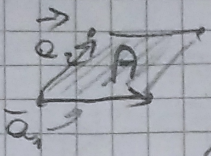
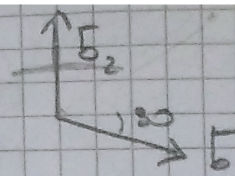
(a) triangular empty lattice

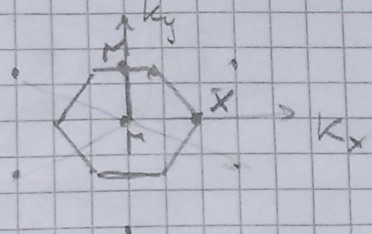




$$\begin{cases}\vec{a}_1 = a(1,0)\\ \vec{a}_2 = a\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)\end{cases} \qquad A = \frac{a^2\sqrt{3}}{2} \qquad \begin{cases}\vec{b}_1 = \frac{2\pi}{\sqrt{3}a}(\sqrt{3},-1)\\ \vec{b}_2 = \left(0,\frac{4\pi}{\sqrt{3}a}\right)\end{cases}$$

(1) $k_F = ?$ free e^-: $2\frac{\pi k_F^2}{\left(\frac{2\pi}{L}\right)^2} = N \Rightarrow \boxed{k_F^2 = 2\pi n}$ $n = \frac{N}{A}$

$Z = 2 \Rightarrow \boxed{k_F^2 = \frac{4\pi\cdot 2}{a^2\sqrt{3}} = \frac{8\sqrt{3}}{3}\frac{\pi}{a^2}}$

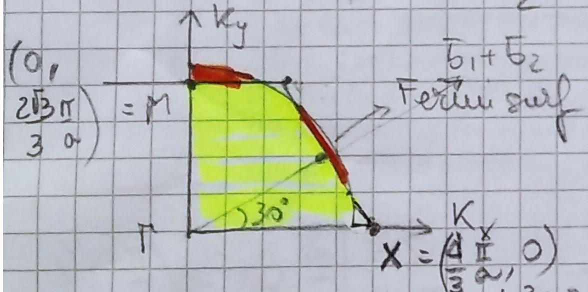


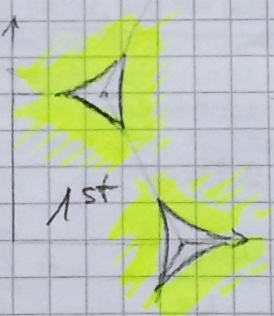
$\overline{\Gamma M} = \frac{|\vec{b}_2|}{2} = \frac{2\pi}{\sqrt{3}a} \to \sim 1.15$

$$\boxed{\left(\frac{k_F}{\overline{\Gamma M}}\right)^2} = \frac{\cancel{8}\sqrt{3}}{\cancel{3}}\frac{\pi}{\cancel{a^2}}\cdot\frac{\cancel{3a^2}}{\cancel{4}\pi^{\cancel{2}}} = \boxed{\frac{2\sqrt{3}}{\pi} > 1} \quad \text{c.v.d.}$$

(2) $\overline{\Gamma X}\cdot\frac{\sqrt{3}}{2} = \overline{\Gamma M} \Rightarrow \overline{\Gamma X} = \frac{2}{\sqrt{3}}\frac{2\pi}{\sqrt{3}a} = \frac{4}{3}\frac{\pi}{a}$

$$\left(\frac{k_F}{\Gamma X}\right)^2 = \frac{\cancel{8}\sqrt{3}}{\cancel{3}}\frac{\cancel{\pi}}{\cancel{a^2}}\cdot\frac{\cancel{9}a^2}{\cancel{16}\pi^{\cancel{2}}} = \frac{3\sqrt{3}}{2\pi} < 1 \Rightarrow \boxed{\overline{\Gamma M} < k_F < \overline{\Gamma X}}$$



