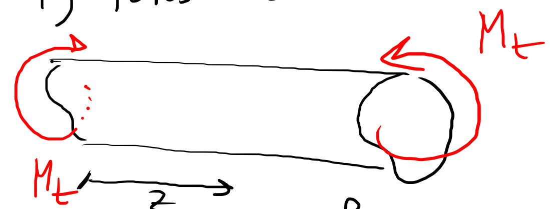
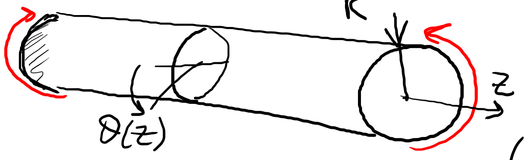
4) TORSIONE $\longrightarrow$ τ_{xz}, τ_{yz} ? $\boxed{\sigma_{zz}=0}$

(diverse da quelle da TAGLIO)

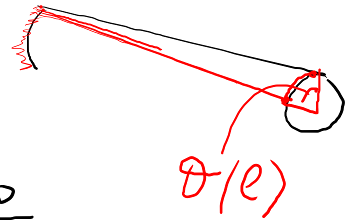
SDC, 15/12/23





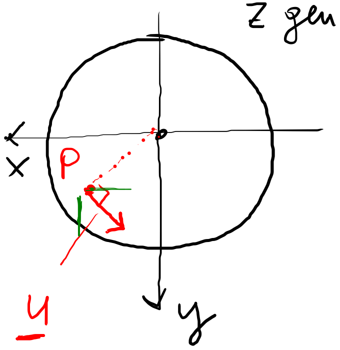
CILINDRO SEZ. CIRCOLARE (COULOMB - 1780)

Θ: ANGOLO UNITARIO DI TORSIONE $\left[\frac{1}{L}\right]$



IPOTESI DI TIPO CINEMATICO

z generica



$\underline{u} \Rightarrow$ prop a $|GP|$

LEGGE LINEARE

$$\begin{cases} u_x = -\theta(z)\, y = -\Theta z y \\ u_y = +\theta(z)\, x = \Theta z x \\ u_z = 0 \end{cases}$$

LEGAME COSTITUTIVO

$$\tau_{xz} = G\,(\underbrace{u_{x,z}+u_{z,x}}_{\gamma_{xz}}) = -G\Theta y$$

$$\tau_{yz} = G\,(u_{y,z}+u_{z,y}) = G\Theta x$$

$\boxed{\text{CDS}}$

$$\underbrace{\int \tau_{xz}\, dA}_{T_x} \overset{?}{=} 0 \;;\; \int -G\Theta y\, dA = -G\Theta \overbrace{\int_A y\, dA}^{S_x} = 0$$

$$\int_A \tau_{yz}\, dA = G\Theta \underbrace{\int x\, dA}_{S_y} = 0$$

Ok

$$\int_A \tau_{yz} x - \tau_{xz} y \, dA = M_t$$ DATO DEL PROBL. $$\Rightarrow G\Theta \underbrace{\int x^2 + y^2 \, dA}_{I_G} = M_t \; ; \; \boxed{G\Theta I_G = M_t}$$

I_G (MOM. D'INERZIA POLARE)

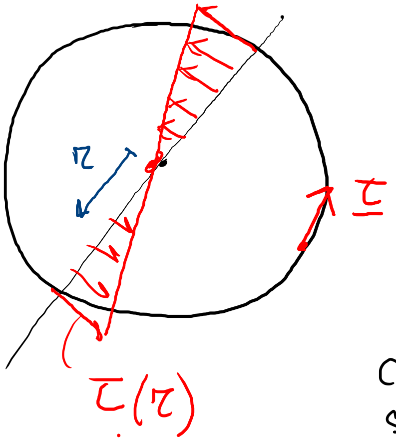
$$I_G = \frac{\pi R^4}{2}$$ SEZ. CIRCOLARE

$$\boxed{\tau_{xz} = -\frac{M_t}{I_G} y \; ; \quad \tau_{yz} = \frac{M_t}{I_G} x}$$

$$|\underline{\tau}| = \sqrt{\tau_{xz}^2 + \tau_{yz}^2} = \frac{M_t}{I_G} \underbrace{\sqrt{y^2 + x^2}}_{|GP| = r} = \frac{M_t}{I_G} r$$

