ES. 1)

3. $\bar{\varphi}(\alpha, \bar{q}) = \begin{pmatrix} \cos\alpha \, q_1 + \sin\alpha \, q_2 \\ -\sin\alpha \, q_1 + \cos\alpha \, q_2 \end{pmatrix}$

$\rightarrow P_{\text{Nöther}} = \dots = M_z$

5. $\left\{ \frac{p_x^2+p_y^2}{2m}, \, x p_y - y p_x \right\} = \left\{ \frac{p_x^2+\cancel{p_y^2}}{2m}, x \right\} p_y - \left\{ \frac{\cancel{p_x^2}+p_y^2}{2m}, y \right\} p_x =$

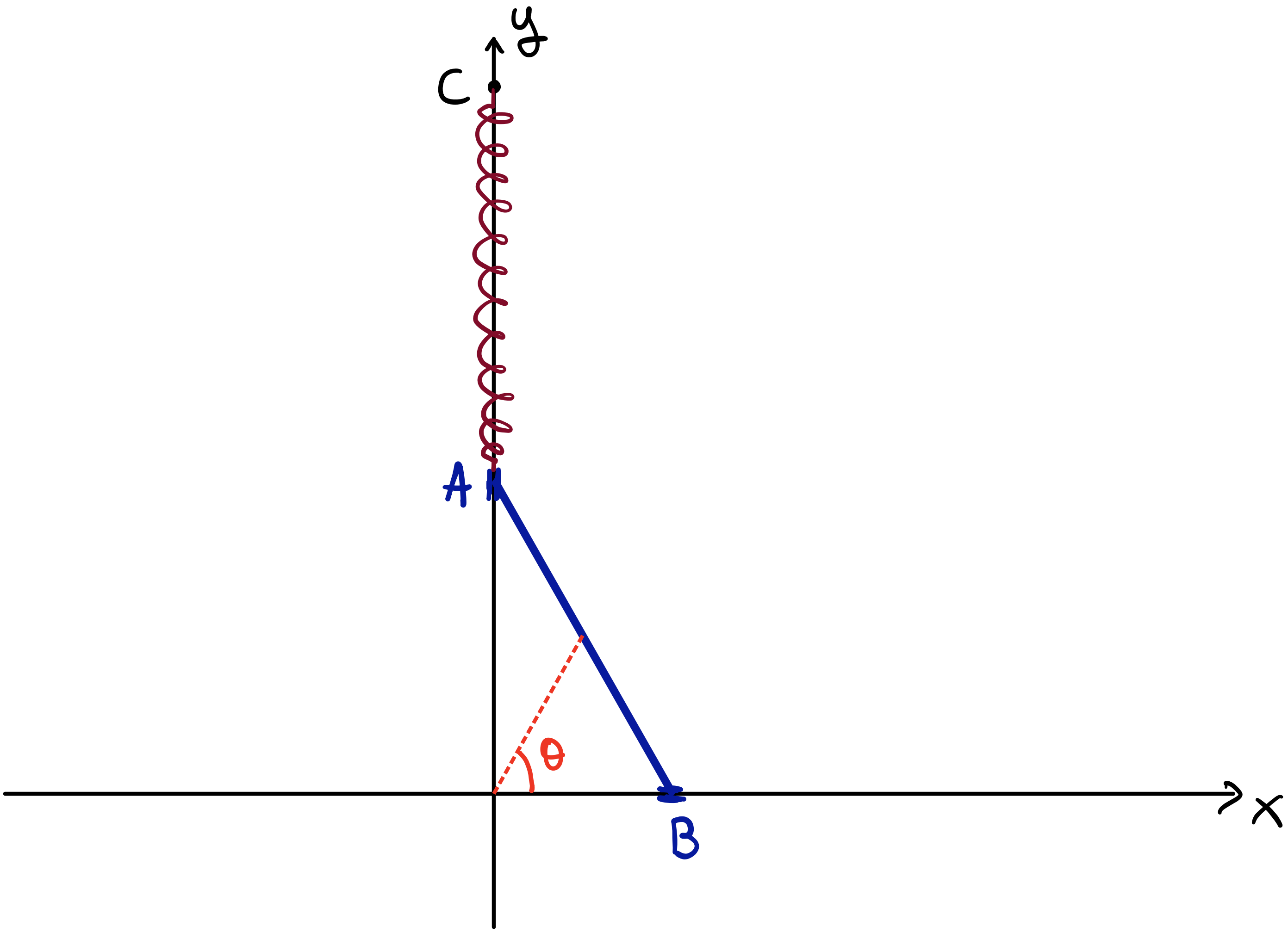
$\{H, M_z\}$

$= \frac{1}{m} p_x \underbrace{\{ p_x, x \}}_{=-1} p_y - \frac{1}{m} p_y \underbrace{\{ p_y, y \}}_{=-1} p_x = 0$

6. H, M_z due cost. del moto in involuz.

$\Rightarrow$ sistema $(n=2)$ è integrabile.

ES. 2



1. $x_G = \frac{\ell}{2}\cos\theta$ $\quad y_A = \ell \operatorname{sen}\theta$ $\quad I_G = \int_{-\ell/2}^{\ell/2} s^2 \rho\, ds = \frac{s^3}{3}\Big|_{-\ell/2}^{\ell/2} \rho = \frac{2\ell^3\rho}{4\cdot 8\cdot 3} = \frac{m\ell^2}{12}$

$y_G = \frac{\ell}{2}\operatorname{sen}\theta$ $\quad x_B = \ell\cos\theta$

$$T = \frac{m}{2}\left(\frac{\ell}{2}\right)^2\dot\theta^2 + \frac{1}{2}\left(\frac{m\ell^2}{12}\right)\dot\theta^2 = \frac{m\ell^2}{6}\dot\theta^2 \qquad \frac{1}{4}+\frac{1}{12}=\frac{1}{3}$$

