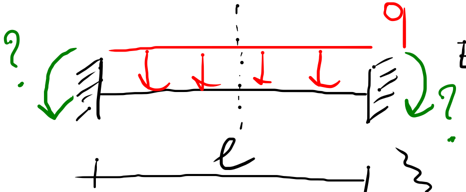
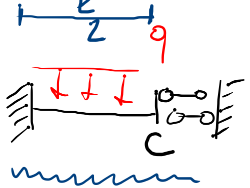
ULTERIORI SCHEMI ELEMENTARI PER IL METODO DEGLI SPOSTAMENTI (MOMENTI DI INCASTRO PERFETTO)

SDC 137N, 21/12/23

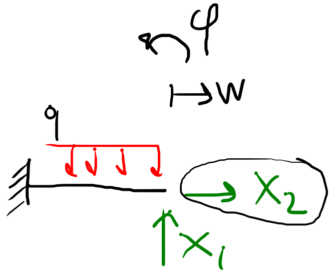


SIM →



2 v. iperstatica

s.p. →



NULLA PERCHÉ IL CARICO q NON INDUCE NESSUNO SPOST. ASSIALE

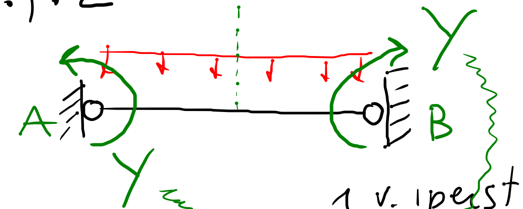
1) $w_C = 0$

$$\frac{X_2 \, \ell/2}{EA} = 0 \Rightarrow X_2$$

2) $\varphi_C = 0 \Rightarrow \underbrace{X_1 = X_1(q)}_{\text{UNICA} \neq 0}$

3 v. iperst. ma con simmetria e quindi studio la str. ridotta per determinare il grado effettivo di iperstaticità

S.P. 2



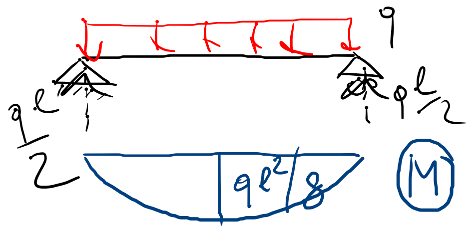
1 v. iperst.

$$\boxed{\varphi_A = 0}$$

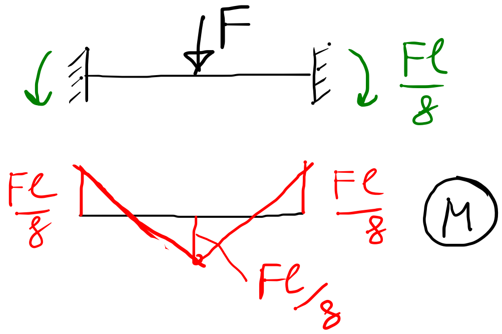
$$\frac{Y\cancel{\ell}}{3E\cancel{I}} + \frac{Y\cancel{\ell}}{6E\cancel{I}} - \frac{q\ell^{\cancel{3}\,2}}{24E\cancel{I}} = 0$$

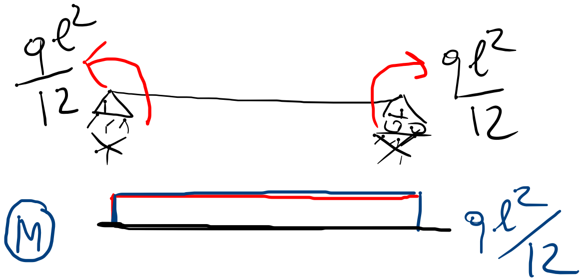
$$\frac{Y}{2} = \frac{q\ell^2}{24} \Rightarrow \boxed{Y = \frac{q\ell^2}{12}}$$

MOMENTO DI INCASTRO PERFETTO....



