THE PHYSICS OF CLUSTER MERGERS

(SARAZIN in ADS) in MERGING PROCESSES
2002

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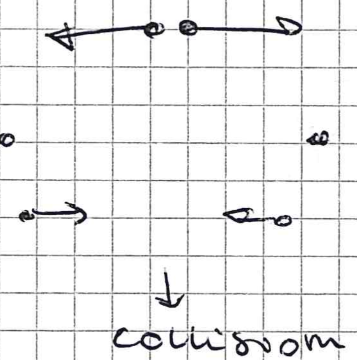
CAP. 2.2

From bimodal model (or Kepler's Third Law)

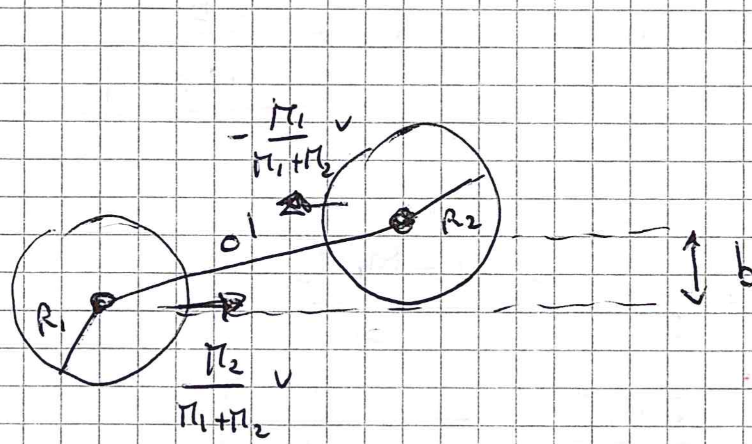


max distance
greatest separation
$d_0 \to t_{max}$
$t_{merge} = 2\, t_{max}$

$$d_0 \sim \left[2G(M_1+M_2)\right]^{1/3} \left(\frac{t_{merge}}{\pi}\right)^{2/3} \sim$$

$$\sim 4.5 \left(\frac{M_1+M_2}{10^{15} M_\odot}\right)^{1/3} \left(\frac{t_{merge}}{10^{10}\,yr}\right)^{2/3} Mpc$$

More complex
From head-on collision



the relative velocity is v

2.2.2 MERGER VELOCITIES

At d_0 zero relative velocity

v_0 is the transverse velocity

eq. A

$$\boxed{v_0 \sim \frac{J_{orb}}{\mu d_0}}$$

orbital angular momentum $\quad J_{orb} \sim \mu v_0 d_0$

orbital energy $\quad E_{orb} \sim \frac{1}{2}\mu v_0^2 - \frac{G M_1 M_2}{d_0} \quad$ (eq. 1.1)

Their reduced mass

$$\mu = \frac{M_1 M_2}{M_1 + M_2} \leftarrow \frac{1}{\frac{1}{M_1}+\frac{1}{M_2}}$$

At separation d

$$J_{orb} \sim \mu v b$$

$$E_{orb} \sim \frac{1}{2}\mu v^2 - \frac{G M_1 M_2}{d}$$

Conserving J_{orb}

eq. B

$$\cancel{\mu} v_0 d_0 = \cancel{\mu} v b \qquad \boxed{v_0 = \frac{v b}{d_0}} \qquad \boxed{b = \frac{v_0}{v} d_0}$$

