

Physical and Thermal Dynamics of Bodies of Magma

FUNDAMENTAL QUESTIONS CONSIDERED IN THIS CHAPTER

1. How do magmas and rocks respond to geologic forces, formulated as states of stress?
2. What physical and compositional properties of bodies of magma are most important in controlling their dynamic behavior?
3. How do dynamic physical processes and heat transfer interact in magmatic systems?
4. How do these physical-thermal interactions influence the compositional evolution of magmas?

INTRODUCTION

Chapters 6 and 7 dealt mostly with atomic-scale chemical dynamics and the resulting kinetic paths in solidifying melts that control the evolution of magmatic fabric. This chapter and the next two deal with larger-scale physical and thermal dynamics of bodies of magma, including their response to states of stress, and the ways they flow, cool, and convect. The basic driving force of magma movement in the Earth is buoyancy. Acting against the buoyant force is viscous resistance to flow. As viscosity is partly dependent on magma temperature, the thermal evolution of the magma system becomes important.

Crystallizing and convectively cooling magma bodies evolve through time, unmixing into compositionally contrasting parts by fractionation of crystals and melt. Mixing of magmas of contrasting composition depends on their contrasting densities and viscosities. Magmas are contaminated by assimilation of wall rock if suffi-

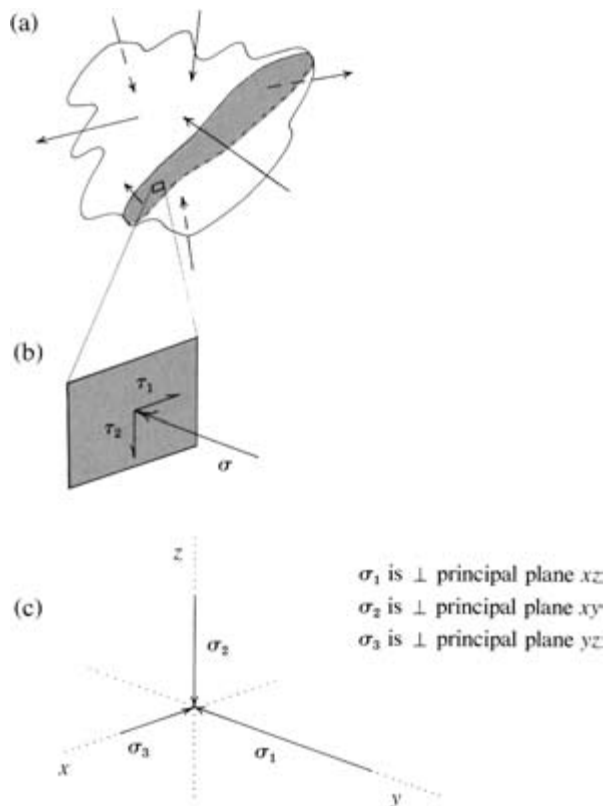
cient thermal energy and time are available. Hence, chemical differentiation of magmas is intimately intertwined with their dynamic physical and thermal behavior.

*8.1 STRESS AND DEFORMATION

8.1.1 Concepts of Stress

In buried rock and magma bodies, forces arise as a result of the weight of overlying rock material and of tectonism, mostly associated with lithospheric plates pushing together or a plate being stretched apart. **Stress** is the magnitude of a force, divided by the area over which it is applied. Hence, both stress and pressure have the same units of pascals or bars. When the same force acts over a smaller area, a greater stress or pressure results.

Suppose a body is acted upon by forces (Figure 8.1) so that it is in a **state of stress**. On any arbitrarily oriented planar surface through the body, there will be, in the general case, one **normal stress**, σ , perpendicular to the plane and two **shear stresses**, τ_1 and τ_2 , perpendicular to one another and tangential (parallel) to the plane. These three orthogonal (mutually perpendicular) stresses constitute the components of the total stress on the plane. For a particular state of stress, the magnitude of these three surface stresses varies with respect to the orientation of the plane through the body. The general state of total stress in the body at a point, which may be considered to be an infinitesimally small cube, is specified by these three surface stresses on each of three orthogonal planes, parallel to the faces of the cube, or nine stresses altogether. However, it can be proved mathematically that it is always possible to orient a coordinate system so that no shear stresses exist



8.1 State of stress in a three-dimensional body. Force vectors (arrows) acting on the body create the state of stress. On an arbitrarily oriented plane (shaded) through the body, forces acting on the area of the plane can be resolved into mutually perpendicular components of **normal**, σ , and **shear stresses**, τ_1 and τ_2 . The normal stress, σ , is perpendicular to the plane, and the two shear stresses, τ_1 and τ_2 , are parallel to the plane.

on the three orthogonal planes parallel to the faces of the cube, reducing the number of stresses needed to define the total state of stress to just three normal stresses. These *three normal principal stresses therefore represent the total state of stress at a point in the body* and are designated with subscripts according to their magnitude, $\sigma_1 > \sigma_2 > \sigma_3$. The specially oriented orthogonal planes on which only the principal stresses, and no shear stresses, prevail are called **principal planes**. Because shear stresses cannot be sustained on the interface between the solid Earth and its atmosphere, this more or less horizontal interface is a principal plane and the other two principal planes perpendicular to it must therefore be vertical. Hence, two principal stresses near the surface of the Earth are commonly considered to be horizontal and the third vertical.

Within the Earth, **compressive** states of stress press material together. **Tensile** stresses act in opposite directions to pull material apart.

A special state of total stress in which principal stresses are of equal magnitude in all directions, $\sigma_1 =$

$\sigma_2 = \sigma_3$, is referred to as a **hydrostatic state of stress**, because such states occur in bodies of water or other liquids. This hydrostatic stress, also called **pressure**, is approached at increasing depths in the Earth where rocks have diminishing strength and flow in the same way as viscous liquids, especially on long geologic time scales. This assumption is implicit in the formulation of **confining pressure**, P (Section 1.2), which is due to gravitational loading, which is referred to throughout this textbook as one of the three intensive thermodynamic variables that defines states of equilibrium in rock systems. This assumption was also made in calculating the geobaric gradient (equation 1.8).

Despite the obvious utility of the assumption of hydrostatic states of stress in rock systems, a nonhydrostatic state of stress is more typical and its effects on the dynamics of magma bodies are one focus of this chapter. In a **nonhydrostatic state of stress** the principal stresses are unequal, $\sigma_1 > \sigma_2 > \sigma_3$.

8.1.2 Deformation

Deformation is the way a body responds to applied stress. The character of the deformation depends on the state of stress and the properties of the body. Individual particles, such as mineral grains, within a stressed rock body are displaced or changed in shape or size until a new state of more stable equilibrium between the imposed forces and the body is attained, according to Le Chatelier's principle. The body, or parts of it, in its final deformed state has experienced one or more of the following components of deformation:

1. Translation (movement, or displacement) from one place to another with respect to Earth coordinates (e.g., in a thrust sheet)
2. Rotation, as in the limbs of a fold
3. Distortion, or **strain**, which can consist of a change in shape and/or volume

A nonhydrostatic state of stress can produce all of these components in folding, faulting, and other types of deformation of rocks in the Earth. On the other hand, a hydrostatic state of stress can generally only produce a change in volume; for example, a magnetite grain is compressed into a smaller grain of the same shape.

In rocks, strain can only be measured with respect to a reference marker whose shape and/or size is known in the initial, undeformed state. Reliable markers in rocks are difficult to find; a more or less ellipsoidal basalt pillow (Figure 7.3a) is a possible marker, but its exact shape and size are uncertain. Strain, ϵ , is measured in terms of the change in some dimensional or angular aspect of the final deformed body relative to the initial undeformed body. Examples of measures of strain include volumetric strain

$$8.1 \quad \varepsilon_v = \frac{(V - V_0)}{V_0} = \frac{\Delta V}{V_0}$$

and linear strain, either an elongation or a flattening,

$$8.2 \quad \varepsilon_l = \frac{(l - l_0)}{l_0} = \frac{\Delta l}{l_0}$$

where V_0 and l_0 represent the initial undeformed state and V and l the final deformed state. Note that strain is a pure number without units, such as 0.45, or, as a percentage change, 45%.

In Figure 8.2, a cubic volume of isotropic rock, which has uniform properties in all directions, is subjected to a nonhydrostatic state of stress, $\sigma_1 > \sigma_2 > \sigma_3$. Consequently, it is flattened perpendicular to σ_1 (the maximal compressive principal stress) and elongated parallel to σ_3 (the least compressive principal stress), which is the direction of least work of deformation. Note that a nonhydrostatically stressed body extends in the directions of least and perhaps intermediate principal stress but not in the direction of the maximal compressive principal stress, σ_1 , as that would require the maximum of work to be done on the surroundings. Nature, being lazy and parsimonious, prefers the least-work alternative.

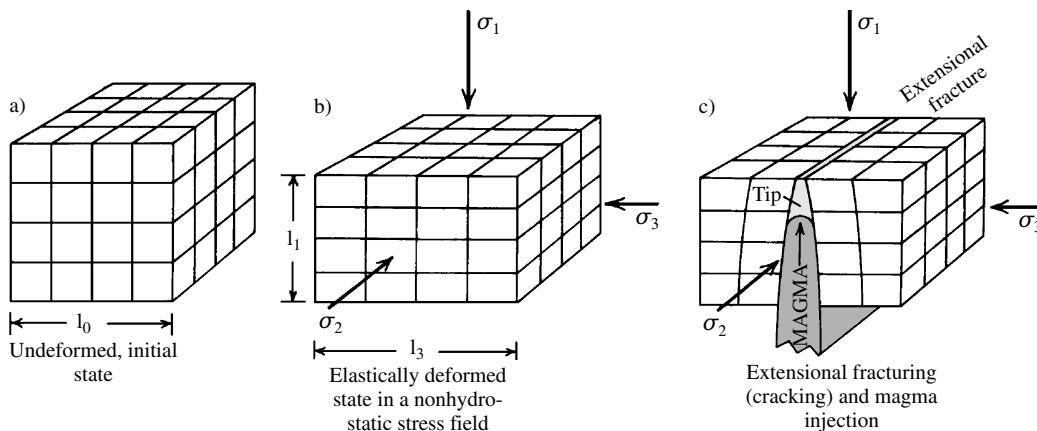
8.1.3 Ideal Response to Stress

Stress and strain are mathematical concepts that apply to any material. The response of *real* rocks and magmas to applied stress can be complex and depends upon

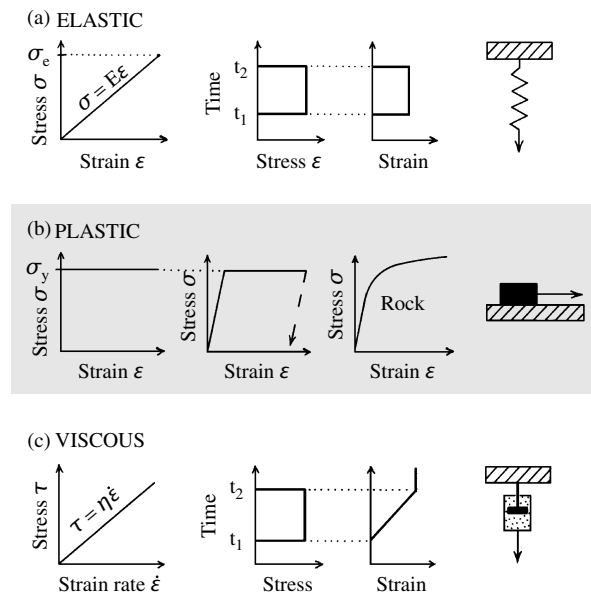
many factors, as will be discussed in the next section. But before dealing with real behavior we first consider *ideal*, or “end-member” elastic, plastic, and viscous behavior, which have simple relations between applied stress and resulting strain.

Elastic behavior is the only ideal response to applied stress that is recoverable, or reversible; the deformed material returns instantaneously to its initial undeformed state when the stress is eliminated (Figure 8.3a). Rubber bands, springs, sheets of window glass, and rocks under certain conditions behave elastically. There is a linear, direct proportionality (Hooke’s law) between stress and elastic strain, $\sigma = E\varepsilon$, where E , Young’s modulus, is a property of the material. The value of E is the slope of the stress-strain line in Figure 8.3a. As the applied stress increases, the material reaches its **elastic strength**, σ_e , and the body ruptures or breaks, losing cohesion and causing permanent deformation. If a pane of window glass is subjected to a small applied stress, it bends slightly and reversibly, but if it is stressed more to exceed its elastic strength, it fractures into pieces.

Elastic strains of glass, minerals, and rocks are much smaller than stretching of a rubber band. Small elastic volumetric strains in geologic solids are a consequence of their very small elastic compressibility. This property, formally defined in Section 8.3.1, is a measure of how much a material body compacts under pressure; even large pressure changes yield only small changes in volume in most rocks, on the order of 1%.



8.2 Deformation of a body under a nonhydrostatic state of stress, $\sigma_1 > \sigma_2 > \sigma_3$. (a) Undeformed body represented by a cubical stack of smaller cubes. (b) Principal stresses of unequal magnitude cause a change in shape—a **strain**. Actual recoverable elastic strain produces only a small percentage of strain, far less than is represented here for illustrative purposes. However, the relative amounts of strain are as expected, that is, flattening perpendicular to the maximal principal compressive stress, σ_1 , and elongation, or extension, parallel to the least principal compressive stress, σ_3 . Strain parallel to σ_2 can be either elongation or flattening or nil. (c) Nonhydrostatic stresses may exceed the brittle (essentially elastic) strength of a rock so that an **extensional fracture** forms perpendicular to the least principal compressive stress, σ_3 . These open cracks may be subsequently filled with magma or fluid. The pressure exerted by the magma or fluid itself may be sufficient to create an extensional fracture by **hydraulic fracturing**. Just beyond the injecting fluid or magma that wedges apart the walls of rock is a tip cavity where transient low pressure can suck out volatiles dissolved in the magma or pore fluids lodged in the wall rock.



- 8.3** Definitions and comparisons of three ideal types of response to applied stress. For each type of ideal behavior, the relation between stress and strain is shown in the left-hand diagram and a mechanical analog is shown in the diagram on the right; the middle diagrams show special attributes. (a) **Elastic behavior** in which stress is linearly proportional to strain. If the stress exceeds the elastic limit, or **elastic strength**, σ_e , the material breaks permanently. Another attribute of elastic behavior is that strain is instantaneous and nonpermanent (reversible). This is shown in the middle diagrams, where a particular stress, σ , is applied between time t_1 and t_2 resulting in an immediate accompanying strain whose magnitude is σ/E . After t_2 when the stress is eliminated there is no longer any strain. Elastic behavior is modeled by a spring. (b) **Plastic behavior** in which no strain occurs until the applied stress reaches the **yield strength**, σ_y , after which permanent strain can accumulate indefinitely. The mechanical model of plastic deformation is a block resting on a table. An applied force cannot move (deform) the block until the frictional resistance (the yield strength) is overcome. Before the stress reaches the yield strength in a material, some amount of reversible elastic strain may result (second diagram); in this case, removal of the stress after an excursion through elastic and then plastic behavior is tracked by the dashed line. A typical response of a stressed rock (third diagram) shows initial elasticlike behavior grading continuously into more plasticlike behavior. The yield strength can only be approximated and the plastic-like region shows “strain hardening” in which increasing stress is required to induce more strain. (c) **Newtonian viscous behavior** in which the time rate of shear strain, $de/dt = \dot{\epsilon}$, and shear stress, τ , are linearly proportional. The proportionality constant is the coefficient of viscosity, η . Any applied shear stress between time t_1 and t_2 results in an immediate, permanent strain that increases until the stress is removed. After t_2 when the stress is eliminated the strain that had accumulated at time t_2 remains permanently. A mechanical model of viscous response is the dashpot—a cylinder containing a viscous liquid, such as honey, in which moves a loose-fitting piston.

In contrast to elastic behavior, ideal **plastic** response involves a nonrecoverable, or irreversible, strain occurring at a stress equal to a critical value known as the **yield strength**, σ_y (Figure 8.3b). For applied stresses less than the yield stress, only reversible elastic strain

occurs. Under some conditions, real minerals and rocks behave more or less plastically, but the stress-strain response is continuously curved and a yield stress can only be approximately determined.

The ideal **viscous** response to applied stress was discussed in Section 6.1 (see also Figure 6.1a) with reference to bodies of melt. Unlike elastic and plastic bodies, a viscous body has no strength and deforms permanently by flow under the smallest applied shear stress. This property is typical of liquids. As shear stress is applied, a proportional strain *rate* results (Figure 8.3c). Such linear flow behavior is referred to as **Newtonian viscosity** and the proportionality constant is the viscosity, η .

