... SUL CARICO CRITICO DI EULERO

SDC, 5/12/23

$$P_{cr} = \frac{EI\pi^2}{l_0^2}$$

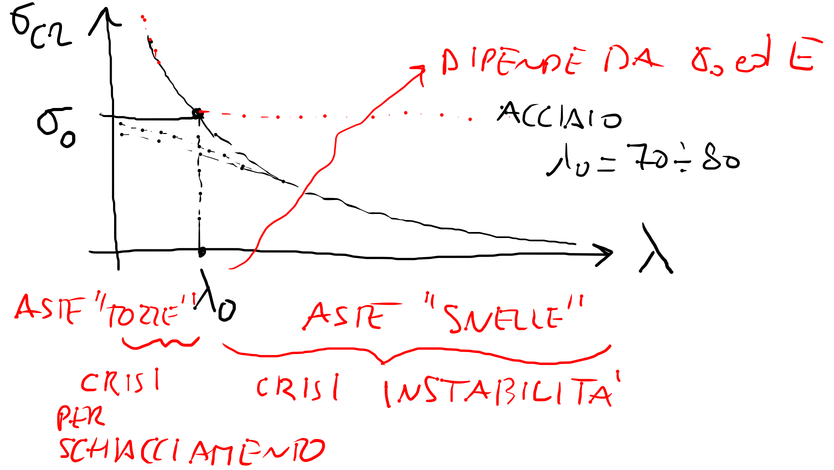
$\leftarrow P_{cr}$ $(N=-P_{cr})$

Quando $P \le P_{cr}$ allora il problema strutturale vede la presenza solo di forza normale $N = -P$

$$\sigma_{zz} = \frac{N}{A} = \sigma$$ → teniamo conto che σ è di compressione, allora

$$|\sigma_{cr}| = \frac{P_{cr}}{A} = \frac{EI\pi^2}{l_0^2 A} = \frac{E\pi^2 \rho_x^2}{l_0^2} = \pi^2 \frac{E}{\lambda^2}$$ IPERBOLE DI EULERO

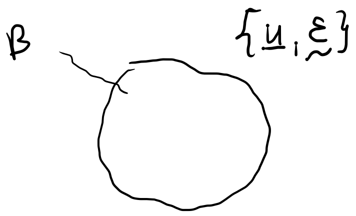
Chiamo $\lambda = \frac{l_0}{\rho_x}$: SNELLEZZA $\quad [\lambda] = [-]$

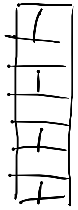


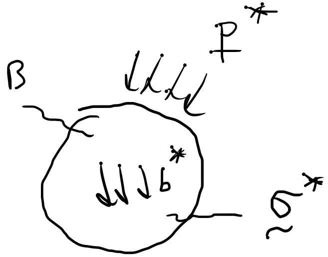
$$\rho_x = \sqrt{\frac{I_x}{A}}$$

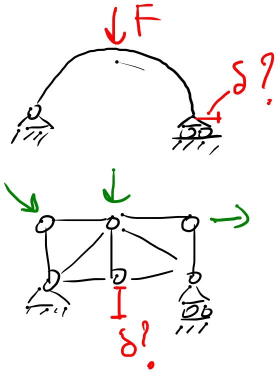
UTILIZZO DEL T.L.V. PER IL CALCOLO DEGLI SPOST. NELLE STR. ISOSTATICHE

MECCANICA DEI SOLIDI









SIST. SPOST.-DEFORM.
CINEMATICAMENTE
AMMISSIBILE

SIST. FORZE-TENSIONI (*)
STATICAM. AMMISSIBILE

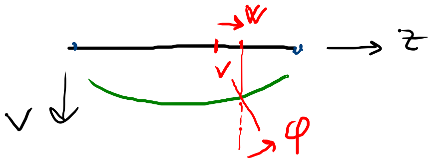
NON ABBIAMO STRUM.
PER CALCOLARE δ

$$L_{ve} = L_{vi}$$

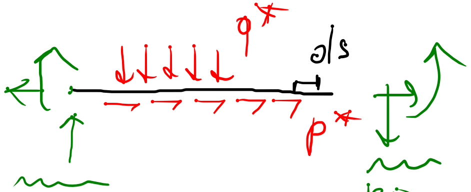
$$\int_{\partial B} \underline{p}^* \cdot \underline{u} \, dS + \int_B \underline{b}^* \cdot \underline{u} \, dV = \int_B \underset{\sim}{\sigma}^* \cdot \underset{\sim}{\varepsilon} \, dV$$

RIFORMULAZ. DELLA STESSA IDENTITÀ PER LE STRUTTURE PIANE e SNELLE.