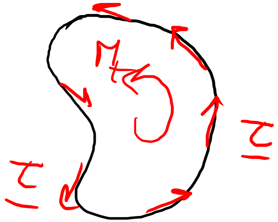
$$\Theta = \frac{M_t}{G I_G}$$ COEFF. DI RIGIDEZZA TORSIONALE (SEZ. CIRCOLARE)

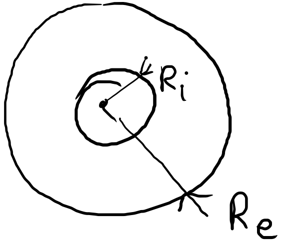
$$\left(\chi = \frac{M_x}{E I_x} \; ; \; \bar{\gamma} = \kappa \frac{T}{GA} \; ; \; \bar{\varepsilon} = \frac{N}{EA} \right)$$



SUL BORDO (∂B) IL VETTORE $\underline{\tau}$ È TANGENTE AL CONTORNO DELLA SEZIONE

(VALE PER TUTTE LE SEZIONI!)

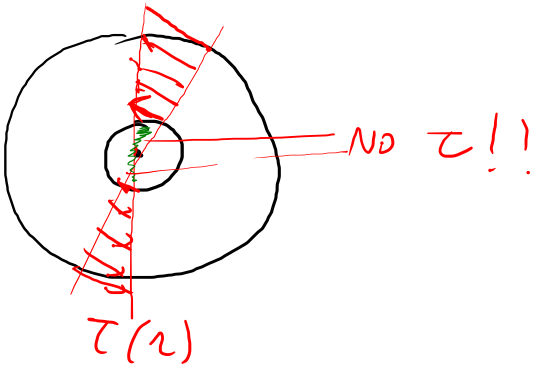




LA SOLUZ. PRECEDENTE VALE IN TUTTO E PER TUTTO; BASTA CALCOLARE I_G PER LA SEZ. ATTUALE

$$I_G = \pi \frac{(R_e^4 - R_i^4)}{2} \Rightarrow \tau_{xz} = -\frac{M_t}{I_G}\, y \; ; \quad \tau_{yz} = \frac{M_t}{I_G}\, x$$

$$\Theta = \frac{M_t}{G\, I_G}$$



VERIFICA CON VON MISES O TRESCA:

VON MISES: $\sqrt{3\tau_{MAX}^2} \le \sigma_0 \Rightarrow \boxed{\tau_{MAX} \le \frac{\sigma_0}{\sqrt{3}}}$

TRESCA: $\sqrt{4\tau_{MAX}^2} \le \sigma_0 \Rightarrow \boxed{\tau_{MAX} \le \frac{\sigma_0}{2}}$

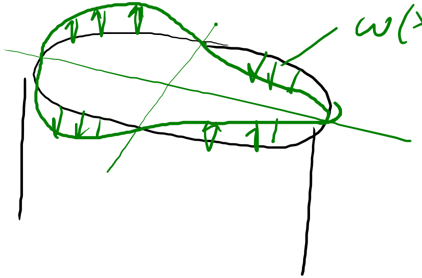
CENNI ALLA TORSIONE IN SEZ. GENERICHE COMPATTE

($\omega(x,y) = 0$ PER LE SEZ. CIRCOLARI)

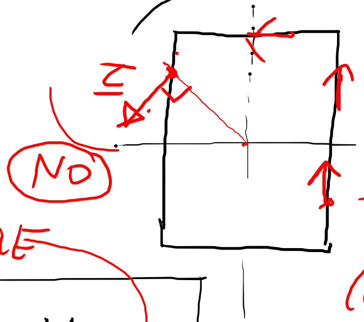
IPOTESI CINEMATICHE

$$\begin{cases} u_x = -\Theta y z \\ u_y = \Theta x z \\ u_z = \Theta \underbrace{\omega(x,y)}_{\text{FUNZ. DI INGOBBAMENTO}} \end{cases}$$

FORM. DI S.V.



secondo COULOMB



τ SOLUZ. ESATTA (GRAZIE A $\omega(x,y)$)

$\omega(x,y)$ SI OTTIENE RISOLVENDO UN PROBLEMA DEL II ORDINE ALLE DERIVATE PARZIALI ($\nabla^2 \omega = 0$ su A)

FATTORE DI RIGIDEZZA TORSIONALE (TIPICO DI OGNI SEZIONE)

$$\boxed{\Theta = \frac{M_t}{G J_t}}$$

$J_t = I_G$ solo per sez.