*8.2 RHEOLOGY OF ROCKS AND MAGMAS

Real rocks and magmas generally deviate, commonly substantially, from the three ideal modes of deformational behavior just discussed. The real deformational behavior of rocks and magmas—their **rheology**—is a combination of the ideally elastic, plastic, and viscous end-member responses that are expressed either simultaneously or under particular conditions. For example, basalt magmas near their liquidus T where few if any crystals exist exhibit Newtonian viscous behavior, whereas with increasing crystallinity the magma rheology becomes non-Newtonian and the magma possesses yield strength; its behavior is a compound of viscous and plastic, or viscoplastic. Accordingly, the applied stress must exceed a critical yield strength in order for permanent viscous flow to occur. For an applied shear stress less than this critical yield strength no permanent deformation occurs. Highly crystalline magmas near their solidus temperatures can fracture more or less elastically as wholly crystalline rock does. Bodies of folded rock layers (Figure 8.4) exemplify compound viscous-elastic behavior. They are viscoelastic. At high T over long periods the compositional layers were folded as if they had low viscosity. Yet if it had been possible to strike the hot body with a hammer when the folds were developing, it would have fractured into pieces. Thus, rocks and magmas have a duality of behavior that differs with circumstances.

Viscoelastic behavior illustrates the importance of the **time rate of strain**, $\Delta\epsilon/\Delta t$, which has units of reciprocal seconds (s^{-1}). A rock subjected to a large strain rate, in which strain accumulates quickly in a small time, such as during a blow with a hammer, results in an elastic response; the rock breaks into pieces. On the other hand, slower strain rates allow a greater amount of permanent but slowly developing viscous flow to occur. The viscoelastic mantle of the Earth transmits elastic seismic waves at speeds measured in kilometers per second (fast strain rate) yet also



8.4 Ductile-flow folds in metamorphosed bedded chert, Sutter Creek, California. This body of rock is mainly quartz and cannot have melted under the crustal conditions of its metamorphism and deformation (Figure 5.1). Hence, folding must reflect low-strength ductile behavior. (Photograph courtesy of L. E. Weiss. Reproduced with permission from Weiss LE. *Minor structures of deformed rocks*. New York: Springer Verlag, 1972. Copyright © 1972 by Springer Verlag.)

convects by viscous flow at rates of centimeters per year (very slow strain rate). Magmas experience strain rates as low as 10^{-8} s^{-1} in slow extrusions of lava to as fast as 30 s^{-1} , in explosive volcanic eruptions.

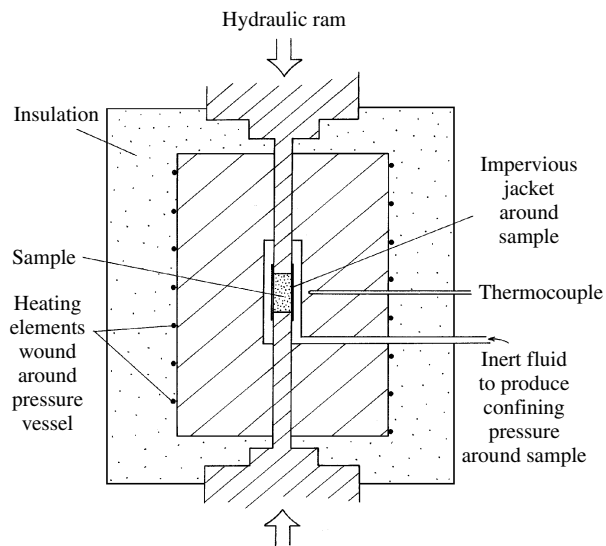
8.2.1 Rheology of Rocks

Brittle Behavior. Rocks near the surface of the Earth subjected to high strain rates where P and T are low respond essentially elastically by fracturing, breaking apart, and losing cohesion when the applied stress reaches the elastic limit; this is **brittle behavior**. Every geologist who has collected a hand sample from a rock outcrop by breaking off a piece with a hammer has witnessed brittle behavior. A more revealing exhibition is to squeeze a small cylinder of rock in a hydraulic testing machine (Figure 8.5). As the axially directed load, in this case the maximum principal compressive stress, σ_1 , is increased under atmospheric pressure ($P = 1 \text{ atm} = \sigma_2 = \sigma_3$), the rock cylinder shortens proportionately in an elastic manner to a maximum of about $<1\%$. Once the load (σ_1) reaches the elastic strength of the rock, the cylinder splits along **extensional fractures** (Figure 8.6a), which are oriented subparallel to σ_1 . These fractures open in the direction of least stress ($\sigma_2 = \sigma_3$) for minimal expenditure of work. Pushing parallel to the pages of this book causes the pages to separate in a manner resembling extensional fracturing.

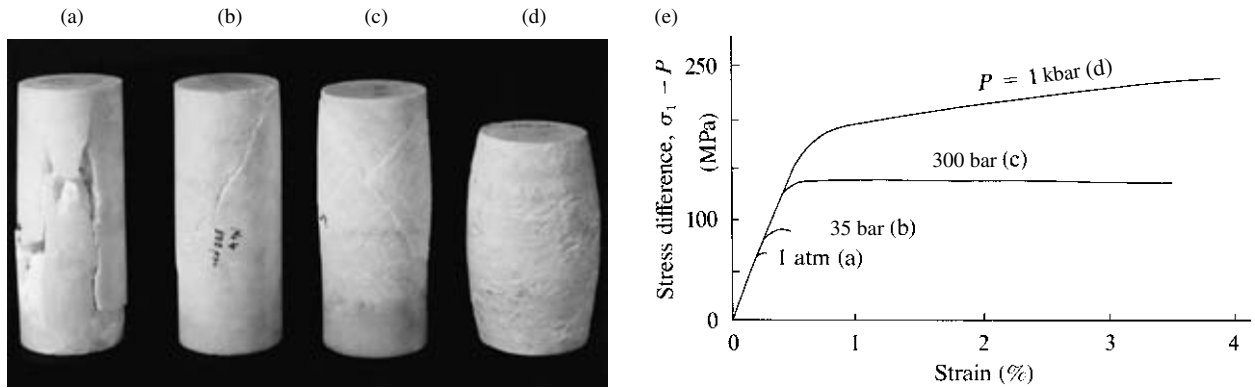
The stress difference required to produce permanent deformation under specified conditions in a body is its **strength**. The brittle strength that was exceeded to produce the permanent deformation manifest in the extensional fractures in Figures 8.2 and 8.6a is equal to the **stress difference** $\sigma_1 - \sigma_3$; it can be as low as 5 MPa

(= 50 bars) for some very weak rocks. It should be realized that open extensional fractures can develop in a volume of rock under a compressive state of stress where $\sigma_1 > \sigma_2 > \sigma_3 > 0$.

In a second experiment at somewhat greater P ($= \sigma_2 = \sigma_3 = 35 \text{ bars}$), corresponding to a depth of about 0.13 km, the axially loaded rock cylinder continues to respond in a brittle way by forming a **fault**, or **shear fracture** (Figure 8.6b), along which parallel displacement occurs and that is inclined at an angle of approximately 30° to σ_1 . Faulting occurs where the shear stress, τ , exceeds the frictional resistance on the potential fracture plane. Frictional resistance is the product of multiplying the normal stress, σ , acting on the fracture (Figure 8.1) by the coefficient of friction of the rock, μ . Friction of rocks does not vary appreciably, but the normal stress that can act on a fracture increases, as does the confining pressure, P , with increasing depth in the Earth. Hence, the frictional resistance, $\sigma\mu$, to faulting increases with depth, necessitating greater shear stresses to cause faulting. In other words, the brittle strength of rocks increases with depth.



8.5 Laboratory deformation apparatus. The behavior of rocks under nonhydrostatic stress can be investigated in the laboratory by using a hydraulic squeezer, illustrated here schematically. Elevated temperatures, measured by the thermocouple, on the deforming sample are produced by the heating wires around the hardened steel pressure vessel (widely spaced diagonal lines). Alternatively, because steel is weak at high T , a small heater can be built immediately around the sample inside the pressure vessel, which may then be cooled by circulating water to maintain its strength. Such heaters may be deformed with the sample and are discarded after an experiment. A confining pressure, P , is created by pumping some inert fluid, such as compressed argon, around the sample. If this fluid pressure is less than the axially directed stress exerted on the ends of the sample cylinder by the hydraulically driven pistons, the axial stress is σ_1 and the confining pressure around the cylinder is $P = \sigma_2 = \sigma_3$, consequently, the sample will be shortened, or compressed, as in Figure 8.6.

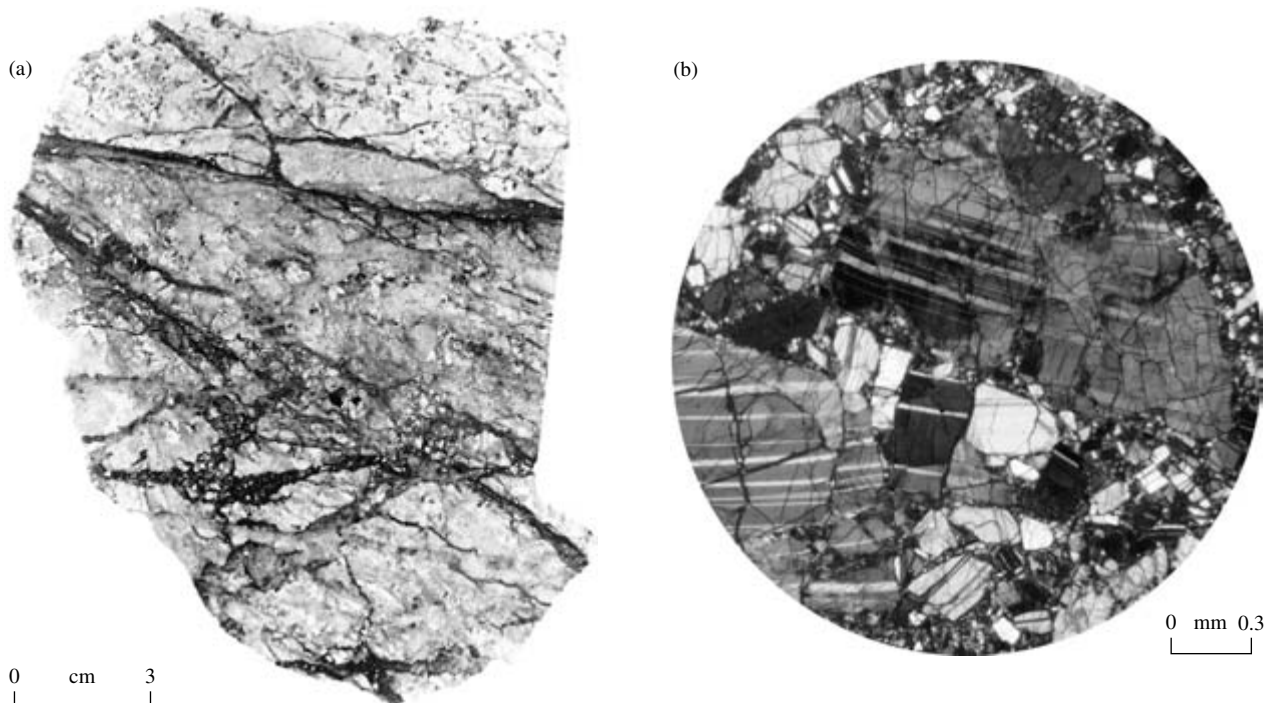


8.6 Cylinders of marble deformed in the laboratory show a transition between brittle and ductile behavior as a function of confining pressure, $P = \sigma_2 = \sigma_3$. Marble cylinders on the left were deformed at 25°C and P indicated in the stress-strain curves on the right. Note the increasing strength (stress difference required to produce permanent, “plasticlike” deformation) of marble with increasing P . See text for further discussion. (Photograph courtesy of M. S. Paterson. Reproduced with permission from Paterson MS. *Experimental rock deformation*. The Brittle Field. New York: Springer Verlag, 1978. Copyright © 1978 by Springer Verlag.)

Not only does increasing depth increase brittle strength, it also retards brittle behavior. This follows because fracturing creates openings in rocks where none was initially present, thus increasing the rock volume for a given mass. **Cataclasis**, creating **cataclastic fabric** (Figure 8.7), involves closely spaced (micro)fracturing, crushing, frictional sliding of broken fragments past one another, and rotation of grains. All of these dilatant processes increase the volume of the rock body

because of the introduced open spaces, which are increasingly prohibited at increasing P .

Liquids play a profound role in deformation. The pressure of a fluid, such as water, or of a silicate melt, that pushes outward on the walls of open spaces in a rock counteracts the normal stress compressing the rock together. The **effective normal stress** is therefore $(\sigma - P_f)$, where P_f is the fluid or melt pressure. Fluid or melt pressure reduces the frictional resistance,



8.7 **Cataclastic fabric.** (a) In polished surface of granitic rock. Note irregularly oriented cracks and wider shear fractures that contain obvious broken mineral grains in a dark matrix, which is more finely crushed grains cemented by silica introduced after the rock lost cohesion during cataclasis. (b) Lunar anorthosite under cross-polarized light. Intense brittle fracturing and brecciation was produced by meteorite impact on the Moon. (National Aeronautics and Space Administration photograph 60215,13.)

$(\sigma - P_f)\mu$. If P_f is sufficiently large in some place in the crust and even in the upper mantle, extensional fracturing can occur parallel to σ_1 and σ_2 and perpendicular to σ_3 . The magma or fluid can then penetrate into the cracks, forming dikes and veins. This phenomenon is called **hydraulic fracturing** (Figure 8.2c).

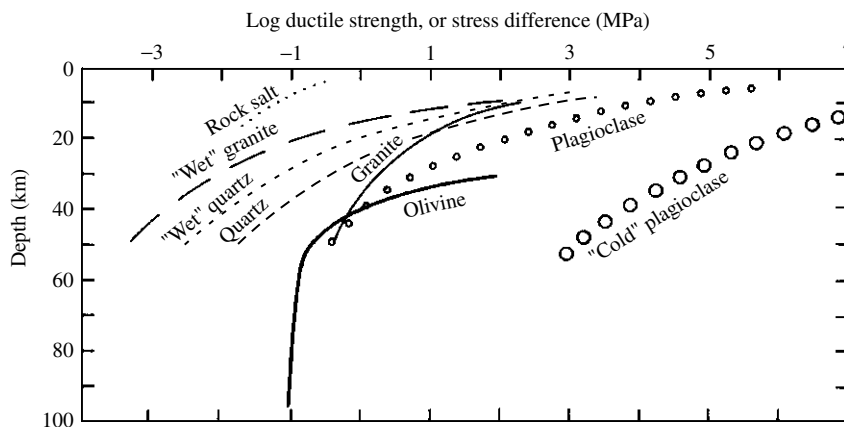
Ductile Behavior. In a third experiment, at still greater P , the marble cylinder in Figure 8.6c displays some permanent bulging or shortening along its axis in addition to faulting. In the fourth experiment, at the highest P ($= \sigma_3 = \sigma_2 = 1$ kbar), the bulging and shortening of the cylinder are greater and clearly involve a more or less uniformly distributed deformation mostly on the scale of individual mineral grains (Figure 8.6d); this is in contrast to the localized deformation along faults cutting many grains that occurred at lower confining pressures. Part of this distributed, permanent deformation in which virtually every grain has changed its shape is brittle microcracking (cataclasis). But another part that does not involve loss of cohesion and dilatancy involves two additional, atomic-scale mechanisms within individual mineral grains, the effects of which can be discerned microscopically. One mechanism involves recrystallization of grains, as atoms diffuse from more highly stressed parts to regions of lower stress. The other mechanism entails intracrystalline plastic slip and twinning, in which groups of atoms, or domains within the grain, move relative to neighboring domains. Both mechanisms produce changes in grain shape without any loss of cohesion, and both are promoted at elevated P and especially at the accompanying higher temperatures in the Earth. Hence, these two mechanisms that involve enhanced atomic mobility at elevated temperatures are said to be **thermally activated**.