$$\frac{1}{8}-\frac{1}{12}=\frac{3-2}{24}=\frac{1}{24}$$





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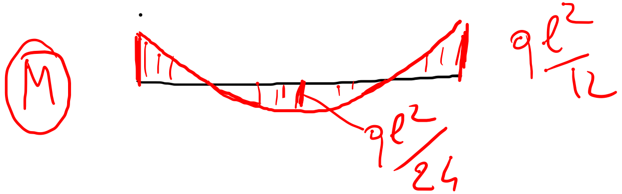
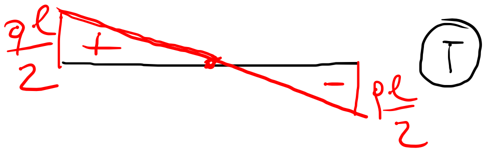
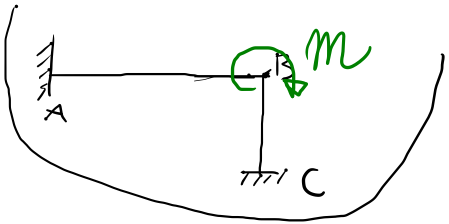
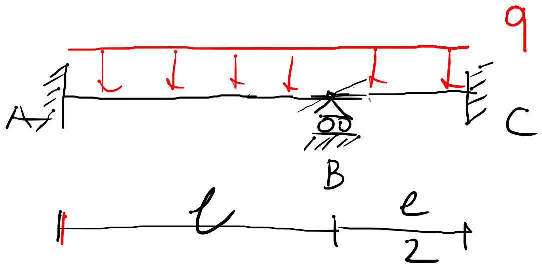




METODO DEGLI SPOSTAMENTI CON CARICHI "NON" NODALI

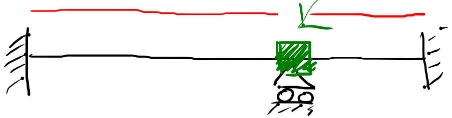




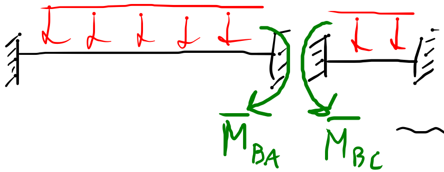
NODO B: NODO DI CUI DOVRÒ CALCOLARE LA ROTAZIONE φ_B

=

MORSETTO (BLOCCA LE ROTAZ.) ($\varphi_B = 0$)



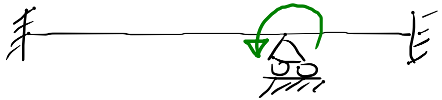
SCHEMA I (INSERIM. DI VINCOLI AUSILIARI PER BLOCCARE I G.D.L. NODALI — MORSETTO PER BLOCCARE φ_B)



$\bar{M}_{BA}$ $\bar{M}_{BC}$ ⟶ $\bar{M}_B = \bar{M}_{BA} - \bar{M}_{BC}$

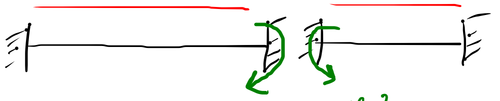
+

$\bar{M}_B$ ⟶ MOM. $\bar{M}_B$ APPLICATO CON SEGNO OPPOSTO



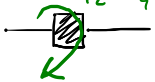
SCHEMA II (RIPART. DEL MOM. $\bar{M}_B$ TRA LE TRAVI E RICERCA DEL G.D.L. φ_B)

QUALUNQUE GRANDEZZA DELLA STRUTTURA INIZIALE SI PUÒ OTTENERE DALLA SOMMA DEI CONTRIBUTI DEI DUE SCHEMI.

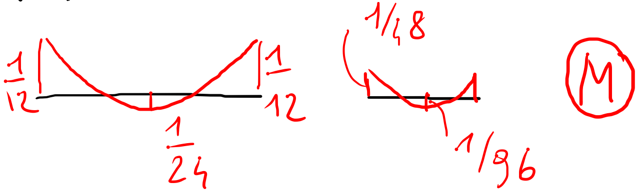


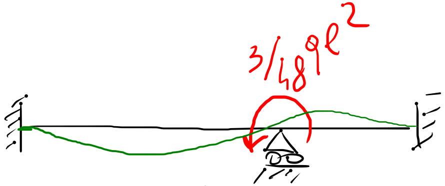
$$\frac{ql^2}{12} \qquad q\left(\frac{l}{2}\right)^2\frac{1}{12} = \frac{ql^2}{48}$$