PRANDTL HA FORMULATO IL PROBL DELLA TORSIONE ATTRAVERSO LA "FUNZIONE DELLE TENSIONI" ($F(x,y)$) PER CUI:

$$\boxed{\nabla^2 F = -2G\Theta \text{ su } A \quad e \quad F = 0 \text{ su } \partial A}$$

BORDO DELLA SEZIONE

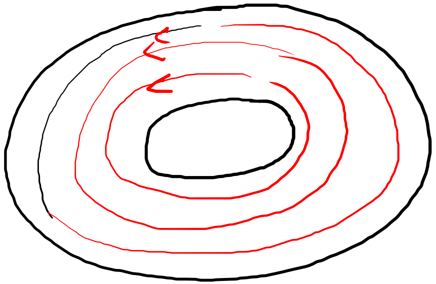
$\exists$ UNA RELAZ TRA $\omega(x,y)$ e $F(x,y)$

SI PUÒ DIMOSTRARE L'ANALOGIA IDRODINAMICA:

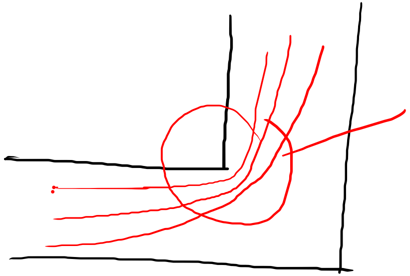
TORSIONE $\longleftrightarrow$ IDRODINAMICA

$\underline{\tau}$ $\qquad\qquad$ $\underline{v}$

LA DISTR DI $\underline{\tau}$ COINCIDE CON IL CAMPO DI VELOCITÀ $\underline{v}$ DI UN FLUIDO PERFETTO CONTENUTO IN UN RECIPIENTE AVENTE LA FORMA DELLA SEZIONE IN MOTO LAMINARE E STAZIONARIO (DI ROTAZIONE).



$\underline{v} \leftrightarrow \underline{\tau}$



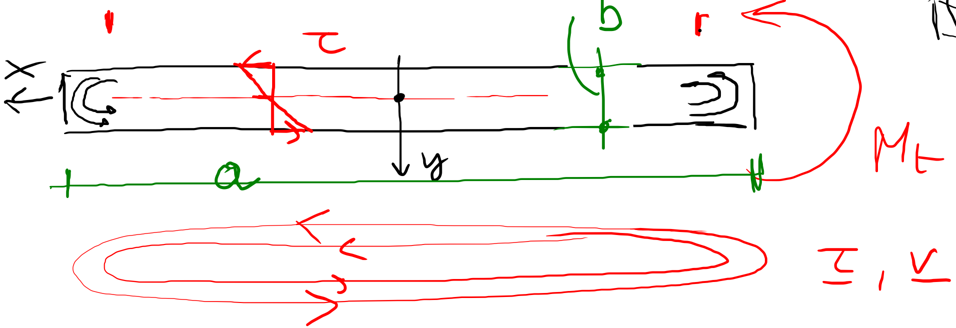
CONCENTRAZIONE DEGLI SFORZI

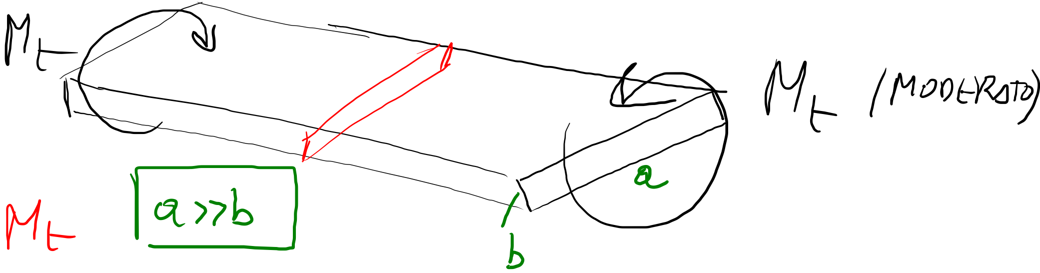
(SPIGOLI DA EVITARE)