The distributed, grain-scale permanent deformation just described is known as **ductile deformation**; it is manifested as folds in metamorphic rocks (Figure 8.4)

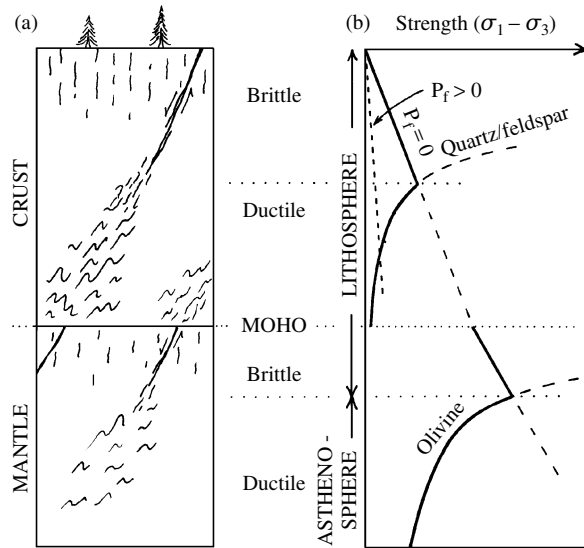
and glaciers (See Figure 14.11b). Because it macroscopically resembles viscous flow, it is also called **ductile flow**.

Because of the thermally activated nature of ductile flow, the ductile strength of minerals and rocks decreases exponentially with depth as T increases (Figure 8.8). The **ductile strength** of felsic minerals and rocks decreases by several orders of magnitude in the continental crust. Higher temperatures combined with slower strain rates make long-term atomic mobility increasingly significant so that ductile rock strengths can be as small as a few megapascals. This vanishingly small strength, comparable to that in toothpaste and cake batter, in hotter rocks in the deep crust and mantle means that large stress differences cannot be sustained; consequently, states of stress are nearly hydrostatic. In Figure 8.8, note that the lesser geothermal gradients prevailing in stable cratonic continental regions cause minerals to be stronger at a particular depth because of lower temperatures, relative to hotter geologically active orogenic regions.

With increasing depth, increasing brittle strength is interrupted by exponentially decreasing ductile strength (Figure 8.9). This phenomenon, together with contrasts in the brittle and ductile strengths of different minerals and rocks, produces a stratification in strength within the lithosphere. Assuming the continental crust to be made of felsic rock and the upper mantle predominantly of olivine, the weak lower continental crust is embedded between the stronger brittle upper crust and uppermost mantle. Where the lower continental crust is made of mafic rock whose ductile strength lies between that of olivine and quartz-feldspar a weak middle crust results. Stratification in strength of the lithosphere has an important bearing on how magmas ascend through it and intrude a particular volume of crustal rock; these topics are considered in the following chapter.



8.8 Ductile strength as a function of depth for some rocks and minerals. All curves are for a strain rate of 10^{-14} s^{-1} and a geothermal gradient appropriate to an active orogenic zone, except the stronger "cold" plagioclase curve, which is for a lesser geothermal gradient in a craton. (Redrawn from Twiss and Moores, 1992, Fig. 18.19.)



8.9 Generalized deformation and strength profiles with respect to depth for the continental lithosphere. (a) Cartoon showing that extensional fractures and faults near surface give way to ductile folds and flow in deeper crust. This same transition in behavior may occur in the upper mantle. (b) Schematic strength profiles for dominant rock-forming minerals in the continental lithosphere that define stratified brittle and ductile regimes. Brittle strength increases essentially linearly with depth, or P , independently of T . At some depth that is shallower for a greater geothermal gradient the brittle strength curve is truncated by the exponentially T - and mineral-dependent ductile strength curve, which is relatively insensitive to P . Ductile strength of the lower crust can be less than a few megapascals, whereas the brittle strength of the upper crust and mantle can be hundreds of megapascals. The solid lines for brittle and ductile strength are the maximum stress difference that can occur in the Earth. Note the dashed line labeled $P_f > 0$, which shows that a rock regime at a high fluid pore pressure has a smaller brittle strength and the realm of brittle fracturing extends to greater depth in the crust. Thus, otherwise ductile rocks can fracture if fluid pressure is high.

8.2.2 Non-Newtonian Rheology of Magma

For the Newtonian viscosity described thus far in this chapter and in Section 6.1, there is a linear proportionality between stress and strain rate (Figure 8.3c). This simple relationship appears to be valid for most crystal- and bubble-free melts, including metastable melts at subliquidus temperatures, which are subjected to moderate applied stress and relatively low strain rates. However, nonlinear behavior, or **non-Newtonian rheology**, prevails in melts subjected to high strain rates that exceed their characteristic relaxation time (see end of Section 6.1) and especially in magmas that contain suspended crystals or bubbles. For such behavior, the **apparent viscosity**, η_a , can be defined by the ratio of shear stress to strain rate at a particular set of conditions. The importance of non-Newtonian rheology in volcanic activity can be readily appreciated from the fact that erupting magmas are vesiculated, are com-

monly partially crystallized, and ascend rapidly at very high strain rates through a feeding conduit.

The following sections explore factors involved in non-Newtonian behavior.

Yield Strength. Many petrologists have noted that some homogeneous, bubble- and crystal-free melts possess a yield strength, σ_y , which makes viscous flow non-Newtonian. (For a contrary viewpoint see Lejeune et al., 1999.) The applied shear stress, τ must exceed σ_y before any viscous deformation can take place. This non-Newtonian viscoplastic rheology obeys the equation

$$8.3 \quad \tau = \sigma_y + \eta_a \left(\frac{dv}{dt} \right)$$

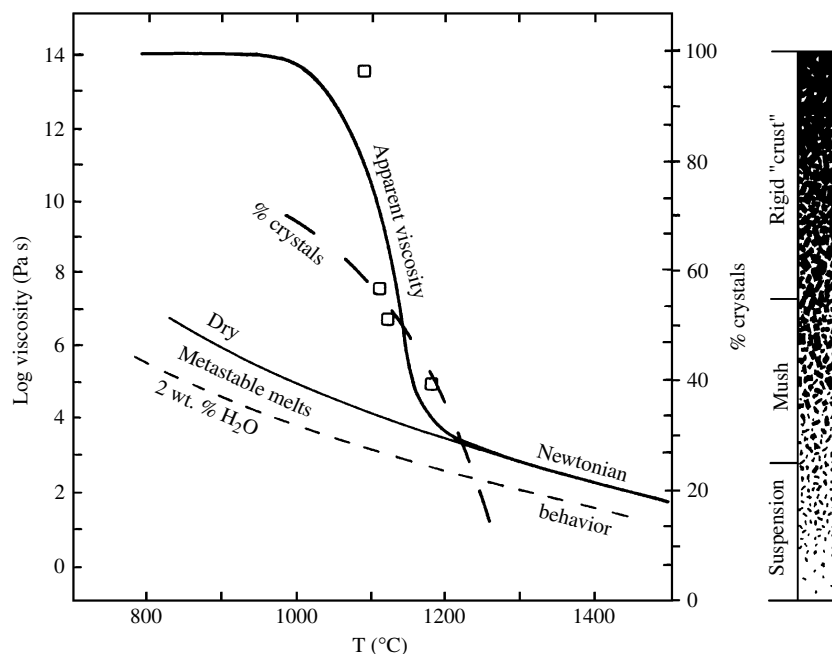
Crystal-Rich Magma. Another reason for non-Newtonian behavior is an increasing volume fraction of crystals (crystallinity) in magmas. Rigid crystals increase the apparent magma viscosity because a more tortuous shear path must be followed in the melt around neighboring crystals, even though they can rotate during flow. Observed yield strengths in crystal-bearing magmas appear to result from grain-to-grain interactions, especially as the crystalline network becomes continuous (Smith, 1997).

The Einstein-Roscoe equation can be adapted (e.g., Marsh, 1981) for the apparent viscosity, η_a , of a crystal-bearing non-Newtonian magma relative to the viscosity of the compositionally equivalent melt, η .

$$8.4 \quad \eta_a = \eta(1 - BX_v)^{-2.5}$$

where X_v is the volume fraction of crystals and $B = 1.35$ for uniformly sized rigid crystals, assumed to be spherical for simplicity. If the fraction of crystals is small, the viscosity is nearly that of crystal-free melt, but as the fraction of crystals increases, the apparent viscosity of the magma increases exponentially (Figure 8.10), until near $X_v = 1/1.35 = 0.74$ it approaches infinity. In real magmas that contain nonspherical crystals of various shapes and sizes, B is larger, about 1.7, so that solidlike behavior occurs where X_v is about 0.59. At greater degrees of crystallinity, magma acts as a solid and can fracture in a brittle manner. Dikes can be rapidly intruded into cracks in high-crystallinity magma as if it were a brittle solid, but subsequently the dike and its host magma can still be deformed, however slightly, by slow viscous flow; no increase in T or applied stress is required to allow flow, only more time.

As magma cools below the liquidus, an increasing non-Newtonian viscosity due to increasing concentration of crystals is compounded by increasing residual melt viscosity because of decreasing T and, in many systems, because of increasing silica concentration of the residual melt. However, in magmas where mostly anhydrous minerals are crystallizing, water concentration



8.10 Dramatic exponential increase in apparent viscosity as the crystallinity of magma increases. Volume fraction (as percentage) of crystals shown by long dashes. Thick solid line is experimentally determined viscosity at 1 atm near the NNO buffer of the dacite lava that formed a dome within the crater of Mount Saint Helens after explosive activity waned in 1981. (From Murase et al., 1985.) Viscosity increases by a factor of 10^9 over less than 200°C as the crystal content increases from about 30% to 70%. Note the simple, nearly linear T -dependent Newtonian viscosity for crystal contents less than about 30%. Light lines show calculated Newtonian viscosities for metastable, crystal-free melts at low temperatures. Note also nearly constant viscosity near the solidus where the magma behaves effectively as a solid rock and has a plastic yield strength of 0.2 MPa. Open squares indicate apparent viscosity calculated by using equation 8.4 and $B = 1.7$. The classification of the magma as to crystal suspension, mush, and rigid “crust” is as occurs in a lava lake (Marsh, 1989.)

increases in the residual melt; thus the melt viscosity might actually decrease. If a residual melt becomes saturated in water so that it exsolves and forms bubbles, the viscosity of the melt increases because of its decreased water concentration, which in turn raises the liquidus T and can result in increased crystallinity, further elevating the viscosity. Exactly how bubbles themselves influence the rheology of the magma body is uncertain. Experiments have revealed no consistent results (Lejeune et al., 1999). These competing factors emphasize the danger of overgeneralization of magma rheology. Each evolving magma system must be considered individually.

Because the viscosity of the melt between crystals and any bubbles can be highly variable and because crystal sizes and shapes are also variable, no universal critical crystallinity value distinguishing between viscous and brittle behavior can be defined for all magmas. Rather, even in a particular magma, there is a crystallinity-dependent transition in behavior.

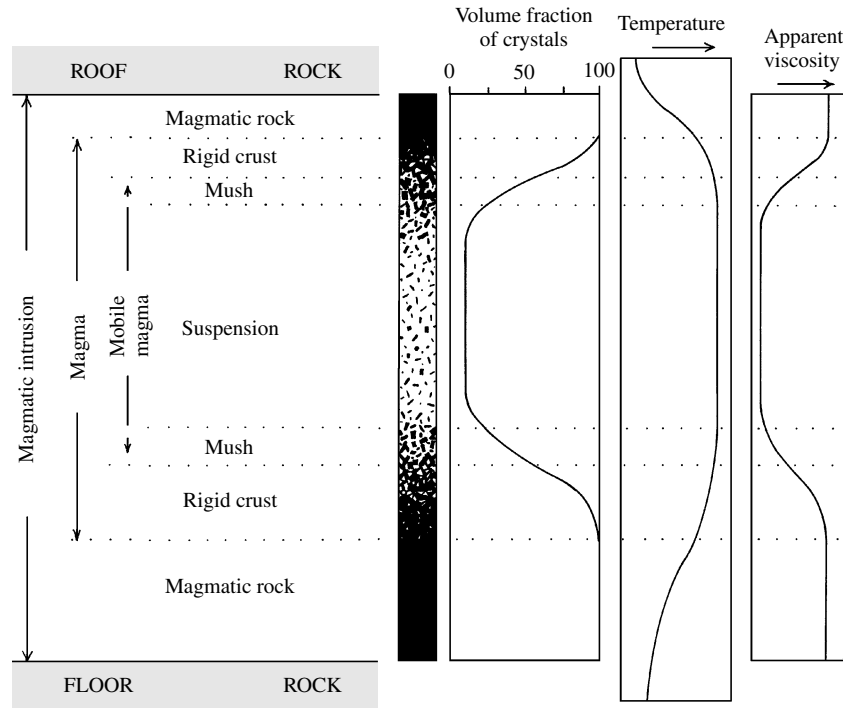
At any time a magma intrusion consists of different rheologic zones—mobile magma, rigid magma still above its solidus, and magmatic rock below the solidus (Figures 8.10 and 8.11). As the magma cools and crystallizes, a diminishing proportion remains mobile. It should also be noted, in Figure 8.11, that the mobile

part of the magma body is smaller than the volume that is still magma, and this magma volume is less than the magmatic intrusion, which includes solidified, wholly crystalline rock.

8.2.3 Deformation and Flow of Magma

Deformation of high-crystallinity magma bodies can produce dilatancy (volume increase) as crystals rotate, shuffle, and interact. This **dilatancy pumping** phenomenon (van der Molen and Paterson, 1979) may redistribute the melt into dikes oriented perpendicular to the least compressive principal stress, σ_3 .

Shaw (1969) drew attention to the possible role of viscous heating and thermal feedback phenomena in magma systems. As a viscous body is sheared, kinetic energy is degraded into thermal energy in an analogous manner to the frictional heating of two solid objects sliding on one another. The T increase in the flowing system depends on the rate of strain, the apparent viscosity and dimensions of the system, and the rate of dissipation of heat out of the system. As T rises during viscous heating, the viscosity of the magma decreases, potentially localizing subsequent flow and heating, causing further reduction in viscosity, and so on, in a process called **thermal feedback**. Magma flow and feedback may become localized along discrete, thin



8.11 Correspondences among rheologic zones, magmatic properties, and definitions of magmatic realms in an intrusive system. Compare Figure 8.10.

surfaces intervening between thicker layers that remain unsheared. This phenomenon may have a bearing on the development of flow layering in viscous lava flows (Section 7.9.2).

In rapidly ascending bubbly magma in a subvolcanic conduit, localized shear may focus more rapid strain, exceeding the relaxation time of the melt (Section 6.1) so that, having made the transition into glass, it ruptures in a brittle manner. This sudden rupturing and release of excessive internal volatile pressure in the bubbles may well be the cause of explosive volcanic eruptions (Section 6.7.2).

Flow of Magma in Channels and Conduits. Rheologic properties of magma influence its movement in extruded lavas, in subterranean conduits such as tabular dikes and circular pipes, and in convecting intrusions.

The nature of the flow regime of a viscous magma, considered here for simplicity to be Newtonian, depends upon the magma properties and the boundary conditions surrounding it. Relevant properties are the viscosity, η , and the density, ρ . Boundary conditions of the magma body include the flow velocity, v , and the *hydraulic radius*, D , which is four times the cross-sectional area perpendicular to flow divided by the *wetted* perimeter of the flowing magma. For flow in a perfectly circular pipe, $D = 4\pi r^2/2\pi r = 2r$ where r is the pipe radius, and for a flow in a dike with a rectangular cross section, $D = 4lw/2(l + w) = 2lw/(l + w)$, where l and w are the length and width; if the dike width is very small compared to its length, $D \sim 2w$.

