ES 1)

5)

Repulsore armonico

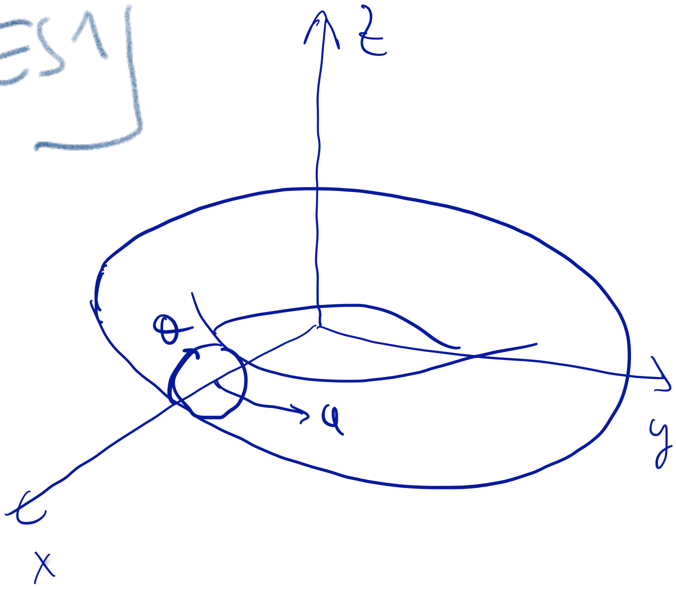
$$S = \int_{t_1}^{t_2} \left(\frac{m}{2}\dot{x}^2 + \frac{m}{2}\omega^2 x^2 \right) dt$$

$$\delta S = \int_{t_1}^{t_2} \left(m\dot{x}\,\delta\dot{x} + m\omega^2 x\,\delta x \right) dt =$$

$$= \int_{t_1}^{t_2} \left(-m\ddot{x} + m\omega^2 x \right) \delta x \; dt = 0 \quad \forall \delta x$$

se $x(t) = e^{\omega t}$

ES 1



$x = (R + r\cos\theta)\cos\varphi$

$y = (R + r\cos\theta)\,\text{sen}\,\varphi$

$z = r\,\text{sen}\,\theta$

$x^2+y^2 = (R+r\cos\theta)^2$

$R > r$

$\dot{x} = -r\,\text{sen}\theta\cos\varphi\,\dot\theta - (R+r\cos\theta)\,\text{sen}\varphi\,\dot\varphi$

$\dot{y} = -r\,\text{sen}\theta\,\text{sen}\varphi\,\dot\theta + (R+r\cos\theta)\cos\varphi\,\dot\varphi$

$\dot{z} = r\cos\theta\,\dot\theta$

$$T = \frac{m}{2}\left(\dot x^2 + \dot y^2 + \dot z^2\right) = \frac{m}{2}\left(r^2\dot\theta^2 + (R+r\cos\theta)^2\dot\varphi^2\right)$$

$$V = \frac{K}{2}(R+r\cos\theta)^2$$

$$a = \begin{pmatrix} mr^2 & 0 \\ 0 & m(R+r\cos\theta)^2 \end{pmatrix}$$

1) $L = T - V \leftarrow$ Non dipende da φ.

2) $$\frac{d}{dt}\frac{\partial L}{\partial\dot\varphi} = \frac{d}{dt}\left(m(R+r\cos\theta)^2\dot\varphi\right) = -2m(R+r\cos\theta)\,r\,\text{sen}\theta\,\dot\theta\,\dot\varphi + m(R+r\cos\theta)^2\ddot\varphi$$

eq. Lag: $$\ddot\varphi = \frac{2r\,\text{sen}\theta}{R+r\cos\theta}\,\dot\theta\dot\varphi$$