$$V = mg\frac{\ell}{2}\operatorname{sen}\theta + \frac{1}{2}k(2\ell - \ell\operatorname{sen}\theta)^2 =$$

$$= \frac{k\ell^2}{2}\operatorname{sen}^2\theta + \left(mg\frac{\ell}{2} - 2k\ell^2\right)\operatorname{sen}\theta + \text{const.}$$

$$L = \frac{m\ell^2}{6}\dot\theta^2 - \frac{k\ell^2}{2}\operatorname{sen}^2\theta - \left(mg\frac{\ell}{2} - 2k\ell^2\right)\operatorname{sen}\theta + \text{cost.}$$

2. $\frac{d}{dt}\frac{\partial L}{\partial\dot\theta} = \frac{m\ell^2}{3}\ddot\theta$ $\qquad \frac{\partial L}{\partial\theta} = -k\ell^2\operatorname{sen}\theta\cos\theta - \left(mg\frac{\ell}{2} + 2k\ell^2\right)\cos\theta$

$$\ddot\theta = -3\frac{k}{m}\operatorname{sen}\theta\cos\theta - \left(\frac{3}{2}\frac{g}{\ell} + 6\frac{k}{m}\right)\cos\theta$$

3. Simm. $\quad x \to -x \quad y \to y$. Non è continua, quindi non si applica il teorema di Nöther.

4. $$V'(\theta) = k\ell^2\operatorname{sen}\theta\cos\theta + \left(mg\frac{\ell}{2} - 2k\ell^2\right)\cos\theta =$$

$$= k\ell^2\cos\theta\left(\operatorname{sen}\theta + \frac{mg}{2k\ell} - 2\right)$$

$= 0$ quando $\quad \theta = \pm\pi/2,\ \theta^*,\ \pi-\theta^*$ $\quad$ con $\operatorname{sen}\theta^* = 2 - \frac{mg}{2k\ell}$