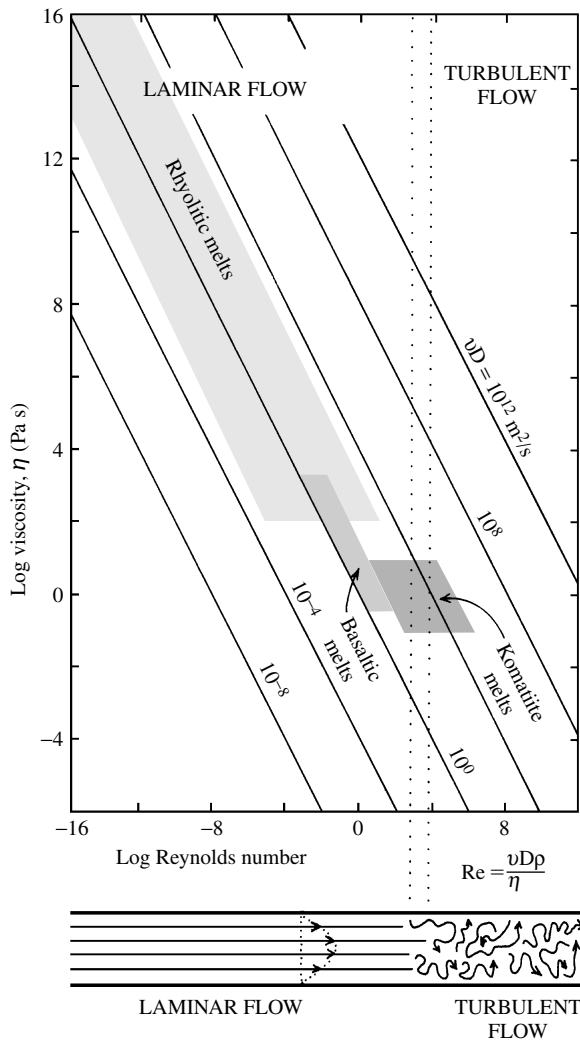
These properties and boundary conditions can be combined into a dimensionless **Reynolds number**

$$8.5 \quad \text{Re} = \frac{vD\rho}{\eta}$$

Flow regimes dominated by a viscous resistance to flow, where η is relatively large and/or v small, have small Re , and the flow is said to be laminar. **Laminar flow** consists of movement in parallel sheets or “pencils” in pipes (see bottom of Figure 8.12). In contrast, flow regimes dominated by inertial forces at high velocities where viscous effects are relatively less significant have large Re and the flow is turbulent. **Turbulent flow** is a rapid and chaotic motion of particles in small eddies superimposed on the overall flow of the moving fluid.

The transition from laminar to turbulent flow depends somewhat on the configuration of the conduit or channel but begins at Re as low as about 500; flow may not be wholly turbulent until $\text{Re} \sim 4000$ (Figure 8.12). In the transitional region, laminar flow is metastable and some disturbance may trigger turbulence. Despite the wide variation in magma viscosity, geologic boundary conditions generally seem to preclude turbulence, except possibly in some very low-viscosity komatiite and carbonatite lavas (Figure 6.2) and gas-charged kimberlite magmas. Fast-moving streams of water are turbulent.

Flowing magma invariably has a velocity gradient near the solid boundary against which it flows. This gradient arises from the viscous drag along the boundary

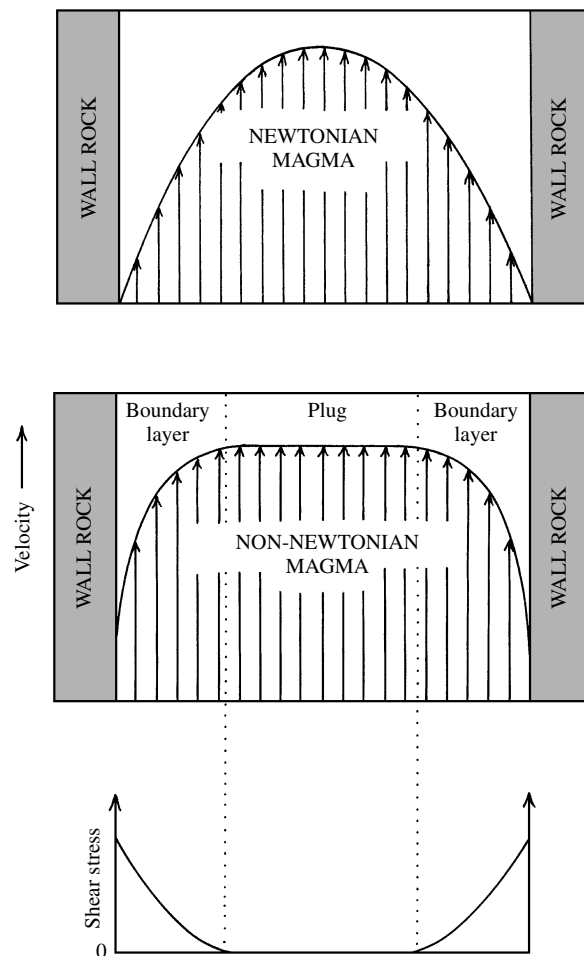


8.12 The Reynolds number, Re , and viscosity, η , govern laminar versus turbulent flow in Newtonian bodies. The transition between laminar and turbulent flows in a pipe, shown schematically at the bottom of the diagram, occurs at $Re = 2000 - 4000$ ($=10^{3.3} - 10^{3.6}$). The range of values of Re and η for komatiite magmas (dark shaded), basaltic magmas (intermediate shaded), and rhyolitic melts (light shaded) are plotted with respect to typical flow velocities and cross-sectional dimensions. Some komatiite lava flows may be turbulent.

where the velocity is nil. In flowing Newtonian magma, the velocity profile is parabolic, whereas flowing non-Newtonian (viscoplastic) magmas have a central stagnant “plug” zone where velocities are uniform and no gradient exists (Figure 8.13). Within the central plug, shear stresses are less than the yield strength of the magma so it is undeformed, whereas in flow margins shear stresses exceed the yield strength and the magma flows viscously. Plug flow is not restricted to lavas but is also observed in muddy volcanic debris flows in which the poorly sorted mixture of clasts as much as several meters in diameter impedes flow just as crystals do in magma. With due allowance for surface crusting of lava flows and other natural perturbations, the flow

profile is a good indicator of the rheology of the mobile material.

Grain dispersive pressure. Another consequence of a velocity-gradient boundary layer is evident in dikes that have a greater concentration of usually larger crystals nearer the center than at the margins (Figure 8.14). As magma containing suspended crystals flows in a laminar manner between the solid conduit walls, a **grain dispersive pressure** in the boundary layer forces grains into the interior of the dike (Komar, 1976). Mechanical interaction between suspended solids in a viscous flow has been studied by engineers in many laboratory experiments and can be observed, for example, in streams of water carrying pieces of wood or leaves. The dispersive pressure is created as more interior grains shear at greater velocity past more slowly moving, more exterior grains. Overtaking grains bump into slower grains in the boundary layer, and, since they can only



8.13 Velocity and shear stress gradients in confined magma flow. Profiles of a lava flow in a confined channel viewed from above or of a subterranean conduit viewed from the side. (a) Parabolic velocity profile (arrows) in flowing Newtonian magma. (b) Plug flow in non-Newtonian (viscoplastic) magma or volcanic debris flow. (c) Schematic profile of shear stress.



8.14 Effect of grain dispersive pressure in a subvertical dike intruded into Pliocene gravels 15 km south of Hoover Dam, Arizona. The highest concentration of largest black amphibole and white plagioclase crystals in the dike lies above and below the rock hammer, where the intrusive flow velocity gradient was negligible. Platy-weathering margin of dike is virtually free of crystals. Compare Figure 8.13. The inward concentration of larger crystals cannot be related to differential cooling rate across the dike because the crystals are anhedral and therefore did not grow in place, because the amphiboles were equilibrated under upper mantle P - T conditions, and because small mantle-derived peridotite xenoliths also occur in the crystal-rich interior of the dike. Thus, the flowing magma contained inward concentrated crystals and rock fragments derived from an upper mantle source as it stalled at this near-surface depth.

move *into* the channel or conduit, away from the fixed solid wall, there is a net component of grain motion into the dike. These mechanical interactions diminish into the interior, causing a gradient in the dispersive pressure. Higher concentration of crystals in the dike interior increases its apparent viscosity, enhancing plug flow. Concentrations of crystals in nonvertical dikes and pipes may be asymmetric as a result of gravitational sinking of denser crystals after flow has ceased but before solidification of the magma.

Other explanations of nonuniform concentration of crystals between the walls of pipes or dikes appeal to special circumstances such as strongly non-Newtonian flow and multiple intrusions.

Grain dispersive pressure, possibly coupled with thermal feedback, in flowing magmas may play a role in development of flow layering, as depicted in Figure 7.40c.

✱8.3 DENSITY OF MAGMA AND BUOYANCY

Melt densities vary mainly between 2.8 and 2.2 g/cm³. Crystal-bearing *magmas* have somewhat larger densities, but the most significant variation occurs in magmas that contain volatile bubbles, in which densities can be as small as 0.05 g/cm³. From the smallest to largest density is a factor of about 50, compared to the enormous variation in viscosity of about 20 orders of magnitude. This would seem to relegate density to the category of insignificant magma properties. However, small variations in density *in large masses* of rock and magma can have significant effects. Unlike in viscosity, whose absolute value dictates flow phenomena, it is seemingly small *contrasts* in density between solids and melt, or between contrasting parcels of magma, that strongly affect dynamic magmatic behavior.

In addition to controlling the rise of magma from deeper sources in the mantle and crust, small density contrasts play major roles in the generation and diversification of magmas. If partial melts in mantle and deep crustal sources had the same densities as their source rock or greater densities, there would be no magma movement to shallower depths and consequently no intrusions and volcanism. Physical incorporation and chemical assimilation of chunks of wall rock into magma depend on their density contrasts. Mixing of contrasting magmas and convection in bodies of magma involve density differences.

8.3.1 Density Determinations

Densities are measured in grams per cubic centimeter or in the less familiar SI units as kilograms per cubic meter. The former is used here.

Melt densities over a range of P and T are generally measured by the falling sphere method using Stokes's law (discussed later).

Measured densities at one particular P and T can be corrected to another P and T by using compressibility and thermal expansion data for melts (Lange, 1994) and crystals (Clark, 1966). The **coefficient of isothermal compressibility**, β , expresses the change in volume or density as P changes with depth in the Earth at constant T

$$8.6 \quad \beta = -\left(\frac{1}{V}\right)\left(\frac{dV}{dP}\right)_T = -\left(\frac{1}{\rho}\right)\left(\frac{d\rho}{dP}\right)_T$$

where V is the molar volume and ρ is the density. Because dV/dP is negative β is positive; it has units of reciprocal pressure. β depends somewhat on P and T and differs slightly in different crystallographic directions in anisotropic crystals, so a hydrostatic state of stress can actually cause an anisotropic strain, or different changes in shape in different directions. For crystalline solids (Clark, 1966), $\beta = 1-2 \times 10^{-11} \text{ Pa}^{-1}$, and for

melts, $\beta \sim 7 \times 10^{-11} \text{ Pa}^{-1}$. The volumetric **coefficient of thermal expansion**, α , expresses the change in volume or density as T changes at constant P

$$8.7 \quad \alpha = -\left(\frac{1}{V}\right)\left(\frac{dV}{dT}\right)_P = -\left(\frac{1}{\rho}\right)\left(\frac{d\rho}{dT}\right)_P$$

α has units of reciprocal degrees. Most minerals and rocks have a thermal expansion in the range of $1\text{--}5 \times 10^{-5}/\text{deg}$ and for many silicate melts, $\alpha \sim 3 \times 10^{-5}/\text{deg}$.

8.3.2 Densities of Minerals and Melts

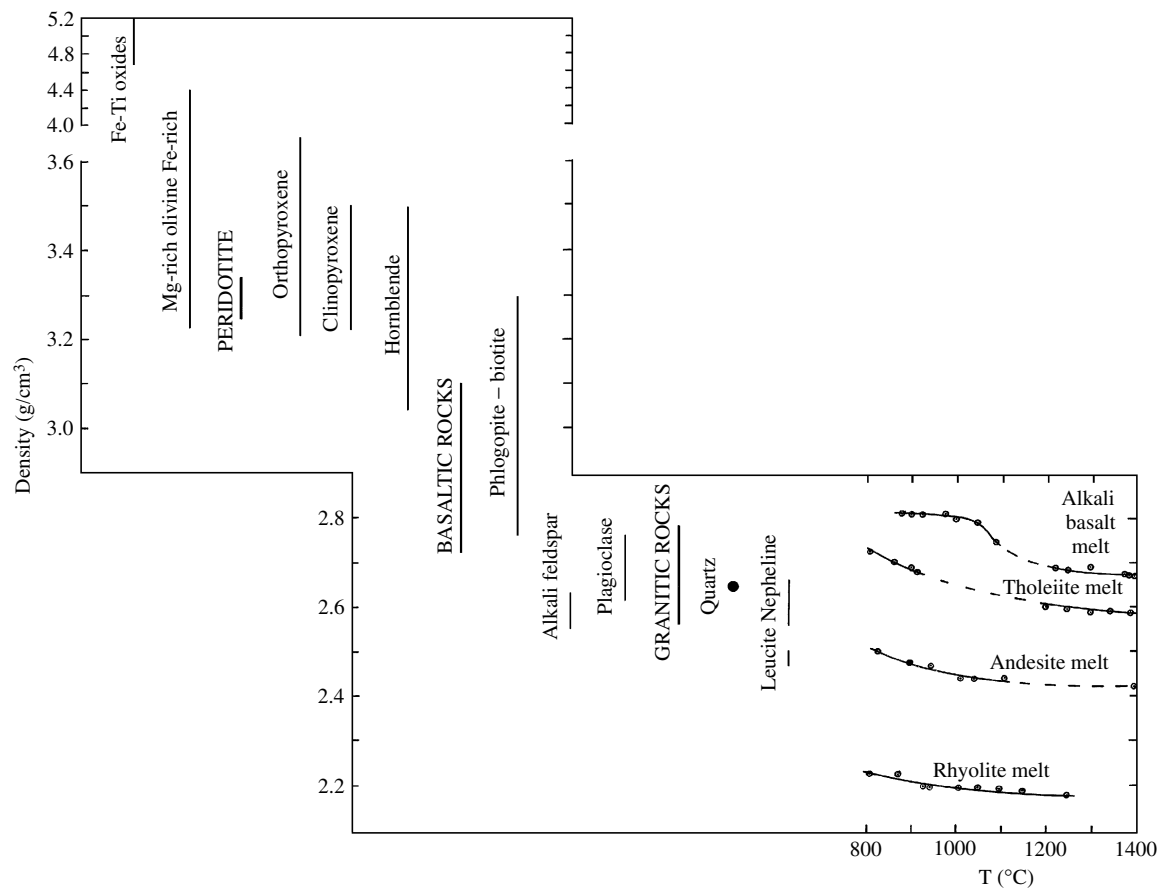
Densities of common rock-forming minerals (Figure 8.15) range from about 5 g/cm^3 for Fe-Ti oxides to about 2.6 g/cm^3 for felsic minerals.

Unlike the familiar, but atypical, liquid water-ice system in which the solid is less dense than liquid, silicate melts are 10–20% less dense than the compositionally equivalent crystalline solids (Figure 8.15). The densities of natural melts depend mostly on their chemical composition, especially the concentration of water, and to a lesser extent on P and T (Advanced Topic Box 8.1). Figure 8.15 shows that the change in density for a particular melt over a range of several hundreds of de-

grees Celsius is only a small fraction of the variation between melts of contrasting major-element composition, such as between basalt and rhyolite. The effect of dissolved water on melt density is also dramatic. For example, dissolving only 0.4 wt.% water in a basalt melt at 1200°C and 700 bars has the same effect as increasing T by 175°C or decreasing P by 2300 bars (Ochs and Lange, 1997). This significant control exerted by water follows because its coefficients of thermal expansion and isothermal compressibility ($4.7 \times 10^{-4}/\text{deg}$ and $1.7 \times 10^{-5}/\text{bar}$, respectively, at 1000°C and 1 kbar) are the most expansive and compressible of those of any component in a melt. It may also be recalled (Problem 4.2) that a modest weight percentage of water translates into a much greater amount on a molecular basis.

Water-oversaturated magmas that contain bubbles of exsolved water can produce even greater variations in density, to less than 0.05 g/cm^3 for some very highly vesiculated melts at low P .

Densities of compositionally changing residual melts during fractional crystallization of magma depend mostly on the concentrations of Fe and dissolved water. In fractionating tholeiitic mafic magma (Figure 8.16), the residual melt increases in density as a result of Fe



8.15 Densities of common rock-forming minerals and rocks at atmospheric P and T and melts at 1 atm. Higher densities for mafic solid-solution silicates are Fe-rich end members; lower densities are Mg-rich end members. Note change in density scale in upper left. Experimentally measured densities for crystal-free melts. (Redrawn from Murase and McBirney, 1973.)

Advanced Topic Box 8.1 Thermodynamic model for determining the density of silicate melts as a function of their bulk composition, P , and T

Bottinga and Weill (1970) found that the molar volume, V , of a silicate melt at a particular P and T can be approximated by simply adding the partial molar volumes of the constituent liquid oxide components for that P and T : $V = \sum_i X_i v_i$ where X_i is the mole fraction of the i oxide and v_i is the partial molar volume (Section 3.4.2) of the i oxide. It is assumed that the partial molar volumes of the oxides are independent of the melt composition. Refinements of this model using new experimental data have been made by I.S.E. Carmichael and coworkers (for example, Lange, 1994). The molar volume can now be calculated with good accuracy for almost any silicate melt and, most importantly, including ones that contain dissolved water, for a wide range of P and T in the lithosphere.