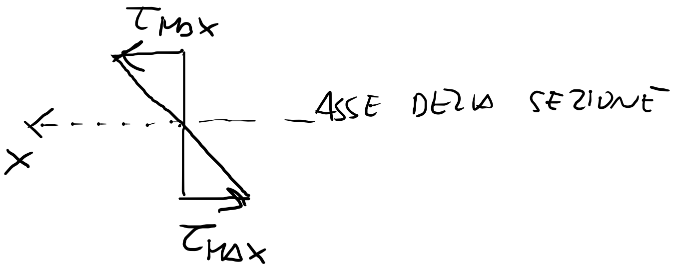
INTAGLIO

CONCENTRAZIONE DELLE $\underline{\tau}$

RETTANGOLO SOTTILE







$$G = \frac{E}{2(1+\nu)}$$

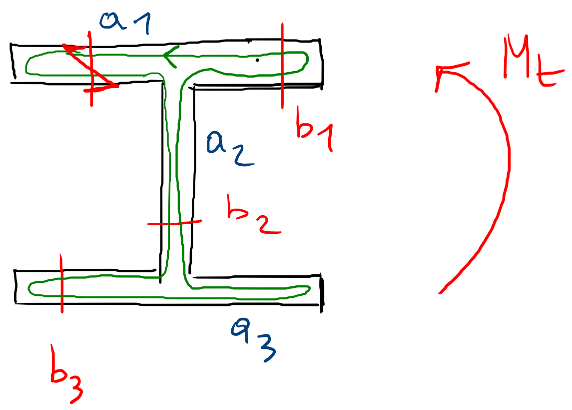
DALLO STUDIO DELLE SEZ. RETTANGOLARI SI OTTIENE CHE PER IL PROFILO SOTTILE:

$$\tau_{max} = 3\,\frac{M_t}{ab^2} = \frac{M_t}{J_t}\,b$$

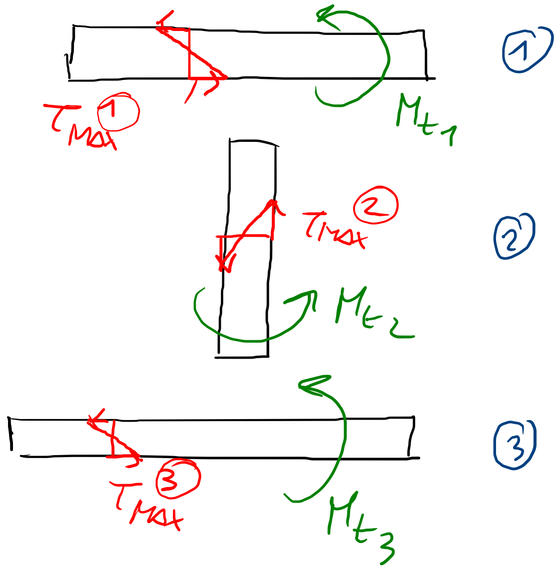
perché $\boxed{J_t = \frac{1}{3}ab^3}$ FATTORE J_t PER I RETTANGOLI SOTTILI

$$\Theta = M_t / G J_t$$

ES



SEZ. SOTTILE "APERTA"



$$\sum_{i=1}^{3} M_{t_i} = M_t$$ EQUIVALENZA

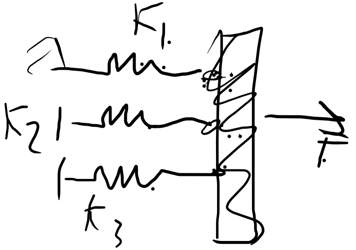
$$\tau_{MAX}^{(i)} = \frac{M_{t_i}}{J_{t_i}} b_i$$

È NECESSARIO RIPARTIRE M_t

NEI 3 RETTANGOLI SOTTILI ?

COME ? $\Theta_1 = \Theta_2 = \Theta_3 = \Theta$

UGUAGLIANZA DEGLI ANGOLI UNITARI DI TORSIONE.



$$\Theta_i = \frac{M_{t_i}}{G J_{t_i}} \quad \text{(NOTO)}$$

È UGUALE A Θ

J_t (TOTALE DELLA SEZIONE)

$$M_t = \sum_i M_{t_i} = \sum_i G J_{t_i} \Theta_i = G \Theta \underbrace{\sum_i J_{t_i}}_{J_t}$$

G COSTANTE

$$\Theta_i = \Theta \Rightarrow \frac{M_{t_i}}{J_{t_i}} = \text{costante} \Rightarrow \boxed{\frac{M_{t_1}}{J_{t_1}}} = \frac{M_{t_2}}{J_{t_2}} = \boxed{\frac{M_t}{J_t}} \; (*)$$

$$M_{t_1} = \frac{M_t}{J_t} J_{t_1} \Rightarrow \boxed{M_{t_i} = M_t \frac{J_{t_i}}{J_t}}$$