SCH - I

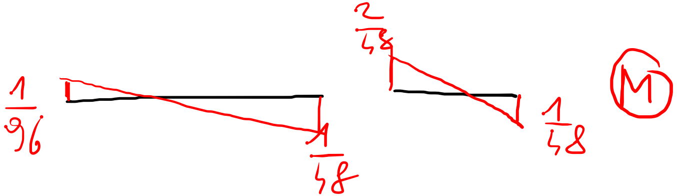


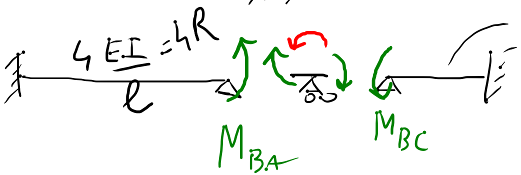
$$\frac{1}{12} - \frac{1}{48} = \frac{3}{48}ql^2 \left(\overline{M}_B\right)$$





SCH - II





$$4\frac{EI}{\frac{l}{2}} = 8\frac{EI}{l} = 8R$$

$$\frac{3}{48}ql^2$$

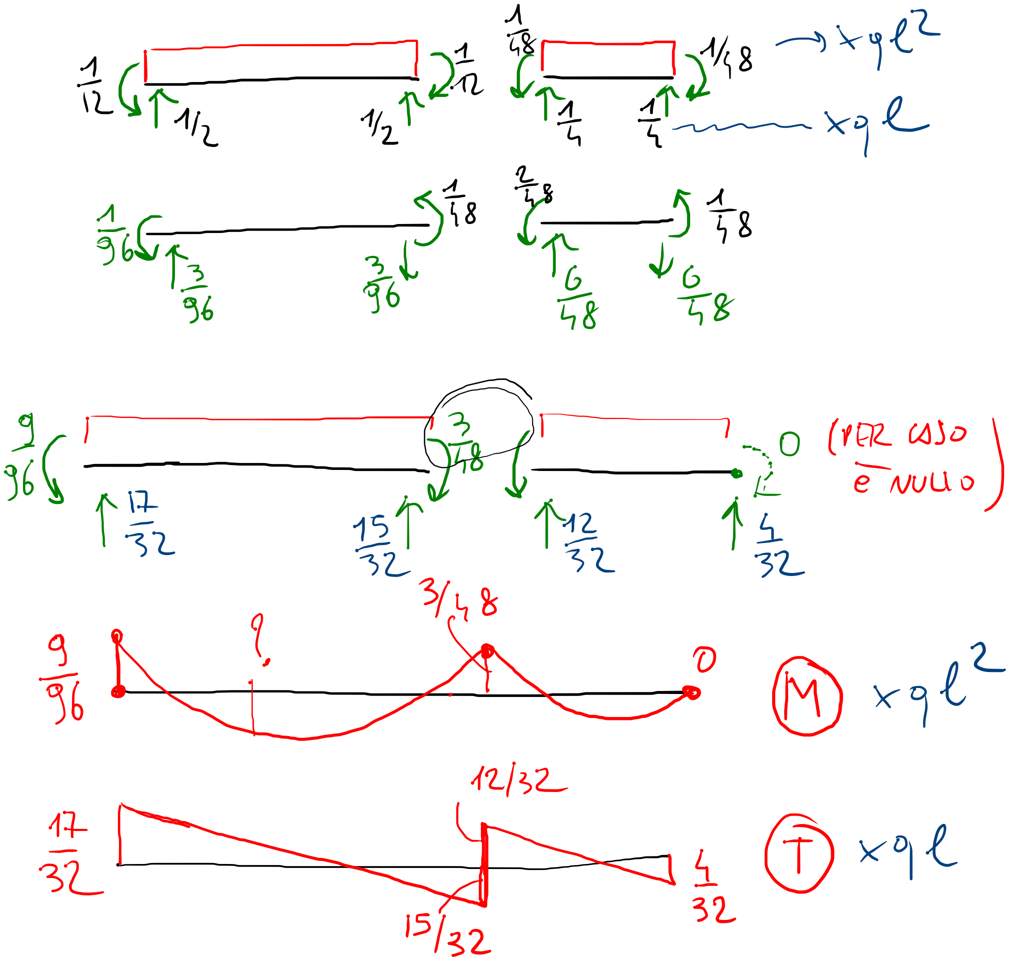
EQUIL.: $M_{BA} + M_{BC} - \frac{3}{48}ql^2 = 0$