The equation for calculation is

$$V = \sum_i X_i \left[v_i + \left(\frac{dv_i}{dT} \right) (T - 1673K) + \left(\frac{dv_i}{dP} \right) (P - 1 \text{ bar}) \right]$$

where dv_i/dT is the thermal expansivity and dv_i/dP is the compressibility of the i oxide. Units for T are in degrees kelvin and for P in bars. Values for v_i , dv_i/dT , and dv_i/dP are given in Table 8.1.

Once the value for V is obtained, the density of the melt, ρ , is found from

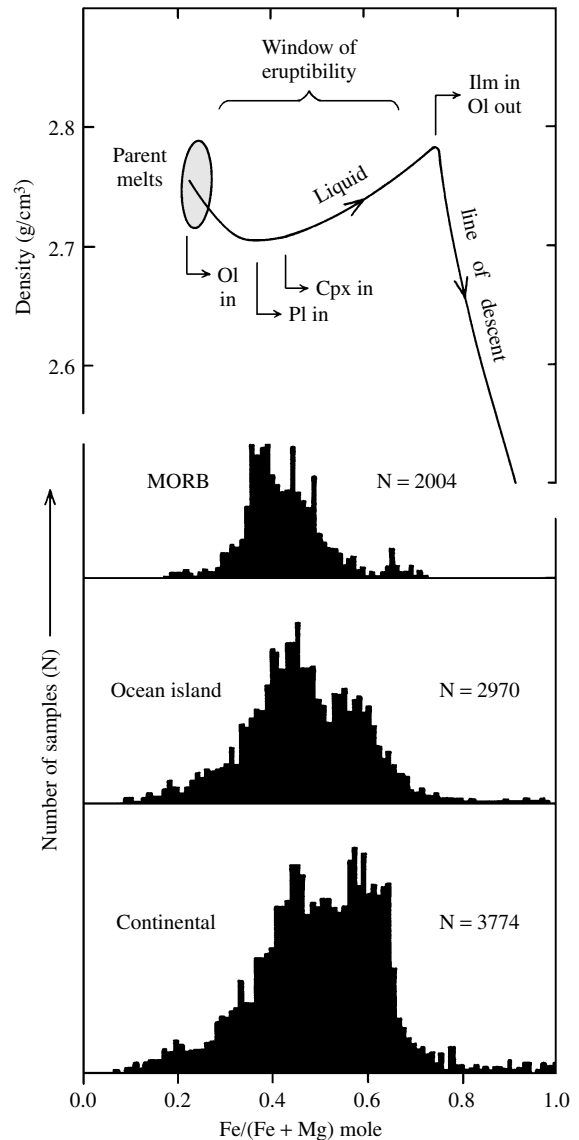
$$\rho = \frac{\sum_i (X_i MW_i)}{V}$$

where MW_i is the molecular weight of the i oxide.

enrichment during crystallization of olivine, pyroxene, and plagioclase then decreases at lower T as Fe-Ti oxides precipitate and fractionate. In fractionating calc-alkaline magmas Fe enrichment is generally limited and residual melt densities diminish as silica, alkalis, and especially water become more concentrated.

8.3.3 Buoyancy

The reason that density contrasts are so important in magma dynamics is their interaction with gravity. In the presence of a gravitational field, density contrasts produce **buoyancy** forces. Bodies denser than their immediate surroundings experience a downward force—**negative buoyancy**—whereas less dense bodies experience an upward force—**positive buoyancy**. A body of the same density as its surroundings has a **neutral buoyancy** and no force prevails. Whether bodies of con-



8.16 Density-composition relations in tholeiitic basalt magma and rocks. Top of diagram shows variation in density and Fe/(Fe + Mg) mole ratio of residual melts following a liquid line of descent as indicated minerals crystallize (e.g., Ol in, cpx in) and fractionate out of a mid-ocean ridge basalt (MORB). Note the minimum density where plagioclase (Pl) and clinopyroxene (Cpx) begin to precipitate. The three histograms below represent Fe/(Fe + Mg) mole ratios in 2004 analyzed samples of MORB, 2970 oceanic island basalts, and 3774 continental basalts. Stolper and Walker (1980) postulate that the density minimum in the residual melts constitutes a “window of eruptibility,” which correlates with the most frequent basalt compositions observed in oceanic and continental crusts. (Reproduced with permission from Stolper EM and Walker D. Melt density and the average composition of basalt. *Contr. Min. Petrol.* 1980;74:7–12. Copyright © 1980 by Springer Verlag.

trasting density actually sink or rise depends upon viscous resistive forces acting against buoyancy forces.

Many magmatic phenomena involve buoyant movement of a crystal, rock, or volatile bubble (hereafter simply referred to as a *particle*) within viscous melts

and magmas of contrasting densities. Buoyancy is also relevant in movement of pyroclasts in volcanic plumes created from exploding volcanoes. Moving particles are driven by a buoyant driving force but retarded by a viscous drag force so that after a steady state is attained, the **terminal velocity** of upward or downward movement is constant. For a smooth, isolated sphere of radius r moving without interference through a Newtonian melt (or other liquid) of viscosity η the terminal velocity is given by **Stokes's law**

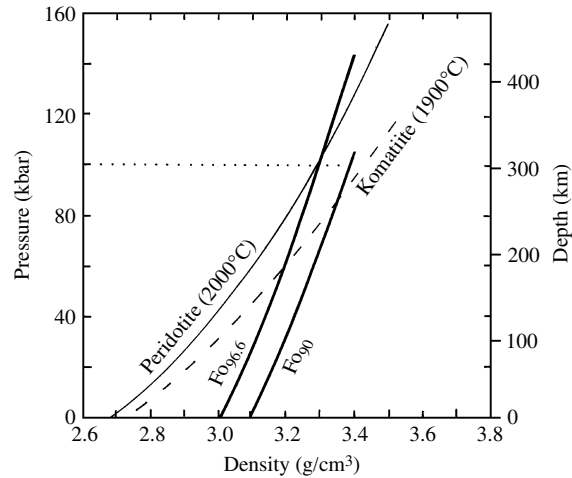
$$8.8 \quad v = \frac{2r^2g\Delta\rho}{9\eta}$$

where $\Delta\rho$ is the contrast in density between particle and melt (or other surrounding media) driving buoyant movement and g is the acceleration of gravity. For slowly moving particles, departures from the smooth sphere model are generally insignificant. Thus, particle terminal velocity is chiefly proportional to the reciprocal of the viscosity, η , and the square of the particle radius, r , both of which can vary over several orders of magnitude. Density contrasts between melts and silicate crystals have less effect on velocity because they are generally small, $\Delta\rho < 1.0 \text{ g/cm}^3$. However, density contrasts between melts and volatile bubbles are greater.

In crystal-rich magmas, movement of an individual particle can be hindered by neighboring particles. However, clumps of numerous particles may move rapidly en masse according to the dimension of the clump and its density contrast. This may be important in some intrusions where crystallization occurs along the walls and cascades of crystals slump into the chamber.

Stokes's law is not valid for non-Newtonian rheologies. Thus, the sinking velocity of dense blocks of roof rock into a chamber of partially crystallized magma cannot be determined. The Stokes model cannot apply to melts that possess a yield strength; crystals of insufficient mass cannot overcome this strength and would remain suspended indefinitely. Nonetheless, sinking of near-liquidus olivines in basaltic melts has been documented experimentally and in Hawaiian lava lakes, suggesting that such melts have a negligible yield strength.

Because of the somewhat greater compressibility of melts than of crystals, a density contrast at low P that makes a crystal negatively buoyant, causing it to sink, might be reversed at high P . One of the first to demonstrate this experimentally was Kushiro (1980), who showed that calcic plagioclase is more dense than tholeiite melt at low P but less than the melt at high P . If plagioclase can float deep in the continental crust in a basaltic melt of negligible yield strength, it may explain the origin of problematic Proterozoic anorthosite intrusions made largely of plagioclase (Section 19.5.2). A similar crossover in densities (Figure 8.17) might occur for olivine in ultramafic melts in the mantle (Stolper et al., 1981). This has possible implications for



8.17 Density relations for ultramafic melts and olivine at high P and $T = 1900 - 2000^\circ\text{C}$. Thicker solid lines represent two olivine compositions; two thinner lines represent ultramafic melts at two different temperatures. Neutral buoyancy between peridotite melt and equilibrium composition olivine, $\text{Fo}_{96.6}$, occurs at 100 ± 10 kbar. (Redrawn from Agee and Walker, 1993.)

the earliest differentiation of Earth, when a primeval “magma ocean” is postulated to have existed.

✳8.4 CONDUCTIVE HEAT TRANSFER

As briefly discussed in Section 1.1.3, heat transfer occurs by radiation, conduction, advection, and convection. Lavas lose heat and cool rapidly because of radiation and convection of heat from the surface into the transparent and very low-viscosity atmosphere. In wet climates, vaporization of rain on the surface of the extrusion absorbs heat, enhancing cooling. In just a few hours, lava flows can form a crust rigid enough to walk on, but complete cooling of flows to ambient atmospheric temperatures takes several years to tens of years for thicker flows. Intrusions of magma have slower rates of heat loss for comparable volumes because rapid radiative and convective transfer does not occur. Instead, slow heat loss is dictated by a complex interplay of transfer processes, including internal convection within the magma, conduction into the enclosing rocks, and advective circulation of hydrothermal fluids that extract heat from near the intrusion and transfer it through the surrounding rock.

With regard to rock and magma properties and boundary conditions for the system, the rate of cooling and the T distribution within a magmatic body depend on its T at emplacement, volatile content, latent heat of crystallization, apparent viscosity, thermal conductivity, density, specific heat, dimensions, and shape, as well as the T_i conductivity, specific heat, volatile content, and permeability of the surrounding rocks. It is impossible

to evaluate all of these variables in any but the most ideal, simplified cases.

Section 6.3 introduced some concepts of conductive heat transfer. In this section, these concepts are applied to the conductive cooling of bodies of magma where heat is transferred into the adjacent rock. Advection and convection are discussed in Sections 8.5 and 8.6.

It is appropriate here to note that the rocks surrounding a magmatic intrusion are commonly referred to as **country rocks**. Depending on their position, these are wall rocks, floor rocks, and roof rocks.

8.4.1 Conductive Cooling Models

Because exact heat conduction, or thermal diffusion, equations are complex and many input variables cannot always be evaluated, approximation models are used.

One such approximation is $t = z^2/\kappa$ (equation 6.7), which indicates that a *thermal transfer time*, t , increases as the square of the dimension in the z direction of the body for a constant thermal diffusivity κ . Thus, doubling the dimension of a body increases its heating or cooling time in that direction by a factor of 4. The **thermal diffusivity**, $\kappa = k/\rho C$ (where k is the thermal conductivity, C is the specific heat, and ρ is the density), is the ratio of the ability of a material to conduct heat relative to its capacity to accumulate thermal energy. The diffusivity is a property of the conducting material and is about 10^{-6} to 10^{-7} m^2/s for melts and common dry rocks but is somewhat less for rocks that contain water or air in pore spaces (Delaney, 1987). Strongly anisotropic rocks such as schists have a greater diffusivity parallel to the foliation than across it.

A more accurate heat transfer approximation was formulated by Jaeger (1968). He proposed a *characteristic*, or *nondimensional, time*, $\tau = \kappa t/a^2$, where a is the radius of a sphere or cylinder or half the thickness of a sheet. For example, if $\kappa = 10^{-6}$ m^2/s , t is in years, and a is in meters, $\tau = 31.5t/a^2$ or

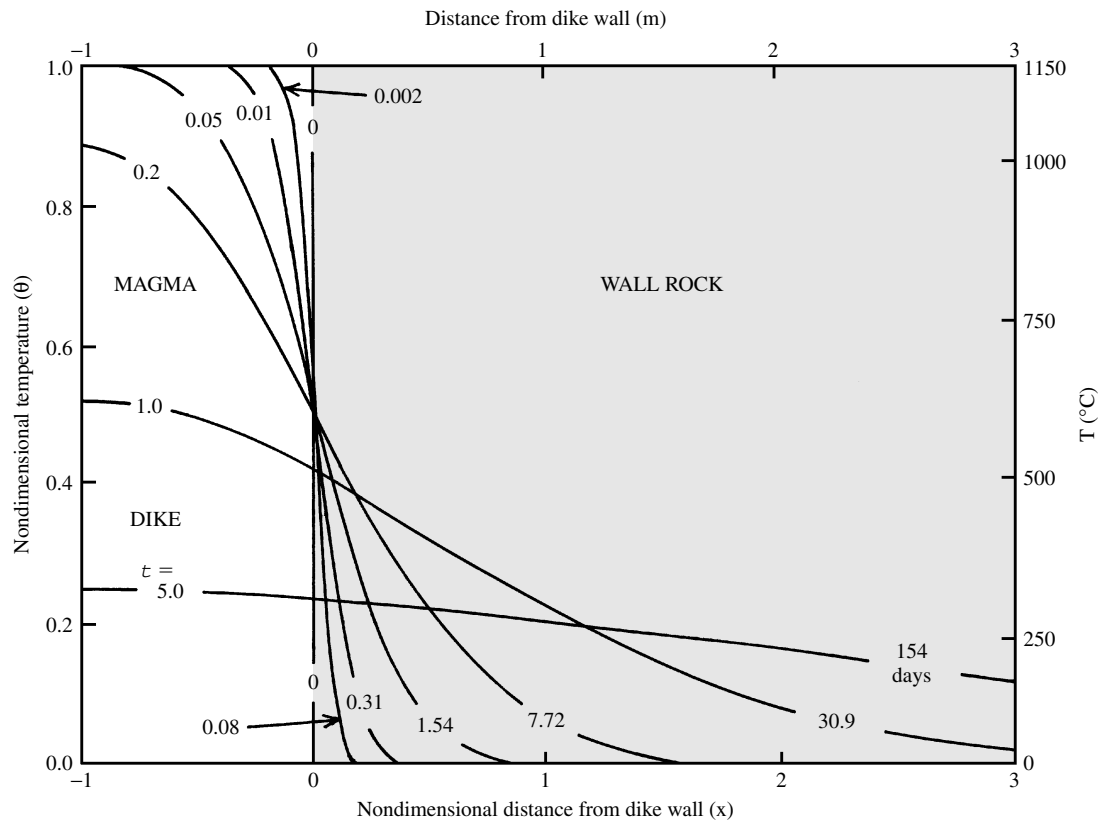
$$8.9 \quad \tau = \frac{\kappa t}{a^2} = \frac{t}{31.5}$$

If $\tau < 0.01$, cooling is superficial; for $\tau \sim 0.1$, cooling will have penetrated to the center of the body; and for $\tau \sim 1$, there is substantial cooling at the center of the body and about as much heat has been lost to the country rocks as remains in the body: That is, its average T is roughly halved. For $\tau > 10$ heat transfer is practically complete.

Equation 8.9 can be used to approximate the time required for substantial cooling, $\tau \sim 1$, and loss of extrusive capability in a hypothetical static circular cylinder of magma 5 m in radius feeding a volcano. Substituting, $\tau = (1 \times 5^2)/31.5 = 0.794$ $y = 290$ days.