$\uparrow\ \nearrow$

$\exists$ se $\ -1 < 2 - \frac{mg}{2k\ell} < 1 \qquad -3 < -\frac{mg}{2k\ell} < -1$

cioè se $\ 2 < \frac{mg}{k\ell} < 6$

Stabilità:

$$V''(\theta) = kl^2\cos^2\theta - kl^2 \operatorname{sen}^2\theta - \left(\frac{mgl}{2} - 2kl^2\right)\operatorname{sen}\theta$$

$$= kl^2 - 2kl^2\operatorname{sen}^2\theta - \left(\frac{mgl}{2} - 2kl^2\right)\operatorname{sen}\theta =$$

$$= kl^2\left(1 - 2\operatorname{sen}^2\theta - \left(\frac{mg}{2kl} - 2\right)\operatorname{sen}\theta\right)$$

$$V''\left(\frac{\pi}{2}\right) = kl^2\left(1 - 2 - \frac{mg}{2kl} + 2\right) = kl^2\left(1 - \frac{mg}{2kl}\right)$$

STAB. se $mg < 2kl$

$$V''\left(-\frac{\pi}{2}\right) = kl^2\left(1 - 2 + \frac{mg}{2kl} - 2\right) = kl^2\left(\frac{mg}{2kl} - 3\right)$$

STAB. se $mg > 6kl$

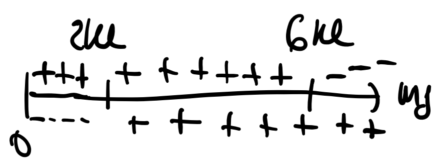
$$V''(\theta^*) = kl^2\left[1 - 2\left(2 - \frac{mg}{2kl}\right)^2 + \left(2 - \frac{mg}{2kl}\right)^2\right]$$

$\operatorname{sen}\theta^* = 2 - \frac{mg}{2kl}$

$$= kl^2\left[1 - \left(2 - \frac{mg}{2kl}\right)^2\right] =$$

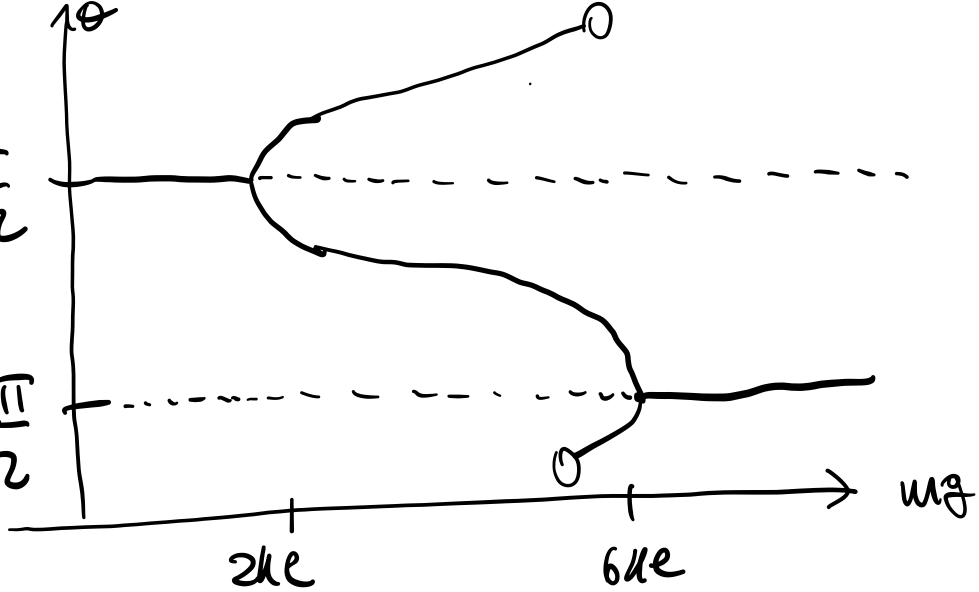
$$= kl^2\left[\left(1 + 2 - \frac{mg}{2kl}\right)\left(1 - 2 + \frac{mg}{2kl}\right)\right] =$$

$$= kl^2\left(3 - \frac{mg}{2kl}\right)\left(\frac{mg}{2kl} - 1\right)$$



STAB per $2kl < mg < 6kl$ cioè quando $\exists$.