M_t SI RIPARTISCE PROPORZIONALM. AL RAPPORTO TRA J_{t_i} e J_t

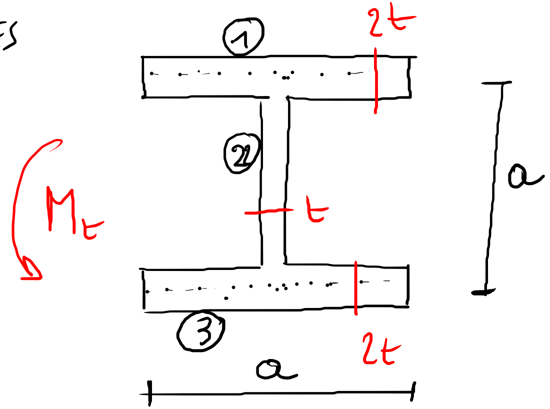
$$\tau_{MAX}^{(i)} = \boxed{\frac{M_{t_i}}{J_{t_i}}}\,(*)\; b_i = \frac{M_t}{J_t} b_i$$

$$\tau_{MAX}(\text{SEZ.}) = \max\{\tau_{MAX}^{(i)}\} = \frac{M_t}{J_t} \max\{b_i\}$$

τ_{MAX} delle MAX

MASSIMO SPESSORE

LES



$$J_t = \sum_i J_{t_i} = \frac{1}{3} a (2t)^3 + \frac{1}{3} a t^3 + \frac{1}{3} a (2t)^3 = \frac{17}{3} a t^3$$

$$J_{t_1} = J_{t_3} = \frac{8}{3} a t^3$$

$$J_{t_2} = \frac{1}{3} a t^3$$

$$M_{t_1} = M_{t_3} = M_t \frac{J_{t_1}}{J_t} = M_t \frac{8}{\cancel{3}} a t^3 \frac{\cancel{3}}{17 a t^3} = \frac{8}{17} M_t$$

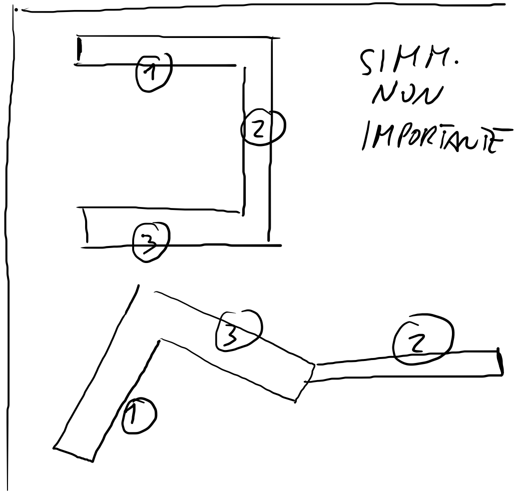
$$M_{t_2} = M_t \frac{J_{t_2}}{J_t} = M_t \frac{1}{3} a t^3 \frac{3}{17 a t^3} = \frac{1}{17} M_t$$

$$\tau_{MAX}^{1} = \frac{M_{t_1}}{J_{t_1}} 2t = \frac{8}{17} M_t \frac{3}{8 a t^3} 2t = \frac{6}{17} \frac{M_t}{a t^2}$$

$$\tau_{MAX}^{3}$$

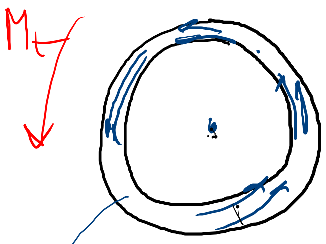
$\tau_{MAX-MAX}$

$$\tau_{MAX}^{2} = \frac{M_{t_2}}{J_{t_2}} t = \frac{1}{17} M_t \frac{3}{a t^3} t = \frac{3}{17} \frac{M_t}{a t^2} \quad \left(= \frac{1}{2} \tau_{MAX}^{1} \right)$$



SIMM. NON IMPORTANTE

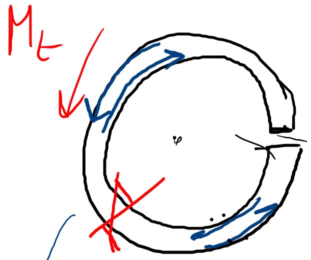
INTRODUZIONE AI PROFILI SOTTILI CHIUSI



SEZ. "CHIUSA"