Conserving energy

$$\frac{1}{2}\mu v_0^2 - \frac{G M_1 M_2}{d_0} = \frac{1}{2}\mu v^2 - \frac{G M_1 M_2}{d}$$

$$\frac{1}{2}\mu v^2 - \frac{1}{2}\mu v_0^2 = \frac{G M_1 M_2}{\mu}\left(\frac{1}{d} - \frac{1}{d_0}\right)$$

$$\frac{1}{2}\mu v^2\left(1 - \left(\frac{b}{d_0}\right)^2\right) = \ldots$$

$$V^2 \simeq \frac{2GM_1M_2}{\frac{M_1M_2}{M_1+M_2}}\left(\frac{1}{d}-\frac{1}{d_0}\right)\left[1-\left(\frac{b}{d_0}\right)^2\right]^{-1}$$

$$V \sim \underline{2930}\left(\frac{M_1+M_2}{10^{15}M_\odot}\right)^{1/2}\left(\frac{d}{1\,\mathrm{Mpc}}\right)^{-1/2}\underbrace{\left[\frac{1-\frac{d}{d_0}}{1-\frac{b}{d_0}}\right]^{1/2}}_{\text{circa } \frac{1}{1}} \ \underline{\mathrm{km/s}} \qquad (15)$$

typical velocity of collision

$\cancel{V_{1D} = \sqrt{3}\,V}$ $\qquad V_{1D} = \frac{1}{\sqrt{3}}V$

yes! obs. $V \sim 2000 - 3000$ km/s max

2.2.3 IMPACT PARAMETER

SPIN PARAMETER (COSMOLOGY)

$$\lambda \equiv \frac{J\,|E|^{1/2}}{G\,M^{5/2}}$$

J = Total angular momentum

E = Total energy

of the HALO

linear theory

$\Longrightarrow \lambda \sim 0.05$

VIRIAL THEOREM $\quad 2T+U=0 \quad U=-2T \quad E=T+U=-T$

$|E| \propto M\sigma_v^2 \qquad M \propto \sigma_v^2 R \propto \sigma_v^3$

$|E| \propto M \cdot M^{2/3} \propto M^{5/3} \qquad |E| \propto M^{5/3}$ for a virialized halo

$\underline{E = -a\,M^{5/3}}$

$$J \propto \frac{\lambda G M^{5/2}}{|E|^{1/2}} \sim \frac{\lambda G M^{5/2}}{a^{1/2}M^{5/6}} \sim \underline{\frac{\lambda G}{a^{1/2}}M^{5/3}}$$

Take the halo the final merged cluster

$$J_{TOT} = J_1 + J_2 + J_{orb}$$

$$J_{orb} = J_{TOT} - J_1 - J_2 = \frac{\lambda G}{a^{1/2}}\left[(M_1+M_2)^{5/3} - M_1^{5/3} - M_2^{5/3}\right] =$$

$$= \frac{\lambda G}{a^{1/2}}(M_1+M_2)^{5/3}\left[1 - \frac{M_1^{5/3}+M_2^{5/3}}{(M_1+M_2)^{5/3}}\right]$$

$$E_{TOT} = E_1 + E_1 + E_{orb}$$

$$E_{orb} = E_{TOT} - E_1 - E_2$$

eq. 17

$$\frac{1}{2}\frac{M_1M_2}{M_1+M_2}v_0^2 - \frac{GM_1M_2}{d_0} = -a\left[(M_1+M_2)^{5/3} - M_1^{5/3} - M_2^{5/3}\right]$$

$$a = \frac{-(\;)}{(M_1+M_2)^{5/3}\left[1 - \frac{M_1^{5/3}+M_2^{5/3}}{(M_1+M_2)^{5/3}}\right]}$$

$$J_{orb} = \frac{\lambda G\,(M_1+M_2)^{5/3\;+\frac{5}{6}}}{\left(\frac{G(M_1+M_2)}{d_0} - \frac{1}{2}v_0^2\right)^{1/2}\left(\frac{M_1M_2}{M_1+M_2}\right)^{1/2}}\left[1 - \frac{M_1^{5/3}+M_2^{5/3}}{(M_1+M_2)^{5/3}}\right]^{1\;+\frac{1}{2}}$$

$$J_{orb} = \frac{\lambda G\, M_1M_2}{(\quad)^{1/2}}\underbrace{\frac{(M_1+M_2)^{3/2}}{M_1^{3/2}M_2^{3/2}}\left[\quad\right]^{3/2}}_{f(M_1,M_2)}$$