MECCANICA DELLE STRUTTURE (TRAVI SNELLE)



CINEM. AMM.



STAT. AMM. (*)

REAZ. VINCOLARI DELLA STRUTTURA R_v^*

$$L_{ve} = \int_{STR} q^* ds\, v + p^* ds\, w$$

$$= \int_{STR} q^* v + p^* w \; ds \; + $$ LAVORO VIRTUALE DELLE REAZ. VINCOLARI (R_v^*)

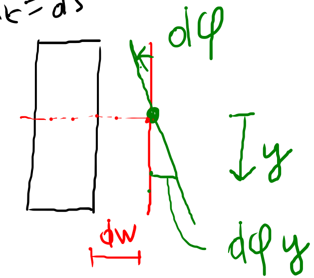
\+ LAVORO VIRTUALE DI FORZE O MOMENTI CONCENTRATI APPLICATI ALLA STRUTTURA

$$L_{vi} = \int_B \underset{\sim}{\sigma}^* \cdot \underset{\sim}{\varepsilon}\, dV \overset{\sim}{=} \int_B \sigma_{zz}^* \varepsilon_{zz}\, dV$$

$$\underset{\sim}{\sigma}^* = \begin{bmatrix} 0 & 0 & \tau_{xz}^* \\ 0 & 0 & \tau_{yz}^* \\ \times & \times & \sigma_{zz}^* \end{bmatrix}$$

TRASCURO IL LORO CONTRIBUTO PERCHÉ $\gamma_{xz}, \gamma_{yz} = 0$

$dz = ds$



SPOST. ASSIALE GENERICO

$$dw(y) = dw + d\varphi\, y$$

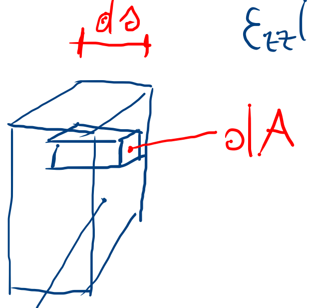
def. dell'asse baricentrico

$$\varepsilon_{zz}(y) = \frac{dw(y)}{dz} = \frac{dw}{dz} + \frac{d\varphi}{dz} y = \bar{\varepsilon} + \chi y \Rightarrow \varepsilon_{zz}(y) = \bar{\varepsilon} + \chi y$$

$$L_{vi} = \int_B \sigma^*_{zz}\, \varepsilon_{zz}\, dV = \int_{TRAVE} \sigma^*_{zz}\, (\bar{\varepsilon} + \chi y)\, dV = \int_{\substack{ASSE\\STR}} \left\{ \int_A \sigma^*_{zz} (\bar{\varepsilon} + \chi y)\, dA \right\} ds =$$

$$\Downarrow$$

$$\varepsilon_{zz}(y)$$



$$= \int \left\{ \int_A \sigma^*_{zz}\, \underset{}{\overset{\text{COST.}}{\bar{\varepsilon}}}\, dA + \int_A \sigma^*_{zz}\, \chi\, y\, dA \right\} ds$$

$$= \int \left\{ \bar{\varepsilon} \underbrace{\int_A \sigma^*_{zz}\, dA}_{N^*} + \chi \underbrace{\int_A \sigma^*_{zz}\, y\, dA}_{M^*_x = M^*} \right\} ds$$

DALLE EQZ CDS

$$\boxed{L_{vi} = \int_{STR} N^* \bar{\varepsilon} + M^* \chi\, ds}$$

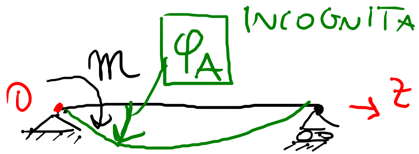
ESTESO ALL' ASSE.

LV INTERNO IN UNA TRAVATURA SNELLA.

$$\boxed{L_{ve} = L_{vi}}$$

CALCOLO DI SPOST. STR. ISOSTATICHE

ES



$M(z)$

SIST. CINEM. AMMISS.

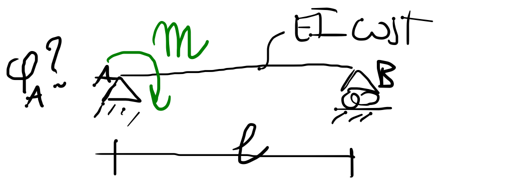
$v, \chi, \bar{\varepsilon}$

STR. REALE

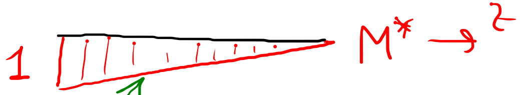
SIST. STAT. AMM. (*)

F^*, M^*, N^*

STR. FITTIZIA



STR "REALE"