3) φ ciclica

$$p_\varphi = \frac{\partial L}{\partial\dot\varphi} = m(R+r\cos\theta)^2\dot\varphi = \ell \rightarrow \dot\varphi = \frac{\ell}{m(R+r\cos\theta)^2}$$

cost. del moto

4)
$$L^* = \frac{m}{2}r^2\dot\theta^2 - \frac{\ell^2}{2m(R+r\cos\theta)^2} - \frac{K}{2}(R+r\cos\theta)^2$$

$$V_{eff} = \frac{\ell^2}{2m(R+r\cos\theta)^2} + \frac{K}{2}(R+r\cos\theta)^2$$

5)

$$V'_{eff} = +\frac{\ell^2}{m}\frac{r\,\mathrm{sen}\theta}{(R+r\cos\theta)^3} - k(R+r\cos\theta)\,\mathrm{sen}\theta = 0$$

$$\frac{\ell^2}{m}\frac{r\,\mathrm{sen}\theta}{(R+r\cos\theta)^3}\left[1 - \frac{km}{r\ell^2}(R+r\cos\theta)^4\right]$$

$$\theta = 0, \pi$$

e $$R + r\cos\theta = \left(\frac{r\ell^2}{km}\right)^{1/4}$$

$$\hookrightarrow \cos\theta^* = \left(\frac{\ell^2}{kmr^3}\right)^{1/4} - \frac{R}{r}$$

$\exists$ se $\uparrow$

$$(R-r)^4 < \frac{r\ell^2}{km} < (R+r)^4$$

$$V''_{eff} = +\frac{3\ell^2}{m}\frac{r^2\,\mathrm{sen}^2\theta}{(R+r\cos\theta)^4} + \frac{\ell^2}{m}\frac{r\cos\theta}{(R+r\cos\theta)^3} - k(R+r\cos\theta)\cos\theta + kr\,\mathrm{sen}^2\theta$$

$$V''_{eff}(0) = \frac{\ell^2}{m}\frac{r}{(R+r)^3} - k(R+r)$$

$\hookrightarrow$ stab. se $\frac{r\ell^2}{km} > (R+r)^4$

$$V''_{eff}(\pi) = -\frac{\ell^2}{m}\frac{r}{(R-r)^3} + k(R-r)$$

$\hookrightarrow$ stab. se $\frac{r\ell^2}{km} < (R-r)^4$

$$V''_{eff}(\theta^*) = \frac{3\ell^2}{m} r^2(1-\cos^2\theta^*)\cdot\frac{km}{r\ell^2} + \frac{r\ell^2}{m}\cos\theta^*\left(\frac{km}{r\ell^2}\right)^{3/4} = \ldots$$

θ^* è STABILE perché V_{eff} è continua e $\theta = 0, \pi$ sono INSTAB. quando $\pm\theta^*$ esistono

6) $k=0 \rightarrow \theta^*$ non esiste come solut. e $\theta=0$ è STAB.

$$\omega^2 = \frac{V''_{eff}(0)}{mr^2} = \frac{\ell^2}{m^2r^2}\frac{r}{(r+R)^3}$$

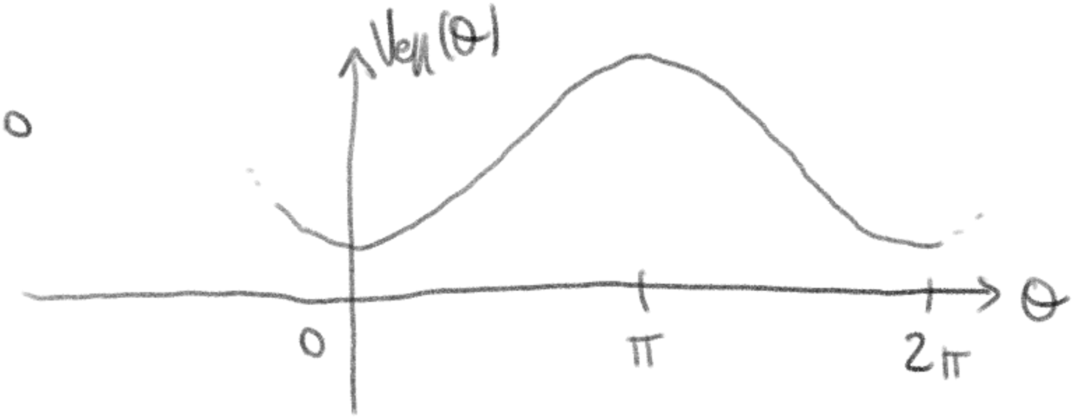
$$L^* = \frac{m}{2}r^2\dot\theta^2 - \frac{\ell^2}{2m(R+r\cos\theta)^2} \qquad \theta = 0 + \delta\theta$$