FLUSSO QUASI COSTANTE CHE RUOTA ATTORNO AL BARICENTRO

FORTE



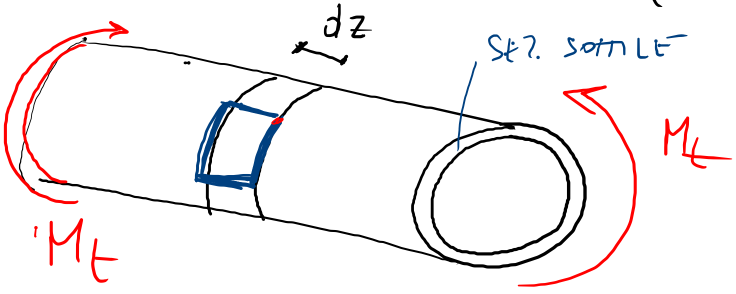
SEZ. "APERTA"

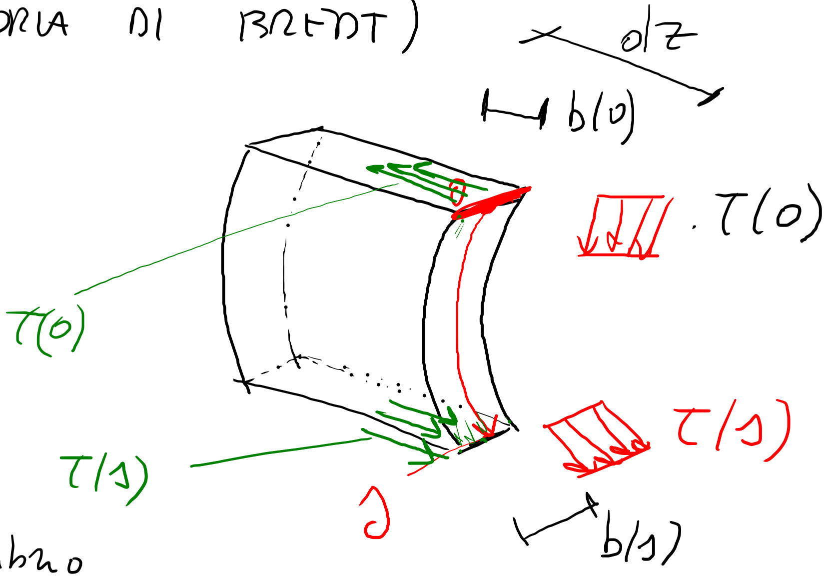
FLUSSO "A FARFALLA" TIPICO DELLE SEZ. APERTE

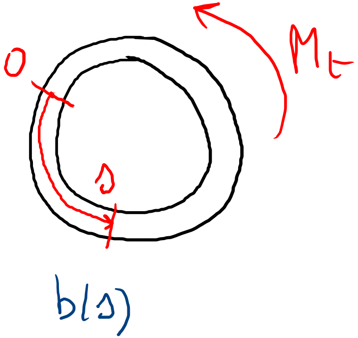
DEBOLE

- COMPORT. MOLTO DIVERSO
- DELLE DUE SEZIONI ALLA SOLLECITAZ. DI TORSIONE

SEZIONI SOTTILI CHIUSE (TEORIA DI BREDT)







$\tau(s)$ (COSTANTI NELLO SPESSORE)

Per l'equilibrio LONGITUDINALE: $dF(0) = dF(s)$ $\quad (dF(0) - dF(s) = 0)$

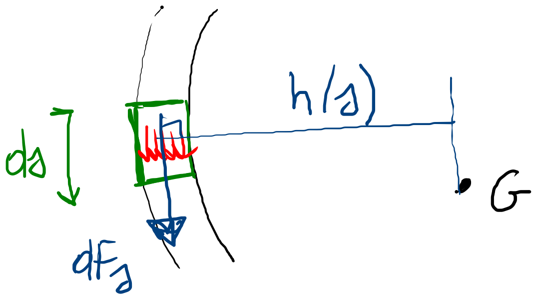
$$\tau(0)\, b(0)\, \cancel{dz} = \tau(s)\, b(s)\, \cancel{dz}$$