5.



6.

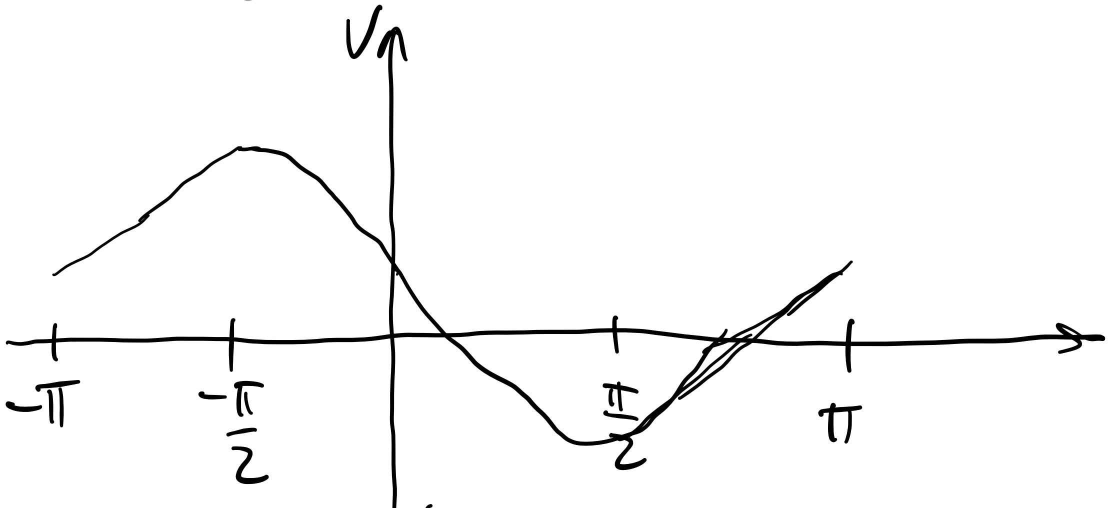
Freq. piccole oscillazioni $\omega^2 = B/A$

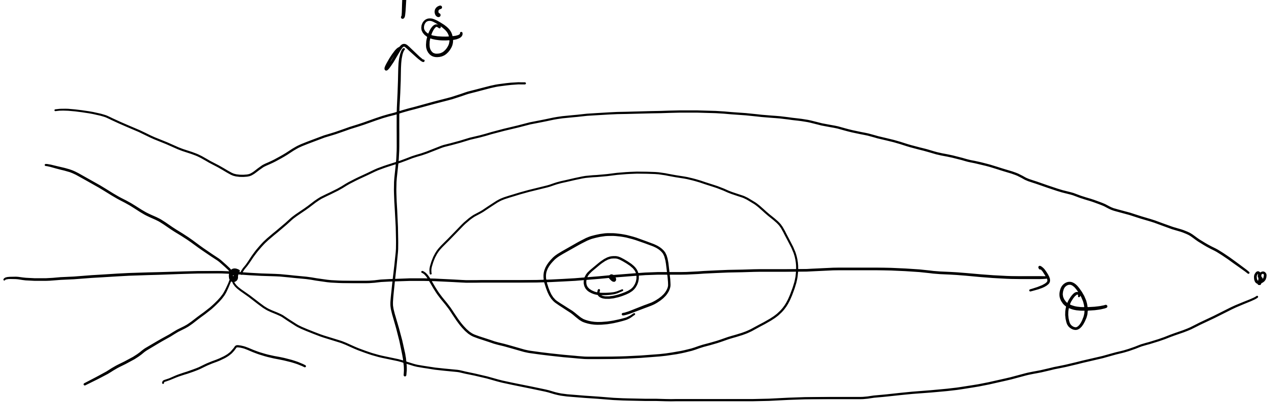
$$B = V''\left(\frac{\pi}{2}\right) = \frac{mgl}{2}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3}{2}\frac{g}{l}}$$

$$A = \frac{ml^2}{3}$$

7.