eq. 18 e 19

$\frac{v_0^2}{2}$ is $2\lambda^2 \sim 1\%$ of the potential energy

$1.8 < f < 2,15$

$f \sim 2$

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$$J_{orb} \sim \lambda M_1 M_2 \sqrt{\frac{G d_0}{M_1+M_2}} \quad f(M_1, M_2)$$

eq. A

$$\hookrightarrow v_0 \sim \frac{J_{orb}}{m\, d_0} \sim \lambda \sqrt{\frac{G(M_1+M_2)}{d_0}} \quad f(M_1, M_2)$$

$$V_0 \sim \underline{93} \left(\frac{\lambda}{0.05}\right)\left(\frac{M_1+M_2}{10^{15} M_\odot}\right)^{1/2} \left(\frac{d_0}{5 M_{pc}}\right)^{-1/2} \frac{f}{2} \quad \underline{km/s}$$

obtained before

ed. B

$$\hookrightarrow b \simeq \frac{v_0}{V_0} d_0 \Rightarrow b \ll d_0 \Rightarrow \frac{b}{d_0} \sim \emptyset \quad \text{in eq. (15)}$$

$$\underline{b} \sim \lambda \sqrt{\frac{d_0 d}{2}} \left(1 - \frac{d}{d_0}\right)^{-1/2} f(M_1, M_2)$$

$$\sim \underline{160} \left(\frac{\lambda}{0.05}\right)\left(\frac{d}{1 M_{pc}}\right)^{1/2} \left(\frac{d_0}{5 M_{pc}}\right)^{1/2} \underbrace{\left(1 - \frac{d}{d_0}\right)^{-1/2}}_{\sim 1,\ d \ll d_0} \left(\frac{f}{2}\right) \underline{Kpc}$$

$\hookrightarrow$ small impact parameter

$\Rightarrow$ collisions are head on

3. THERMAL PHYSICS (ICM)

CLUSTER MERGERS WILL DRIVE SHOCK WAVE

V_s velocity of the shock

sound speed of the preshock gas is $C_s = \sqrt{\frac{5}{3}\frac{P}{\rho}}$

Mach number of the shock $\mathcal{M} \equiv \frac{V_s}{C_s}$

simulations $\rightarrow$ $\mathcal{M} \lesssim 3$

ok with obs.

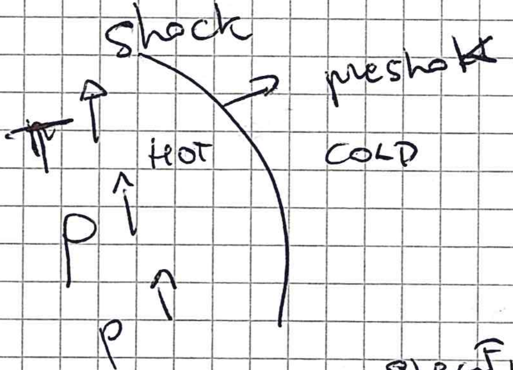
$V_s \sim V_{merger}$

$C_s \sim \sigma_v$ in cluster

$\mathcal{M} \sim \frac{V_{merger}}{\sigma_v} \sim \frac{2\text{-}3 \cdot 10^3}{10^3} \rightarrow 2-3$

obs. ok

MERGER



$S\uparrow$

specific entropy

$$s = \frac{3}{2} k \ln\left(\frac{P}{\rho^{5/3}}\right) = \frac{3}{2} k \ln\left(\frac{T}{\rho^{2/3}}\right)$$

$$ds = \frac{dQ}{T} = n C_v \frac{dT}{T} + p\frac{dV}{T} \dots \qquad C_v = \frac{3}{2}R$$

$$p \propto T V^{-1} \propto T\rho$$

COLD FRONT

cool core is moving ?