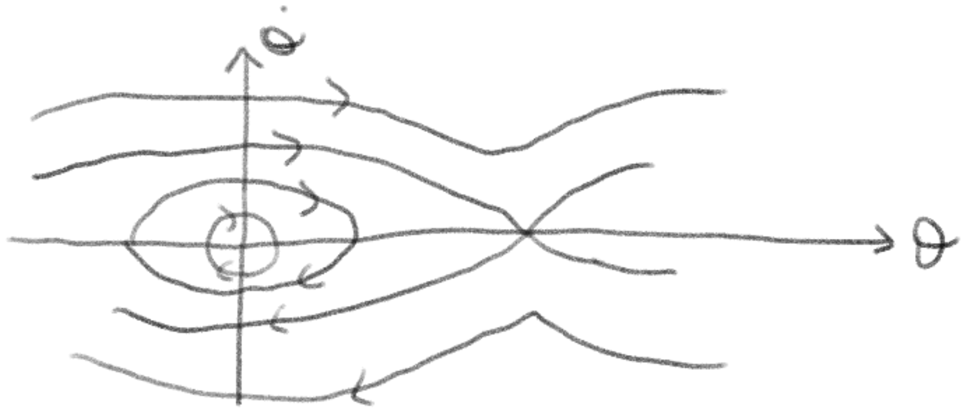
$$\simeq \frac{mr^2}{2}\delta\dot\theta^2 - \frac{\ell^2}{2m(R+r)^2\left(1-\frac{r\delta\theta^2}{2(R+r)}\right)^2} \simeq$$

$$\simeq \frac{mr^2}{2}\delta\dot\theta^2 - \frac{\ell^2}{2m(R+r)^2}\left(\cancel{1} + \frac{r\delta\theta^2}{R+r}\right)$$

$$= \frac{mr^2}{2}\delta\dot\theta^2 - \frac{1}{2}mr^2\omega^2\delta\theta^2$$

7) $k=0$





Es. 3 $\quad \varphi_0(q) = a_0 e^{-q^2/2}$

$$A\varphi_0 = \frac{a_0}{\sqrt{2}}\left(q + \frac{d}{dq}\right)e^{-q^2/2} = \frac{a_0}{\sqrt{2}}(q - q)e^{-q^2/2} = 0$$

$$A^+A\,\varphi(q) = \frac{1}{2}\left(q - \frac{d}{dq}\right)\left(q + \frac{d}{dq}\right)\varphi(q) =$$

$$= \frac{1}{2}\left(q - \frac{d}{dq}\right)\left(q\varphi + \varphi'\right) =$$

$$= \frac{1}{2}\left(q^2\varphi + \cancel{q\varphi'} - \cancel{q\varphi'} - \varphi - \varphi''\right) =$$

$$= \frac{1}{2}\left(-\varphi'' + q^2\varphi\right) - \frac{\varphi}{2} = \frac{1}{\hbar\omega}\hat{H} - \frac{1}{2}\mathbb{1}$$

$$H\psi = -\frac{\hbar^2}{2m}\psi'' + \frac{1}{2}m\omega^2 x^2\psi = -\frac{\hbar^2}{2m}\frac{m\omega}{\hbar}\varphi'' + \frac{1}{2}m\omega^2\frac{\hbar}{m\omega}q^2\varphi =$$

$$= \hbar\omega\left(-\frac{\varphi''}{2} + q^2\varphi\right)$$

$$\Rightarrow \quad A^+A\,\psi_1 = \left(\frac{1}{\hbar\omega}\hat{H} - \frac{1}{2}\mathbb{1}\right)\psi_1 = \left(\frac{1}{\hbar\omega}\hbar\omega\left(1 + \frac{1}{2}\right) - \frac{1}{2}\right)\psi_1$$

$$= \psi_1$$