8. $V = V_{in}\big|_{k=0} + F x_B = V_{in}\big|_{k=0} + F\ell\cos\theta =$

$$= mg\frac{\ell}{2}\,\mathrm{sen}\,\theta + F\ell\cos\theta$$

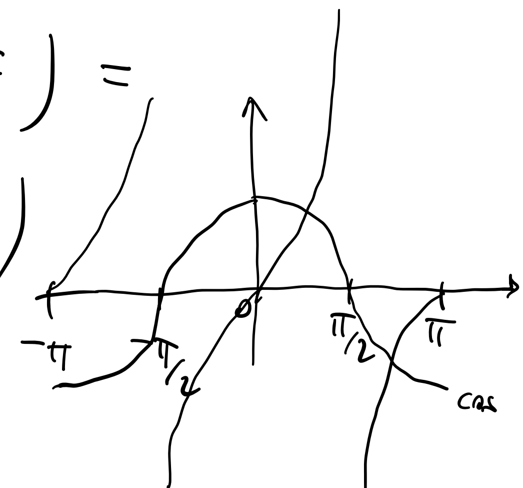
$$V' = mg\frac{\ell}{2}\cos\theta - F\ell\,\mathrm{sen}\,\theta = 0$$

$$\mathrm{tg}\,\theta = \frac{mg}{2F} \rightarrow 2 \text{ soluz.}$$

$$V'' = -mg\frac{\ell}{2}\,\mathrm{sen}\,\theta - F\ell\cos\theta =$$

$$= -\ell\cos\theta\left(\frac{mg}{2}\,\mathrm{tg}\,\theta - F\right) =$$

$$= -\ell F\cos\theta\left(\left(\frac{mg}{2F}\right)^2 - 1\right)$$



$\downarrow$ una sol. è posta a $\cos\theta > 0$ e una a $\cos\theta < 0$

da cui la stabilità di una sola delle due.

ES 3)

$$E_n^\theta = \frac{\hbar^2}{2mR^2}\left(n - \frac{\theta}{2\pi}\right)^2 \qquad n \in \mathbb{Z}$$

3) Sì, poiché $\dfrac{e^{in\varphi}}{\sqrt{2\pi}} \in L^2(\underset{\substack{\uparrow \\ \text{cerchio}}}{S^1})$ $(\neq \mathbb{R})$

4) $$\psi(\varphi) = \frac{1}{\sqrt{\pi}} \operatorname{sen}\varphi = \frac{1}{\sqrt{2}\, i}\left(\underset{\substack{\updownarrow \\ n=1}}{\frac{e^{i\varphi}}{\sqrt{2\pi}}} - \underset{\substack{\updownarrow \\ n=-1}}{\frac{e^{-i\varphi}}{\sqrt{2\pi}}}\right)$$

$$E_{n=1}^\theta = \frac{\hbar^2}{2mR^2}\left(1 - \frac{\theta}{2\pi}\right)^2$$

$$E_{n=-1}^\theta = \frac{\hbar^2}{2mR^2}\left(1 + \frac{\theta}{2\pi}\right)^2$$

$$\psi(\varphi, t) = \frac{1}{\sqrt{2}\, i}\left(\frac{e^{-iE_{n=1}^\theta t/\hbar}\, e^{i\varphi}}{\sqrt{2\pi}} - \frac{e^{-iE_{n=-1}^\theta t/\hbar}}{\sqrt{2\pi}}\right)$$

5) Stato fondamentale in $\theta = 0 \Longleftrightarrow n = 0$

$$\psi_0(\varphi) = \frac{1}{\sqrt{2\pi}} \quad \text{(normalizzato)}$$

$$\langle \tan\varphi \rangle_{\psi_0} = \int_0^{2\pi} |\psi_0(\varphi)|^2 \tan\varphi \, d\varphi =$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \tan\varphi \, d\varphi = 0$$