The spatial relations of T and t in a conductively cooling static magma body and its country rocks are conveniently represented by a family of **isotherms**. The configuration of these three-dimensional isothermal surfaces in real rock bodies can be represented as isothermal lines in two-dimensional sections through the body. Figure 8.18 shows evolving isotherms in and adjacent to a common intrusion—a basaltic dike—in which the wall rocks initially have a uniform $T = 50^\circ\text{C}$. Consequences of this intrusive situation, which also applies to any sheetlike intrusion (Delaney, 1987), may be generalized as follows:

1. Within a day or two, all but the center of a 2-m-thick dike has cooled to $<90\%$ of its initial $T = 1150^\circ\text{C}$. This cooler T ($<1035^\circ\text{C}$) is near or below the solidus of the magma (Figures 5.10 and 5.11); therefore, any further magma flow is impossible. Hence, the time available for subsequent transport of magma is significantly less than overall dike cooling times. The observation that a basaltic fissure eruption commonly evolves into a focused central eruption after days to a few weeks of activity implies inward solidification of magma and sealing in thin feeder dikes, except in one subvertical cylindrical conduit, where magma transport is thermally most efficient. The efficiency follows from the smaller surface area/volume ratio in the cylinder relative to the dike (Section 6.3.1). The preceding calculation suggests that a 5-m-radius conduit can remain viable for several months.
2. Immediately after instantaneous intrusion, the maximum T of the wall rock at the contact is half the T of the magma, regardless of the thickness of the dike. However, if the latent heat of crystallization of the magma is taken into consideration, the peak wall rock T can be as much as 100°C greater and the rate of inward movement of solidification into the dike as much as three times less. Also, prolonged transport of new magma past a particular area of wall rock introduces more heat into it.
3. Because $t = z^2/\kappa$ (equation 6.7), the velocity of an isotherm moving away from a contact is $v = dz/dt = 0.5(\kappa/t)^{1/2}$. Therefore, shortly after emplacement, isotherms advance rapidly but then slow. Thus, margins of a magma body chill rapidly, but the interior cools at a slower rate, a conclusion in agreement with observed variations in grain size in relation to cooling rate in typical dikes. The outer few centimeters of the margin of thin basaltic dikes less than a few meters in thickness are typically quite glassy. Internal parts have partly glassy intersertal to wholly crystalline intergranular texture. Thicker dikes may be of ophitic-textured diabase.



8.18 Isotherms in a magma dike and its wall rock. Thermal properties of magma and wall rock are assumed to be identical and constant. Latent heat released during crystallization of the magma is ignored. Upper and right axes are distance in meters and T in degrees, respectively, for a dike 2 m wide in which the magma initially was 1150°C and the wall rocks 50°C. The labeled curves show T with respect to distance from the dike contact at times of 1.54, 30.9, 7.72, and so on, days. Bottom and left axes are nondimensional distance and temperature, respectively, that permit the cooling history of a planar body of any width to be represented. For example, a dike whose width is 8 m has its center at $x = -1$ and a distance of 4 m into the wall rock is at $x = 1$. The nondimensional temperature, $\theta = (T - T_{wri}) / (T_{mi} - T_{wri})$, where T_{wri} is the initial wall rock T and T_{mi} is the initial magma T , varies between 1 and 0. The curves are also labeled in nondimensional time, $t = t\kappa/a^2$ where a is the dike half width, κ is the thermal diffusivity, and t is time; therefore, for a dike 2 m in width at $t = 0.2$, $t = 7.72$ days, and for a dike four times that width (8 m) at $t = 0.2$, $t = 7.72 \times 16 = 123.5$ days. (Redrawn from Delaney, 1987.)

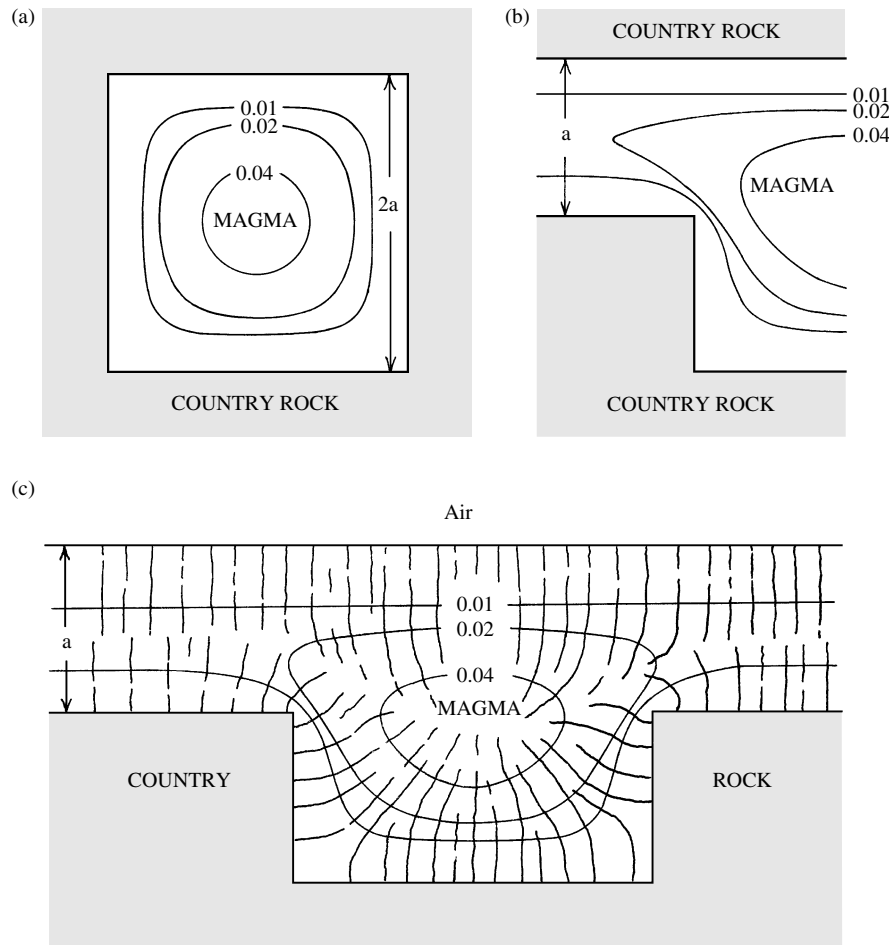
An important factor in the conductive cooling of a magma body is its shape (Section 6.3.1). Isotherms in Figure 8.19 illustrate the influence of the shape of a magmatic body on its conductive cooling. Heat is conducted away from outside corners faster than along planar sides because the corner volume has a considerable area (roughly twice that of the sides) through which heat can be conducted. Therefore, after some cooling, isotherms are located farther into the body near corners. In contrast, near reentrant corners where country rock projects into the magma body (Figure 8.19b), heat from the two adjacent sides conducts into the same mass of country rock, which thermally “saturates,” reducing the country rock thermal gradient, thus impeding cooling of the magma in that vicinity; therefore, isotherms are crowded together. In an extrusive body (Figure 8.19c), more heat conducts from the surface than from the base; consequently the part of the body that remains hottest longest is not at the center but is displaced downward. Actually, the downward

displacement is more extreme in real lava flows, where radiation and convection also dissipate heat into the atmosphere. Beneath relatively thin lava flows, especially those having insulating rubbly bases, not much heat is conducted into rock material beneath the extrusion before it cools from above so that thermal effects on the substrate are minimal.

It must be emphasized that these conductive cooling models only apply to bodies of static, or motionless, magma and ignore other heat transfer processes. Nonetheless, these models serve as a valuable “baseline” against which to evaluate thermal histories of more dynamic advective and convective systems where movement of liquids facilitates heat transfer.

✱8.5 ADVECTIVE HEAT TRANSFER

Rocks in the brittle upper crust are fractured. Any outcrop or roadcut is laced with cracks, usually of two or more orientations and spaced centimeters to no more



8.19 Isotherms in cooling magmatic bodies. Magma bodies extend indefinitely in the third dimension perpendicular to the page. Isotherms are $0.8T_0$ at nondimensional times (see caption Figure 8.18) $t = 0.01, 0.02$, and 0.04 after emplacement of magma initially at a uniform temperature T_0 . The pattern of a family of isotherms at different temperatures at one instant of time would be similar. Country rocks (shaded) are initially at a uniform temperature. (a) and (b) are intrusions and (c) is an extrusion. In (a), a magma conduit that is square in cross section and 8 m on a side, the $0.8T_0$ isotherm has advanced more than two-thirds of the way into the conduit and is a circle (cylinder in three dimensions) after a time $t = \tau a^2/\kappa = [0.04 \times (4\text{m})^2]/10^{-6} \text{ m}^2/\text{s} = 6.4 \times 10^5 \text{ s} \times 1.16 \times 10^{-5} \text{ days/s} = 7.4 \text{ days}$. In (c), contraction-induced joint columns oriented perpendicular to isotherms are shown schematically by irregular lines. (Redrawn from Jaeger, 1968.)

than a meter or so apart. These fractures are commonly filled with fluid, usually aqueous. Deep drilling and study of exhumed once-buried rocks reveal the presence of fluids in cracks to depths of at least 10 km. Even in the deeper crust and upper mantle, where rock flows as a viscous or ductile material, extensional fractures must still form by hydraulic fracturing, as evidenced by sheetlike magma intrusions and veins (Plate VI) found in such rocks now exposed at the surface.

The significance of these facts is this: Movement of magma and hydrothermal fluids in the Earth through interconnected openings in rock can transfer heat much more rapidly than by conduction alone. The dry country rock model assumed for simplicity in conductive calculations is inappropriate for many geologic environments. **Advection** of liquids through passageways in rocks “short-circuits” heat transfer from a cooling

magma intrusion via a lower resistance path into the cooler country rocks. Far traveling advecting liquids can move faster than heat can conduct through solid rock. Magma derived from a central intrusive mass can quickly invade fractures in the surrounding rock, forming dikes and sills. Swarms of such sheetlike intrusions are common over large plutons (see Figure 9.5) and testify that heat has been transferred far into the country rocks. Exsolved aqueous fluids expelled from a crystallizing magma intrusion might mix with larger volumes of meteoric water lodged in country rock openings to form huge advective systems (Section 4.3.3).

In the absence of preexisting fractures, or where preexisting fractures are not suitably oriented in the local stress field to be open, high-pressure magmas and hydrothermal fluids can move into self-generated

hydraulic extensional fractures (Figure 8.2c). At shallow depths of the crust, wholesale brecciation can occur where roof rocks rupture over an intrusion if excessive fluid pressures develop (last part of Section 4.3.1). Magma and fluids advecting into widely distributed cracks and breccia openings rapidly lose heat by conduction to the large contacting surface areas of cooler rock.

Advective transfer of heat associated with magmas ascending from the mantle elevates geothermal gradients in the crust above subduction zones and in rifts. As a result, deeper crustal temperatures locally reach the solidus and cause magma generation.

Advection requires permeable rock, as either preexisting channels or self-induced hydraulic extensional fractures created by the advecting magma or fluid. **Permeability** is the ease with which fluid can move through interconnected openings. **Porosity** is the proportion of openings available in the rock to hold fluid: that is, pore volume/total rock volume. In a lava flow, vesicles and cracks formed during thermal contraction and perhaps by continued flow of the still-mobile interior create porosity. In a plutonic rock, such as granite, porosity lies in cracks and perhaps vugs produced by exsolved fluids. Porosity and permeability generally decrease with depth in the Earth, especially if related to cracks.

*8.6 MAGMA CONVECTION

Convection is a more efficient mode of heat transfer, by one to two orders of magnitude, than conduction in cooling magma bodies. Convecting magma bodies transfer more heat to the roof rocks, where solidus temperatures can be exceeded, causing partial melting, especially above large mafic intrusions. Convection also influences the way a magma body crystallizes and unmixes into compositionally diverse parts, or remains homogeneous.

The essence of **convection** is transfer of heat within a body by buoyant movement of thermally contrasting parts of it. (In contrast, advection is movement of a liquid through openings in a solid.) In magma systems, convection may be initiated by injection of hotter magma into the base of a chamber filled with cooler, denser magma, causing a convective overturn to restore gravitational equilibrium. Convection also occurs during foundering of slabs of cooler, dense roof rock into a magma chamber, causing cool magma adjacent to the slab to be dragged into the hotter interior. The convection discussed in the remainder of this section is driven by internal density differences within the liquid body caused by contrasts in T and/or composition.

Purely thermal convection in a homogeneous melt is considered first to establish some concepts of con-

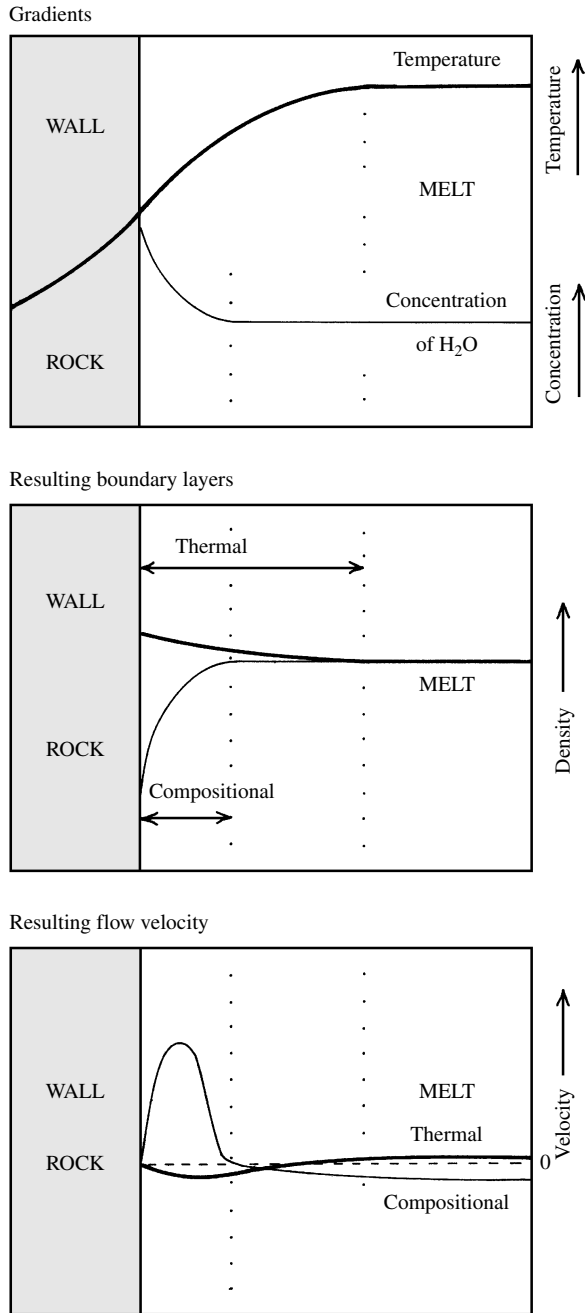
vection. Then follows a discussion of convection in more typical crystallizing magmas, where contrasts in both T and composition drive thermochemical convection.

8.6.1 Thermal Convection in a Completely Molten Body of Melt

Convective motion of water in a pot on a hot stove is a familiar example of thermal convection. Gravitational stabilization occurs when cooler, more dense fluid at the top sinks and hotter, less dense fluid at the bottom rises. The released gravitational potential energy is consumed in overcoming viscous resistance to flow.

As a consequence of conductive heat loss into the wall rock, the T of a melt decreases exponentially toward a vertical contact of an intrusion (Figure 8.20). This cooler and, therefore, denser melt in the thermal boundary layer is negatively buoyant and potentially able to sink. A gradient in the concentration of water in the melt near the contact may develop as a result of inward diffusion from water-bearing wall rock. This more hydrous melt is less dense, positively buoyant, and potentially able to rise. *Any* density contrast in a steeply inclined thermal or compositional boundary layer (Figure 8.20) is gravitationally unstable, but whether the buoyancy is sufficient to cause movement depends on melt rheology. If movement of the boundary layer does occur, return flow must necessarily occur farther into the closed body of melt. These complementary flows establish convection.