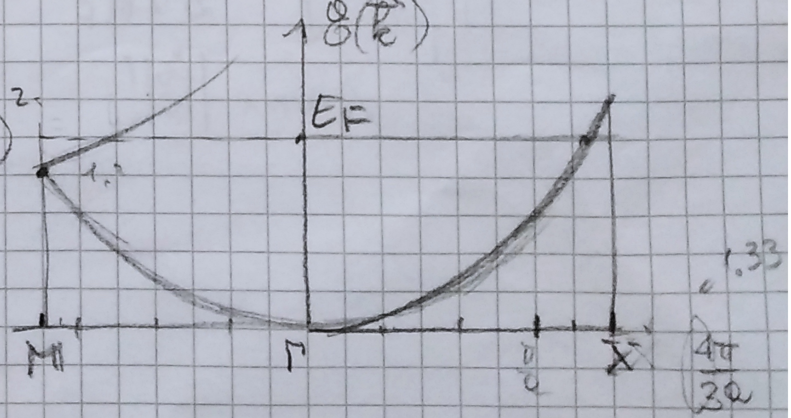


$\mathcal{E}_1(\vec{k}\in\Gamma M) = \frac{\hbar^2 k^2}{2m}$

$\mathcal{E}_2(\vec{k}\in\Gamma M) = \frac{\hbar^2}{2m}\left(k - \frac{4\pi}{\sqrt{3}a}\right)^2$

$\mathcal{E}_1(\vec{k}\in\Gamma X) =$ 1.47

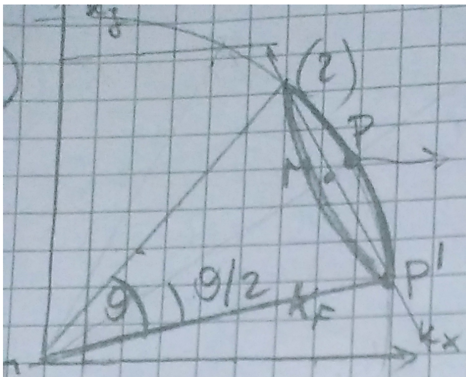
$E_F = \frac{\hbar^2 k_F^2}{2m} = \frac{\hbar^2}{2m}\frac{8\sqrt{3}}{3\pi}\frac{\pi^2}{a^2}$



$\mathcal{E}_X = \frac{\hbar^2}{2m}\frac{16\pi^2}{9a^2}$, $\mathcal{E}_M = \frac{\hbar^2}{2m}\frac{4}{3}\frac{\pi^2}{a^2}$

1.77 1.33

(3) Close orbits: 1st: h-like ; 2nd: e-like



$$\overline{\Gamma P'} = \overline{\Gamma P} = k_F = \left(\frac{8\sqrt{3}}{3}\,\frac{\pi}{a^2}\right)^{\frac{1}{2}}$$

$$\overline{\Gamma \Pi} = \frac{b_i}{2} = \frac{4\pi}{\sqrt{3}\,a}$$

$$\text{perimeter}^{(2)} = 2 * k_F \theta \quad , \quad \theta = ?$$

$$\overline{\Gamma P'} \cos\frac{\theta}{2} = \overline{\Gamma\Pi} \Rightarrow \cos\frac{\theta}{2} = \frac{\overline{\Gamma\Pi}}{\overline{\Gamma P'}} = \frac{\cancel{4}\pi}{\cancel{\sqrt{3}}}\left(\frac{3}{8\sqrt{3}}\right)^{\frac{1}{2}} \cdot \frac{1}{\sqrt{\pi}}$$

$$= \frac{\sqrt{\pi}}{\sqrt{2}\,(\sqrt{3})^{\frac{1}{2}}} = \left(\frac{\pi}{2\sqrt{3}}\right)^{\frac{1}{2}}$$

$$\cong 0.9523$$

$$\Rightarrow \frac{\theta}{2} \cong 17.76^\circ$$

$$\Rightarrow \theta \cong 35^\circ$$

$$\boxed{\text{perimeter}^{(2)} = 2\cdot\left(\frac{8\sqrt{3}}{3}\,\frac{\pi}{a^2}\right)^{\frac{1}{2}} 2 \arccos\left(\frac{\pi}{2\sqrt{3}}\right)^{\frac{1}{2}}}$$

$$T_{free} = \frac{2\pi}{\omega_c} = 2\pi \frac{mc}{eH}$$

$$\text{Ratio } \frac{T^{(2)}}{T_{free}} = \frac{\text{perimeter}^{(2)}}{2\pi k_F} = \frac{2\theta \cancel{k_F}}{2\pi \cancel{k_F}} = \frac{\theta}{\pi} = \boxed{\frac{2}{\pi} \arccos\left(\frac{\pi}{2\sqrt{3}}\right)^{\frac{1}{2}}}$$

same $|\vec{k}|$!

$$\cong \frac{35^\circ}{180^\circ} \cong \boxed{0.197}$$