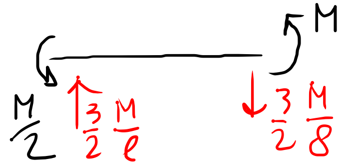
$$4R\varphi_B + 8R\varphi_B = \frac{3}{48}ql^2 \quad ; \quad \varphi_B = \frac{3}{48}ql^2\frac{1}{12R} = \frac{1}{192}\frac{ql^3}{EI}$$

$$M_{BA} = 4R\varphi_B = \frac{4R}{12R}\frac{3}{48}ql^2 = \frac{1}{48}ql^2$$

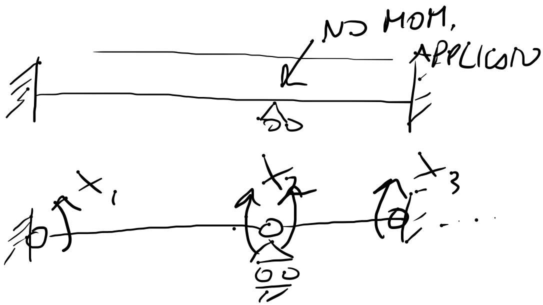
$$M_{BC} = 8R\varphi_B = \frac{8R}{12R}\frac{3}{48}ql^2 = \frac{2}{48}ql^2$$

STUDIAMO GLI S.C.L. EQUILIBRATI DA DUE SISTEMI (I e II)

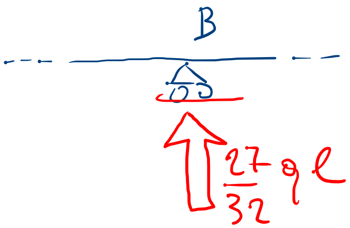




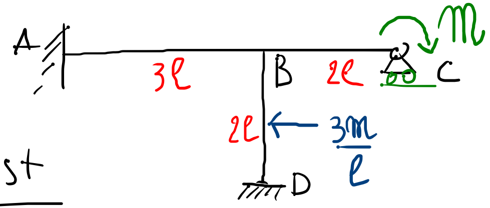
$$\left(\frac{2}{48}+\frac{1}{48}\right)\frac{1}{l/2}=\frac{3}{48}\cdot 2=\frac{6}{48}$$



3 v iperst. FLESSIONALM.



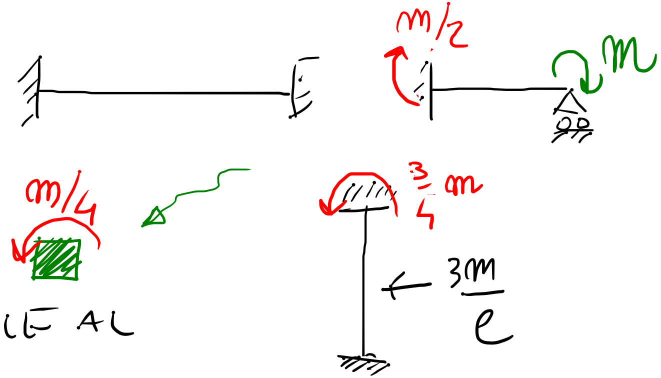
ES



EI cost

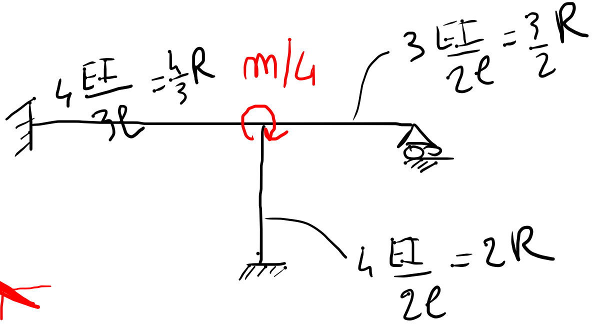
I

MORSETTO IN B

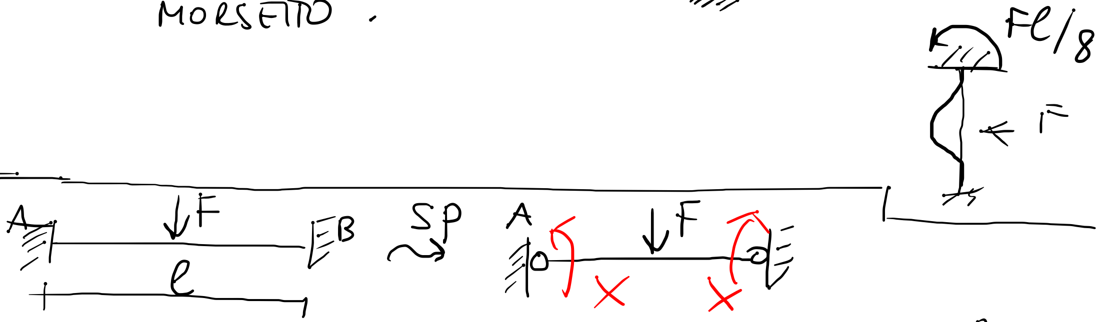


MOM. TOTALE AL MORSETTO.