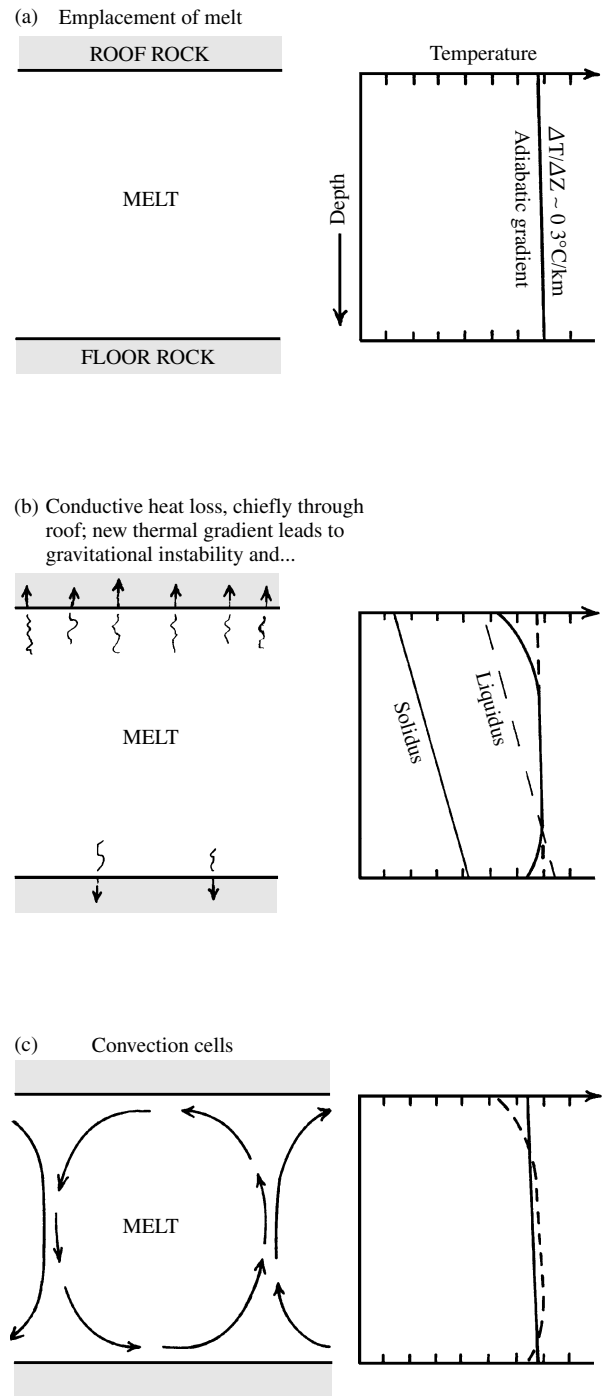
What happens at more or less horizontal contacts between melt and country rocks, such as at the margins of a flat sill (Figure 8.21)? For simplicity, assume that the body of melt, *initially*, has a uniform T above the liquidus. Self-compression in the insulated intrusion due to the downward increasing weight of the melt creates an adiabatic T gradient that is only a few tenths of a degree per kilometer. (Recall in Section 3.2.1 that compressive work done on a system transforms into thermal energy.) Whether gravitational instability develops in the melt near the roof as it cools and becomes denser depends on the contrast in T between roof melt, T_r , and floor melt, T_f , or $(T_f - T_r)$. More heat tends to be lost by conduction through the roof than the floor of a thick horizontal sheet because shallower rocks are cooler on an ordinary geotherm and conductive heat transfer is greater where the thermal gradient (equation 1.5) is greater. Additionally, for shallow intrusions, groundwater in fractured roof rocks absorbs heat and advects it away. After conductive, and possible advective, heat loss (Figure 8.21b), the cooler melt at the roof might be dense enough to overcome viscous resistance to flow, causing convective overturn (Figure 8.21c). Cooler, denser melt sinks and deeper, hotter, less dense



8.20 Thermal and compositional gradients create gravitationally unstable boundary layers along the vertical wall of a body of melt. Conductive transfer of heat to the wall rock creates a broad thermal gradient. Heavier lines in middle and bottom diagrams pertain to thermal boundary layer and thinner lines to compositional boundary layer. Actual boundary layers differ in breadth more than shown here. In thermochemical convection, the effect of decreasing T at the wall is counteracted by water enrichment in residual melts produced by sidewall crystallization, which creates a less dense, positively buoyant boundary layer that can float upward.

melt moves to the top of the body. The T gradient in the gravitationally restabilized body of melt is again adiabatic. Further heat loss through the roof rocks would initiate further convective overturn, either in episodes or in a steady state.

Gravitational stability in this sort of fluid body can be quantified in terms of $(T_f - T_r)$, with other relevant factors cast into a ratio of buoyancy forces driving convection (in numerator) to opposing resistive drag forces (in denominator)



8.21 Thermal convection in a horizontal slab of melt cooled mostly at the roof by conduction. The melt and roof and floor country rocks extend indefinitely to the right and left of the three vertical cross sections shown here. Thermal gradient shown by solid line, pre-existing gradient by dashed line.

$$8.10 \quad Ra = \frac{[\rho g \alpha (T_f - T_r) h^3]}{\eta \kappa}$$

where ρ is the density, g is the acceleration of gravity, α is the coefficient of thermal expansion, h is the vertical thickness of the convectable fluid, η is the viscosity, and κ is the thermal diffusivity. The **Rayleigh number**, Ra is a dimensionless number that prescribes whether convection occurs. For magma chambers, Ra must be > 500 – 2000 , depending on the exact shape of the melt body. The larger the ratio of buoyant to resistive forces, the more vigorous is convection. Convection occurs in roughly equidimensional convection cells (Figure 8.21c). Occurrence and vigor of thermal convection are most sensitive to four factors:

1. Thickness of the melt body, h ; doubling h increases Ra by a factor of 8.
2. $(T_f - T_r)$ the difference in T between the bottom and top of the magma body.
3. Viscosity, η , which ranges over many orders of magnitude.
4. Density, ρ , which is most sensitive to composition, especially the concentration of dissolved water or to the proportion of exsolved volatile bubbles in a volatile saturated magma.

Superliquidus bodies of homogeneous low-viscosity basaltic melt of virtually any vertical dimension convect. Thick bodies of more viscous water-rich granitic melts with large values of $(T_f - T_r)$ might convect, but dry granitic melt bodies less than a kilometer or so thick probably do not thermally convect.

Because the liquidus T of minerals increases about 3°C per kilometer depth whereas the adiabatic gradient in melts is about an order of magnitude less, crystallization occurs at the base of a uniform melt body of considerable vertical thickness (kilometers, rather than meters), even though most of the cooling occurs through the roof (Figure 8.21b). However, in magma chambers that have a vertical compositional gradient, the preferential bottom crystallization may not occur.

8.6.2 Thermochemical Convection in Crystallizing Magmas

Beginning with the pioneering studies of Shaw (1965), numerous theoretical and experimental studies have shown that compositional buoyancy is far more significant in driving convection than that resulting from thermal gradients alone (e.g., McBirney, 1980; Sparks et al., 1984). Whereas the variation in density from 800°C to 1200°C for a particular melt composition is only about 0.1 g/cm^3 or less, common volatile-free melt densities at 1 atm and, say, 1000°C range from 2.2 to 2.8 g/cm^3 depending on composition (Figure 8.15). Density variations in volatile-bearing crystallizing magmas can be much greater.

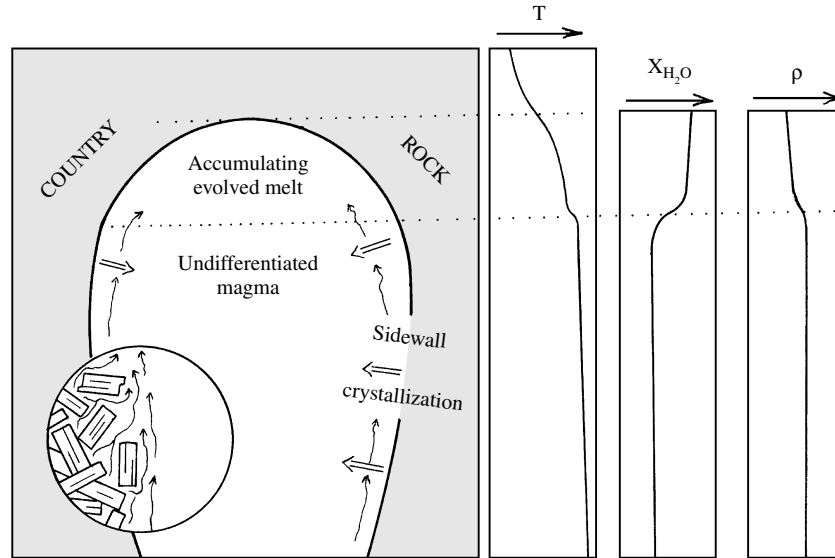
In crystallizing magmas, the residual melt in equilibrium with precipitating crystals is always different in composition—and, therefore density—from the initial melt. (This principle is a central theme of Chapter 5.) For example, the residual melt just above the solidus in basalt magmas (see the Makaopuhi basalt in Plate IIIId) is enriched in silica and alkalis and is approximately of rhyolite composition; the density contrast between rhyolite and basalt melts is about 0.4 g/cm^3 . Residual melts in fractionating mid-ocean ridge basalt magma vary by more than 0.2 g/cm^3 (Figure 8.16) and in basaltic andesite magma by about 0.1 g/cm^3 . If a melt becomes water-saturated, the bubbles of exsolved water can substantially lower the density of the vesicular melt.

Convection driven by compositional differences depends on a density difference between different parcels, A and B, of magma; that is, a gradient in density ($\Delta\rho = \rho_A - \rho_B$). However, since gradients in both T and composition occur in crystallizing magmas, the dynamic process is known as **thermochemical convection**, or, because the relative rates of thermal and chemical diffusion govern these dynamic systems, **double-diffusive convection**.

Magmas can be envisaged to crystallize in two end-member chamber shapes, namely, bottle-shaped vertical cylinders in which subvertical walls dominate the external contacts and flat slabs dominated by a sub-horizontal roof and floor. Two end-member magma compositions may also be considered:

1. Calc-alkaline magmas, such as andesite and dacite, in which residual melts are generally more enriched in silica, alkalis, and water, so that they have lower density and are positively buoyant relative to the initial parent magma. Dissolved water has the greatest effect in promoting buoyancy.
2. Basalt magmas, most commonly tholeiitic, in which residual melts tend to be more Fe-rich, more dense, and negatively buoyant.

Bottle-Shaped Magma Chambers. At near-vertical wall-rock contacts, cooler temperatures in the thermal boundary layer of the magma produce **sidewall crystallization**. In calc-alkaline magmas, less dense, positively buoyant residual melt can free itself from the crystal mush in the boundary layer and rise, collecting into a pool at the top of the magma chamber (Figure 8.22). The chamber, filled initially with what may be compositionally uniform magma, unmixes into contrasting parts, a cap that is enriched in silica, alkalis, and water and an interior that is less evolved in composition. Although this unmixing cannot be directly observed, the geologic record of countless compositionally zoned pyroclastic deposits (see, for example, Figure 10.38) is widely interpreted to have resulted by eruption of magma from a chamber subjected to thermochemical convection driven by sidewall crystallization. Moreover, model studies in tanks of room- T , multicomponent



8.22 Thermochemical convection in a crystallizing bottle-shaped calc-alkaline magma chamber. Pronounced compositional stratification can be produced in an initially homogeneous magma chamber. **Sidewall crystallization** (double-line arrows) yields less dense, silica- and water-enriched residual melt that can separate from the associated mush of crystals adhering to the chamber wall (inset circular diagram at enlarged scale). This melt buoys upward and accumulates in a gravitationally stable, growing cap zone. Schematic properties of the stratified chamber are shown in three graphs on right. Continued heat loss from the magma body allows inward advance of the crystallizing wall so that the final solidified pluton can be concentrically and vertically zoned in composition. Explosively erupted silicic magmas are derived from the upper volatile-rich parts of such stratified chambers.

saline solutions show the phenomenon to be viable. Alternatively, one is left with the dilemma of how to create compositional differentiation in highly viscous magma chambers in which rates of atomic diffusion are exceedingly slow, purely thermal convection cells may be precluded because of high viscosities, and through-chamber crystal settling is very slow, or nonexistent, because of non-Newtonian viscosity.

In addition to, or in lieu of, sidewall crystallization, a positively buoyant compositional boundary layer at a vertical wall might originate in two other ways:

1. Hot magma may raise the wall rock T to above its solidus, generating a low-density partial melt that segregates and buoys upward.
2. Relatively dry melt can absorb water from wet wall rock, reducing the melt density. However, because the rate of chemical diffusion of water is orders of magnitude slower than thermal diffusion—about 10^{-10} m²/s versus 10^{-7} m²/s—the chemical boundary layer is thinner than the thermal (Figure 8.20).

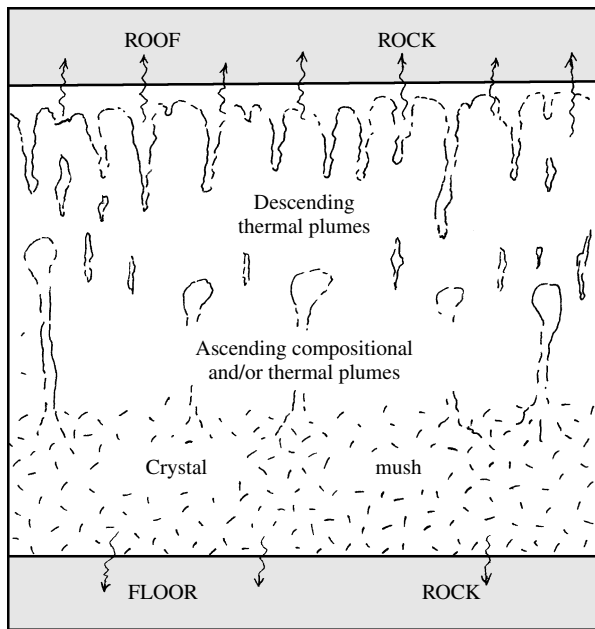
A vertical compositional gradient in a magma chamber has implications not only for differentiation of magmas but also the way they erupt.

In bodies of mafic tholeiitic magmas with predominantly vertical walls, residual melts resulting from crystallization are enriched in Fe and, if not also too enriched in volatiles, are more dense (Figure 8.16). This residual melt, or possibly a dense crystal-laden magma, may sink en masse along the wall of the magma cham-

ber and onto its floor as a density current, not unlike sediment-laden turbidity currents in standing bodies of water.

Flat Slabs. The dynamics of thermochemical convection differ in magma chambers lacking extensive sidewalls and dominated instead by a subhorizontal floor and roof. One possible situation is shown in Figure 8.23, which may be compared with Figure 8.21, where it was shown that most of the heat loss is through the roof but most of the crystallization may be at the floor. Cooling melt at the roof becomes denser and sinks, whereas floor crystallization of calc-alkaline magmas could release compositionally buoyant residual melt that might also be thermally buoyant because of the release of latent heat during crystal growth. Residual melts from fractionating tholeiitic basalt magmas could be more Fe-rich and more dense unless compensated by latent heating and water enrichment. Depending on the contrasts in viscosity of different magma parcels, varying degrees of magma mixing and homogenization may occur by ascending and descending plumes, retarding differentiation of the chamber magma. This contrasts with compositional differentiation, or magma “unmixing,” which can occur in bottle-shaped magma chambers.

These contrasts in the convective dynamics of bottle-shaped and flat-slab magma chambers demonstrate how a seemingly irrelevant factor such as chamber shape can influence the compositional evolution of magmas (de Silva and Wolff, 1995).



8.23 Thermochemical convection in a flat slab of crystallizing magma. Compare Figure 8.21. In contrast to magma unmixing in a bottle-shaped chamber (Figure 8.22), the magma in this slab tends to be homogenized and mixed by the descending and ascending plumes.

Special Interest Box 8.2 Controversial origin of layering in the Skaergaard Intrusion, Greenland

The Skaergaard Intrusion, magnificently exposed in fjords near the Arctic Circle in eastern Greenland, is briefly described in Section 12.4.2. The Skaergaard has served for decades as a supreme example of the effects of convection in a basaltic magmatic body, creating a wide variety of layering (Figure 7.45) together with an extreme compositional differentiation.

The classic investigation of Wager and Deer (1939) and subsequent studies by Wager and his associates concluded that the 2.5-km-thick sequence of subhorizontally layered rocks making up most of the intrusion was, for the most part, a result of magmatic sedimentation. They envisaged convection currents of crystal-laden magma descending from the roof and walls and sweeping across the floor, sorting and depositing the crystals according to their differing densities. Repeated currents were believed to have created the rhythmic modally graded layers (mafic minerals more concentrated downward and plagioclase upward in each layer) that dominate the layered sequence.

However, Bottinga and Weill (1970) pointed out that during the fractional crystallization of the Skaergaard magma the plagioclase crystals should have floated in the increasingly Fe-enriched residual

melts. McBirney and Noyes (1979; see also an updated discussion of the intrusion in McBirney, 1996) attempted to reconcile this paradox of how apparently floating plagioclases could form the major mineral constituent in the rhythmic layers in terms of a non-Newtonian magma. McBirney and coworkers proposed that other processes, mainly of a kinetic character (Section 7.9.2) in more or less static floor zones, were involved in creating the various types of layering.

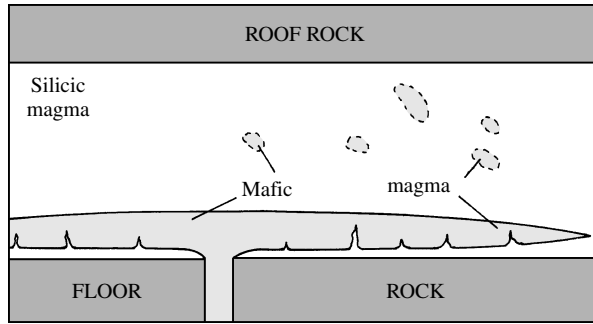
It was not until the 1990s that sufficient data became available to determine the densities of hydrous melts of any composition with reasonable accuracy (Lange, 1994). Prior conclusions regarding plagioclase flotation were necessarily based on anhydrous melt models using less accurate data for partial molar volumes of condensed components (CaO, MgO, etc.). The influence of dissolved water on melt densities is significant, and it can be shown (Problem 8.13) that modest water concentrations in Fe-rich residual melts can allow sinking of plagioclase in the Skaergaard. But whether these modest concentrations actually prevailed cannot be answered because of the lack of independent information. The widespread absence of biotite and amphibole in the evolved Skaergaard differentiates only precludes water concentrations of less than about 3–4 wt. %.