INCOGNITA

VINCOLI PERFETTI

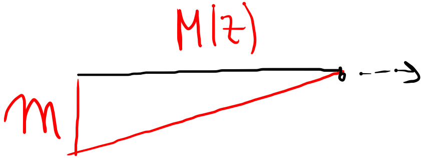
$L_{ve} = L_{vi}$

$$1^* \cdot \varphi_A + \underset{\text{(A)}}{\frac{1}{l} \cdot 0} + \underset{\text{(B)}}{\frac{1}{l} \cdot 0} = \int_0^l M^*(z)\,\chi(z)\,dz$$

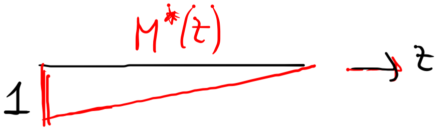
$\chi(z)$: OSSERVO CHE LA STR. REALE È ELASTICA LINEARE E QUINDI

$$\chi(z) = \frac{M(z)}{EI}$$

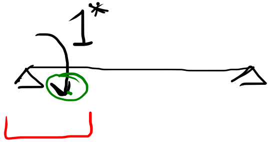
$$= \int_0^l M^*(z)\,\frac{M(z)}{EI}\,dz \Longrightarrow \boxed{\varphi_A = \int_0^l M^* \frac{M}{EI}\,dz}$$



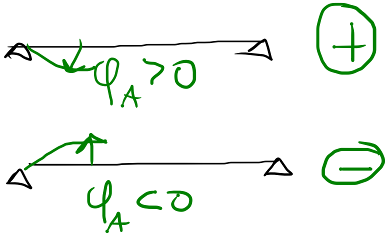
$$M(z) = m - \frac{m}{\ell} z$$



$$M^*(z) = 1 - \frac{1}{\ell} z$$



VERSO SCELTO ARBITRARIAMENTE ALL'INIZIO



$$\varphi_A = \frac{1}{EI}\int_0^\ell m\left(1-\frac{z}{\ell}\right)\left(1-\frac{z}{\ell}\right)dz$$

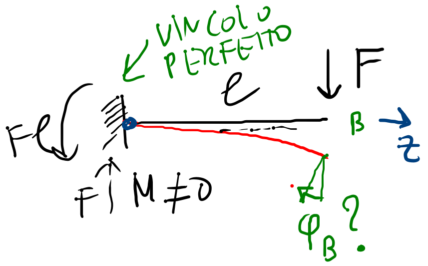
$$= \frac{m}{EI}\int_0^\ell 1+\frac{z^2}{\ell^2}-\frac{2z}{\ell}\,dz$$

$$= \frac{m}{EI}\left[z+\frac{z^3}{3\ell^2}-\frac{2z^2}{2\ell}\right]_0^\ell = \frac{m}{EI}\left[\cancel{\ell}+\frac{\ell}{3}-\cancel{\ell}\right] = \overset{+}{\frac{m\ell}{3EI}}$$

OK

1^* È STATO SCELTO CON IL VERSO GIUSTO!

ES



STR REALE

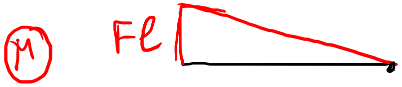
SIST. CINEM. AMM.

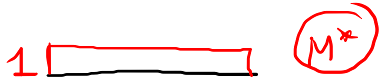
STR FITTIZIA (*)

SIST. STAT. AMM.

$M^* \neq 0, \; N^* = 0$

$L_{ve} = L_{vi}$: $1^* \cdot \varphi_B = \int_{STR} N^* \bar{\varepsilon} + M^* \chi \, dz \Rightarrow \varphi_B = \int_{STR} M^* \chi \, dz \Rightarrow \varphi_B = \int_0^\ell M^*(z) \frac{M(z)}{EI} \, dz$



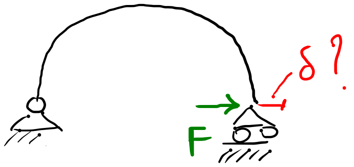


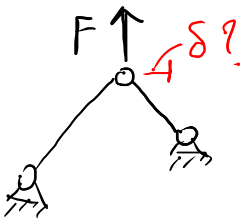
$M(z) = -F\ell + Fz$

$M^*(z) = -1$

$$\varphi_B = \frac{F}{EI} \int_0^\ell (-1)(z-\ell) \, dz = -\frac{F}{EI} \left[\frac{z^2}{2} - \ell z \right]_0^\ell = -\frac{F}{EI} \left[\frac{\ell^2}{2} - \ell^2 \right] = +\frac{F\ell^2}{2EI}$$

φ_B CON VERSO CONCORDE A QUELLO DI 1^*





ESERCIZI
DOMANI.