II

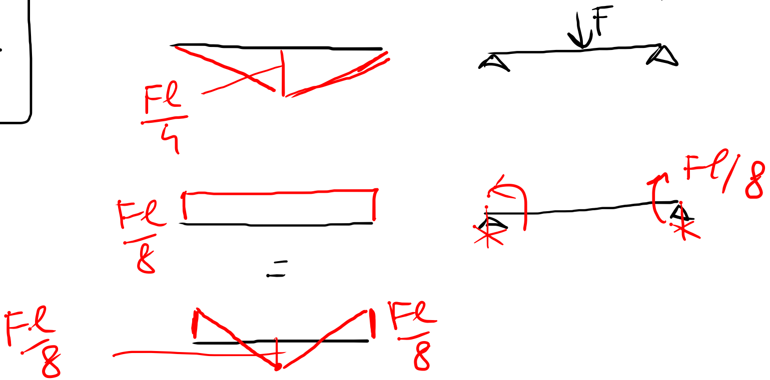


$$\frac{EI}{\ell} = R$$

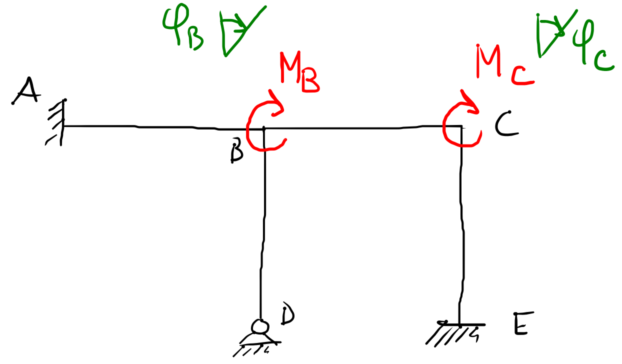


$$+\circlearrowleft\ \varphi_A = 0 : \quad \frac{X\ell}{3EI} + \frac{X\ell}{6EI} - \frac{F\ell^2}{16EI} = 0$$

$$\frac{X}{2} = \frac{F\ell}{16} \Rightarrow X = \frac{F\ell}{8} = \overline{M}$$