The origin of compositional layering in the Skaergaard (Irvine et al., 1998) and, by implication, many other layered intrusions remains controversial.

8.6.3 Replenishment in Evolving Magma Chambers

Lifetimes of magma chambers are commonly extended by episodes of **replenishment**. New draughts of introduced magma are usually hotter, commonly denser, and nearly always less viscous than the resident evolving magma already in the chamber. Consequently, as the new magma is injected from below, its upward momentum and buoyancy carries it well into the chamber, leading to mixing and possible eventual homogenization. This scenario may occur at oceanic spreading ridges as primitive basalt magma from a mantle source replenishes somewhat evolved basalt magma already in the chamber (see Figure 12.12).

In another scenario, slowly moving new magma of more mafic composition rises into an evolving continental silicic chamber and spreads across the floor. Model experiments using a tank of aqueous solutions of contrasting density and viscosity (Snyder and Tait, 1995) reveal that the invading magma traps a layer of the less dense magma beneath it (Figure 8.24). The invading magma moves laterally as subparallel fingers,



8.24 Replenishment of a silicic chamber by basaltic magma that has been introduced from a feeder below the chamber. The basalt magma (light shaded) forms a lens near the base. A thin underlying layer of less dense silicic melt is gravitationally unstable and has penetrated into the lens, initiating convective breakup. Overlying blobs of basalt from a previous episode of replenishment and convective disruption are dispersed through the silicic intrusion. After complete crystallization these blobs may be preserved as mafic inclusions in the granitic host.

resembling a hand inserted into some liquid. The gravitationally unstable layer of resident silicic magma entrapped at the base of the chamber subsequently forms buoyant plumes that pierce upward through the denser layer of new recharging magma, disrupting it into pillowlike blobs, a possible example of which is shown in Figure 8.25. Convective motion in the chamber due to the heating of the silicic magma by the hotter invaded basaltic magma disperses the blobs. Alternatively, or in addition, as the basaltic magma cools and crystallizes, its melt may become volatile-saturated, reducing magma density and causing convective mingling of basalt magma into the silicic magma. Whatever the dispersive mechanism, the blobs of basalt may become the



8.25 Pillowlike bodies of dark gabbro in syenite that appear to indicate coexisting gabbro and syenite magmas. If the syenite magma had intruded into fractured solid gabbro, pieces would be angular. The mingled magmas may have originated by replenishment of a syenite magma chamber by gabbro magma intruding as a basal lens and subsequent breakup of the lens, in the manner of Figure 8.24. (U.S. Geological Survey photograph courtesy of R. E. Wilcox and Louise Hedricks.)

ubiquitous mafic inclusions in granitic intrusions (Section 7.10). Hybridization and other effects of the replenishment significantly impact the compositional evolution of the silicic-mafic magma system (e.g., Wiebe, 1996).

SUMMARY

Forces acting on a body create a state of stress that can be conveniently represented by three orthogonal principal stresses, σ_1 , σ_2 , σ_3 . Near the surface of the Earth, one of the principal stresses is vertical and the other two are horizontal. Hydrostatic states of stress, where $\sigma_1 = \sigma_2 = \sigma_3$, prevail in fluid bodies and in the deep ductile crust and asthenospheric mantle because rock strengths are small, especially at slow strain rates, and cannot sustain large stress differences of a nonhydrostatic state. The strain resulting from hydrostatic stress is a change in volume, whereas nonhydrostatic states of stress (where $\sigma_1 > \sigma_2 > \sigma_3$) produce changes in volume and shape of a body.

The rheologic response of most magmas and rocks to applied stress involves combinations of ideal elastic, plastic, and viscous behavior. Rocks and nearly crystallized magmas near their solidus deform by brittle (essentially elastic) fracturing at rapid strain rates. Extensional fractures form perpendicular to σ_3 . Hydraulic fractures are self-generated by magmas and hydrothermal solutions, whose pressure counteracts P . At increasing depth in the crust the brittle strength of rocks increases, but the ductile strength diminishes exponentially as thermally activated mechanisms at the atomic scale take the place of fracturing. Viscouslike ductile flow is favored over brittle fracturing by low strain rates, which allow more time for atomic movement; by elevated T , which promotes atomic mobility; and by high P , which increases frictional resistance and impedes brittle dilatant deformation. Ductile strength under any of these conditions also depends upon the mineral composition of the rock and this varies with depth in the lithosphere; thus, rheology and strength are stratified in the lithosphere.

During crystallization of a body of magma its rheologic behavior progressively changes from Newtonian, through non-Newtonian with exponentially increasing apparent viscosity, to that of a brittle solid. The Newtonian viscosity of near-liquidus melts depends mostly on T and concentration of silica, water, and fluorine. More crystal-rich, non-Newtonian magmas possess a plastic yield strength, and applied stress below this cannot produce flow.

The rheology and flow velocity of most magmas result in laminar flow. Grain dispersive pressure concentrates suspended particles in a flowing magma

into the interior of the channel or conduit. Plug flow occurs in viscoplastic magmas.

Compared to the many orders-of-magnitude range in viscosities of magmas, densities vary at most by only a factor of about 50. Most of this variation is in volatile-oversaturated magmas in which bubbles can reduce the density to as low as 0.05 g/cm^3 , compared to $2.8 - 2.2 \text{ g/cm}^3$ in bubble-free melts. The small range in bubble-free melt densities is primarily a result of their composition, especially the concentration of dissolved water; thermal expansion and compressibility of melts are small. The density of magmatic rocks is 10–20% greater than their corresponding melts. Despite these relatively small density variations and correspondingly subtle contrasts in density between melt, magma, and rocks, they nonetheless provide a significant buoyant force in large volumes that can overcome viscous resistive forces to drive magma ascent, convection, mixing, unmixing, and other dynamic phenomena in magma systems.

The efficiency of conductive cooling is related to the surface area over which heat can be lost relative to the volume. For a particular shape of body, cooling times increase as the square of a critical dimension. Although conductive cooling (thermal diffusion) rates are so slow that even modest-size plutons a few kilometers in diameter may require tens of thousands of years to crystallize, chemical diffusion rates are orders of magnitude slower still; therefore, little chemical transport is possible in conductively cooling, static bodies.

Advection of liquids through openings in permeable country rock is an efficient heat transfer process that hastens cooling of intrusions. Advecting magma and hydrothermal fluids in country rock can produce economically viable ore deposits and geothermal reservoirs.

Thermal and especially compositional gradients within bodies of magma create internal density contrasts. Gravitational instability of contrasting density parcels can drive convection if viscous resistance can be overcome. Convecting bodies generally cool more rapidly than by conduction and transfer more heat into roof rocks, in some cases sufficient to melt the roof. Despite the generally greater heat loss at the roof, crystallization can occur primarily at the base of a vertically extensive body because the liquidus T increases with depth at a greater rate than the adiabatic gradient.

Thermochemical convection is likely to be more common than purely thermal convection because density contrasts in melts in cooling crystallizing magma are larger than can be produced by any differences in T . It is widely believed that thermochemical convection occurs in tall bottle-shaped calc-alkaline magma bodies by sidewall crystallization. This creates a buoyant boundary layer of silica- and water-enriched residual

melt that can separate from the growing crystals and float upward, ponding at the top of the compositionally stratifying magma chamber. This more evolved, commonly water-oversaturated capping magma can erupt explosively. Other thermochemical convective systems are possible in magma chambers having different shapes, such as horizontal slabs, and different compositions, such as mafic tholeiitic magma that yields denser Fe-rich residual melts. Depending on magma composition and chamber shape, thermochemical convection can lead to magma differentiation and unmixing of an initially homogeneous magma or to stirring and mixing, precluding differentiation.

The physical and thermal dynamics of magmas as well as the geometry of their chambers play a significant role in the compositional diversity of igneous rocks.

CRITICAL THINKING QUESTIONS

- 8.1 Contrast shear, normal, and principal stresses and hydrostatic, nonhydrostatic, and total states of stress.
- 8.2 What is the difference between strain and strain rate? What are their units?
- 8.3 Distinguish among ideal elastic, plastic, and viscous responses to applied stress and composite behavior such as viscoelastic and viscoplastic behavior. Draw mechanical analogs, in the manner of Figure 8.3, for the two types of composite behavior.
- 8.4 Contrast brittle and ductile behavior in rock with regard to mechanisms and geologic conditions.
- 8.5 How do extensional fractures develop? Discuss with respect to geologic conditions under which they form, especially the role of liquids, and to state of stress in the Earth.
- 8.6 Sketch how, and explain why, rock strength varies with respect to depth in the lithosphere.
- 8.7 Discuss factors governing the rheology of magma, and contrast Newtonian and non-Newtonian behavior.
- 8.8 Why do most magmas flow in a laminar manner?
- 8.9 What does plug flow tell about the rheology of the flowing material? Why?
- 8.10 How can flow of a partially crystallized magma impact its composition?
- 8.11 Discuss factors that govern melt density. Magma density.
- 8.12 How do viscosity and density interact in dynamic magma systems to control particle buoyancy?

- 8.13 Contrast the control on buoyant motion of large blocks of foreign rock immersed in a body of magma versus with buoyant motion of an isolated crystal.
- 8.14 Characterize factors governing heat transfer by conduction.
- 8.15 Discuss relevant factors and processes in advective heat transfer.
- 8.16 What drives convection and what retards it?
- 8.17 How do advective and convective transfer of heat differ?
- 8.18 How do thermochemical and thermal convection differ?
- 8.19 How can the geometry (size and shape) of a body of magma influence its chemical evolution?

PROBLEMS

- 8.1 In Figure 8.2a and 8.2b calculate the linear strain in the directions of σ_1 and σ_3 by carefully measuring lengths l_0 , l_1 , and l_3 with a millimeter scale. (*Partial answer:* In the σ_1 direction $\epsilon_1 = -0.2 = -20\%$.)
- 8.2 What is the change in volume of an assumed isotropic crystal in moving it isothermally from the surface of the Earth to a depth of 35 km in the continental crust? (*Answer:* 1%.)
- 8.3 Show that the state of stress in the deep crust is virtually hydrostatic by comparing the ductile strength of “wet” granite at a depth of 30 km (Figure 8.8) with the magnitude of the confining pressure, P , at that depth. (*Answer:* The strength, which is the maximum stress difference possible at that depth, is 0.001% of P .)
- 8.4 Draw a stress-strain rate diagram for a viscoplastic body that obeys equation 8.3.
- 8.5 Using MELTS software (Advanced Topic Box 5.3) explore the counteracting effects on residual melt viscosity of increasing concentrations of silica and water in a fractionating andesite magma whose initial water content is 1.5 wt.%. Using equation 8.4 determine the apparent viscosity of the magma at T increments of 25°C below the liquidus. Discuss your results and any assumptions made.
- 8.6 Determine and discuss the flow regime (laminar versus turbulent) of (a) the upper mantle whose apparent viscosity is approximately 10^{21} Pa s; (b) a river of water whose velocity is as much as 20 m/s; (c) a river of basalt lava near its liquidus T flowing at a velocity of 8 m/s in a channel 2 m deep and 5 m wide. For each situation explain or justify your choice of viscosities and boundary conditions.
- 8.7 Calculate the approximate density of an andesite magma at 1000°C and 6 bars that contains 75 volume % water bubbles. Use values for water and melt density read from Figures 4.4 and 8.15.
- 8.8 Determine the laminar flow velocity of a pahoehoe “tongue” of basalt melt 1 m wide and 0.3 m thick. Justify assumptions made in this determination.
- 8.9 Compare the relative effect on the density of a silicate melt by increasing P versus increasing T in the crust of the Earth. Do melt densities increase or decrease with depth? Assume the geothermal gradient is $\Delta T/\Delta z = 25^\circ\text{C}/\text{km}$ and the geobaric gradient is $\Delta P/\Delta z = 27 \text{ MPa}/\text{km}$. (*Answer:* The compressive effect of P per 1-km increase in depth is 2.5 times that of the expansive effect of T per kilometer.)
- 8.10 Compare the distance traversed by isolated crystals 1 mm in diameter of plagioclase ($\rho = 2.65 \text{ g}/\text{cm}^3$) and pyroxene ($\rho = 3.3 \text{ g}/\text{cm}^3$) in a granite melt ($\eta = 10^7 \text{ Pa s}$; $\rho = 2.3 \text{ g}/\text{cm}^3$) and a basalt melt ($\eta = 10^2 \text{ Pa s}$; $\rho = 2.7 \text{ g}/\text{cm}^3$). Indicate any assumptions made in your calculations. (*Partial answer:* The pyroxene would settle 100 m in 1 year in the basalt melt.)
- 8.11 Using the 1-atm density of tholeiite melt in Figure 8.15 and a 1-atm density for An_{60} of $2.7 \text{ g}/\text{cm}^3$ determine the P at which the plagioclase crystals are neutrally buoyant at 1200°C. Indicate assumptions made in this determination.
- 8.12 A famous basalt flow on Hualalai volcano on the Island of Hawaii contains mantle-derived peridotite xenoliths that are as much as 70 cm in diameter. What minimum ascent velocity would have been required to lift these xenoliths to the surface of the Earth so they would not sink en route? For the acceleration of gravity use $980 \text{ cm}/\text{s}^2$ and note that $1 \text{ Pa} = 10 \text{ g}/\text{cm s}^2$. Justify your choice of parameters and explain any assumptions made in the calculation.
- 8.13 Read Special Interest Box 8.2. Using the Lange density equations in Advanced Topic Box 8.1 and the partial molar volumes in Table 8.1 show that crystals of plagioclase An_{58} ($\rho = 2.70 \text{ g}/\text{cm}^3$) would float in a tholeiitic basalt melt equivalent to the LZb Skaergaard composition in Table 12.6 because its density at 1160°C and 2160 bars is calculated to be $2.76 \text{ g}/\text{cm}^3$. For this calculated melt density, partition the 12.84 wt.% total Fe as FeO listed in Table 12.6 into 2.14 wt.% Fe_2O_3 and 10.91 wt.% FeO and assume

Table 8.1. Partial molar volumes, thermal expansivities, and compressibilities of oxides at 1673 K and 1 bar. Data from Lange (1994) except for H₂O which is from Ochs and Lange (1997).

i	v_i cm ³ /mole	dv_i/dT 10 ⁻³ cm ³ /mole K	dv_i/dP 10 ⁻⁴ cm ³ /mole bar
SiO ₂	26.90	0.00	-1.89
TiO ₂	23.16	7.24	-2.31
Al ₂ O ₃	37.11	2.62	-2.26
Fe ₂ O ₃	42.13	9.09	-2.53
FeO	13.65	2.92	-0.45
MgO	11.45	2.62	0.27
CaO	16.57	2.92	0.34
Na ₂ O	28.78	7.41	-2.40
K ₂ O	45.84	11.91	-6.75
H ₂ O	27.75	10.86	-3.82

0.5 wt. % dissolved water. Also show that the same plagioclase would be neutrally buoyant in the same melt but contains about 1.2 wt. % dissolved water.

- 8.14 Using the “characteristic dimensionless time” of Jaeger (1968) in Section 8.4.1 compare the cooling history of a granite batholith 20 km in diameter, a granite stock 1 km in diameter, and a basaltic dike 0.2 m in width. Discuss assumptions made in this comparison. (*Partial answer:* The dike is almost entirely cooled after 28 hours.)
- 8.15 Calculate the Rayleigh number, Ra , in terms of $(T_f - T_r)$ for a 200-m-thick horizontal sill of uniform basalt melt. What vertical thermal gradient is necessary for thermal convection to occur? Is this reasonable? Discuss. Assume the values of $g = 9.8 \text{ m/s}^2$, $\alpha = 3 \times 10^{-5}/\text{deg}$, and $\kappa = 10^{-6} \text{ m}^2/\text{s}$, $\eta = 10 \text{ Pa s}$, and $\rho = 2.6 \text{ g/cm}^3$.