$$\tau(0)\, b(0) = \tau(s)\, b(s)$$

$$\tau(s)\, b(s) = \text{COST}, \quad \forall s$$

COSTANZA DEL FLUSSO DELLE τ

Come legare M_t e $\tau(s)$? POSSO APPLICARE L'EQ. CDS del M_t



$$\boxed{M = \oint \underbrace{\tau(s)\, b(s)}_{\text{COSTANTE}}\, h(s)\, ds = M_t}$$

$$\tau(s)\, b(s) \underbrace{\oint h(s)\, ds}_{2\Omega} = M_t$$

SULLA SUPERFICIE $b(s)ds$ INSISTE UNA FORZA $\tau(s)\, b(s)\, ds = dF_s$

$$dM(s) = \tau(s)\, b(s)\, ds \cdot h(s)$$

MOM INFINITESIMO RISPETTO A G

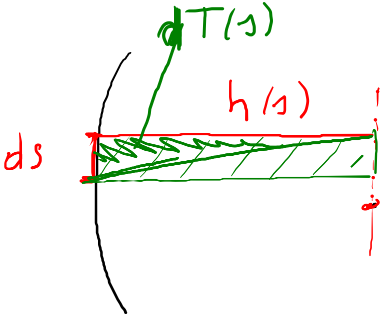
$$\boxed{\tau(s) = \frac{M_t}{2\Omega\, b(s)}}$$

FORMULA DI BREDT

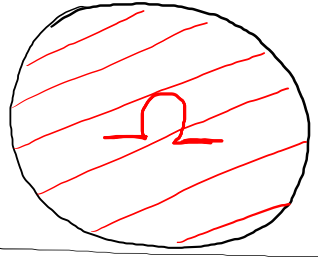
Ω: AREA RACCHIUSA DAL CIRCUITO CHIUSO (COMPRESO ALL'INTERNO DELLA LINEA MEDIA)

$\tau = \text{cost}$ se $b(s) = \text{cost}$

$$\boxed{\tau = \frac{M_t}{2\Omega b}}$$



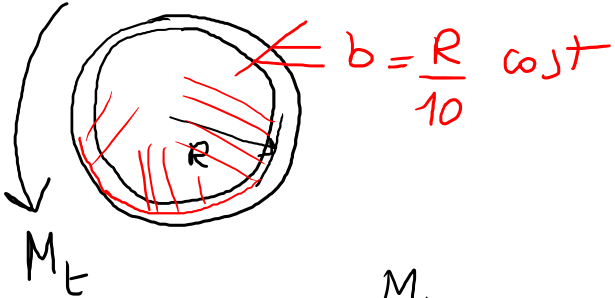
$h(s)\,ds$



$$\Omega = \oint dT(s)$$

$$= \oint \frac{1}{2} h(s)\,ds$$

$$2\Omega = \oint h(s)\,ds$$

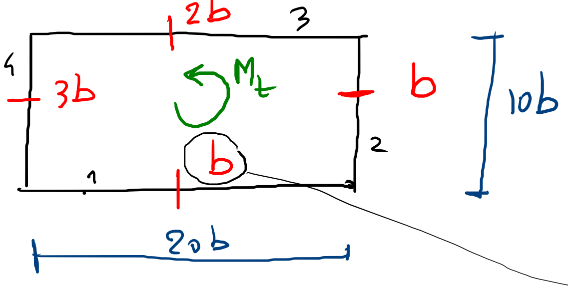


$\tau(s)$?

b : costante ; τ costante

$$\Omega = \pi R^2$$

$$\tau = \frac{M_t}{2\pi R^2 \cdot \frac{R}{10}} = \frac{5}{\pi} \frac{M_t}{R^3}$$

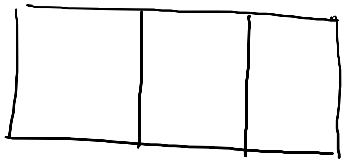


τ_{Max} ? τ_1 o τ_2

$$\tau_1 = \frac{M_t}{2\,\underbrace{(20 \cdot 10\, b^2)}_{\Omega}\, b} = \frac{M_t}{400\, b^3}$$

SE HO UN PROFILO PLURICELLULARE

LA TEORIA DI BREDT È INCOMPLETA



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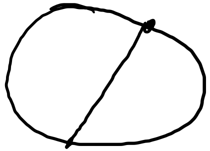

2

CIRCUITI CHIUSI