}$ such that $\vec{r}\cdot\hat{K} = 2\pi m$)
and equispaced by $d = 2\pi / K$ (spacing)
(since, if we consider $\vec{r}\,' = \vec{r} + \frac{2\pi}{K}\hat{K}$, we have:
$$e^{i\vec{K}\cdot\vec{r}\,'} = e^{i\vec{K}\cdot\vec{r}} \cdot e^{i\vec{K}\cdot 2\pi\frac{\hat{K}}{K}} = e^{i\vec{K}\cdot\vec{r}} = 1 \;)$$

By def of $\vec{K} \in$ rec. lattice, all the Bravais lattice vectors $\vec{R}$ are such that $\boxed{e^{i\vec{K}\cdot\vec{R}} = 1}$

Hence, they must lie on these planes $\perp \vec{K}$ and equispaced by d.
But they can be a <u>subset</u> of the set in the continuous space equispaced by a certain distance d'



distance d'

$\Downarrow$

For (a), they are defined by $\vec{K}^* = \frac{2\pi\hat{n}}{d'}$

Everything is consistent if $\vec{K} = n\,\vec{K}^*$,
i.e., $\vec{K}^*$ is the shortest vector, $\in$ reciprocal lattice, $/\!/ \vec{K}$.