CALCOLO ROTAZIONI NODALI IN UN TELAIO A 2 (n) NODI LIBERI DI RUOTARE



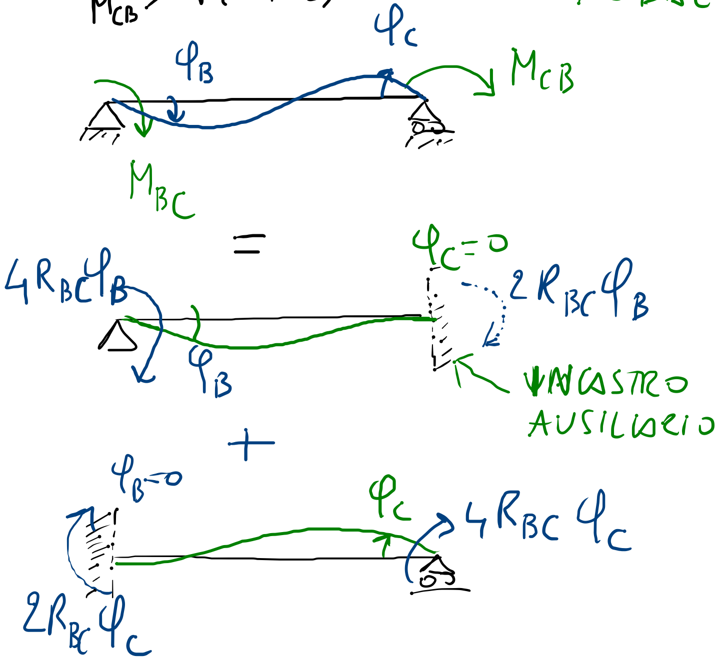
$$M_{BA} = 4 R_{BA} \varphi_B$$

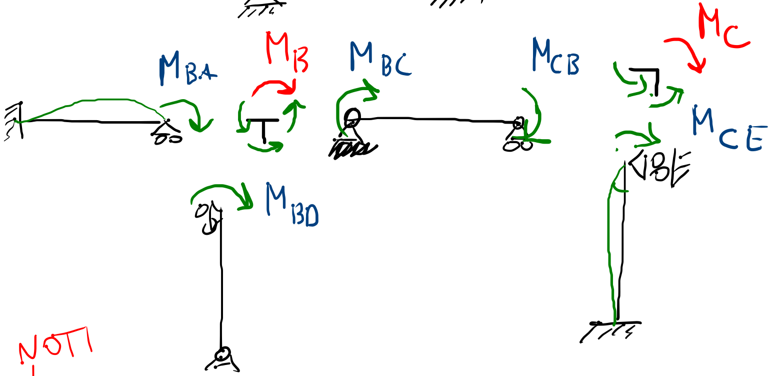
$$M_{BD} = 3 R_{BD} \varphi_B$$

$$M_{CE} = 4 R_{CE} \varphi_C$$

$$\left.\begin{matrix} M_{BC} \\ M_{CB} \end{matrix}\right\rangle = f(\varphi_B, \varphi_C)$$

(φ_B, φ_C: GDL NODALI)





NOTI

EQUIL.

$$\begin{cases} M_B - M_{BC} - M_{BD} - M_{BA} = 0 \\ M_C - M_{CE} - M_{CB} = 0 \end{cases}$$

$$M_{BC} = 4 R_{BC} \varphi_B + 2 R_{BC} \varphi_C$$

$$M_{CB} = 2 R_{BC} \varphi_B + 4 R_{BC} \varphi_C$$

EQ DI EQUIL.

$$\begin{cases} 4 R_{BC} \varphi_B + 2 R_{BC} \varphi_C + 3 R_{BD} \varphi_B + 4 R_{BA} \varphi_B = M_B \\ 4 R_{CE} \varphi_C + 4 R_{BC} \varphi_C + 2 R_{BC} \varphi_B = M_C \end{cases}$$

$$\underbrace{\begin{bmatrix} 4R_{BC} + 3R_{BD} + 4R_{BA} & 2R_{BC} \\ 2R_{BC} & 4R_{CE} + 4R_{BC} \end{bmatrix}}_{\text{MATRICE DI RIGIDEZZA } \underset{\sim}{K}} \begin{bmatrix} \varphi_B \\ \varphi_C \end{bmatrix} = \begin{bmatrix} M_B \\ M_C \end{bmatrix}$$

$$\left[\underset{\sim}{K} \, \underline{\varphi} = \underline{M} \right]$$

$\underset{\sim}{K}$: SIMMETRICA $\left(\underset{\sim}{K} = \underset{\sim}{K}^T \right)$

$\underset{\sim}{K}$: DEFINITA POSITIVA $\begin{cases} \underset{\sim}{K} \underline{v} \cdot \underline{v} > 0 \quad \forall \underline{v} \neq \underline{0} \\ \quad '' \quad = 0 \iff \underline{v} = \underline{0} \end{cases}$

(anche $\det \underset{\sim}{K} \neq 0 \Rightarrow \underline{\varphi} = \underset{\sim}{K}^{-1} \underline{M}$)

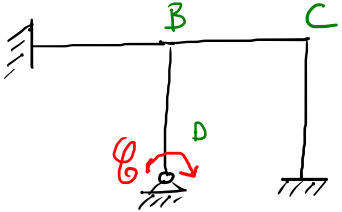
SI DIMOSTRA ATTRAVERSO IL TEOREMA DI CLAPEYRON

... IL TEOREMA DI BETTI (o RECIPROCITA')

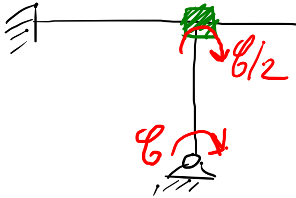
TEORIA E TECNICA DELLE STRUTTURE (P. POZZATI)
VOL 2-1 UTET

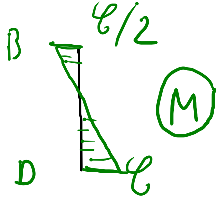
ES

R = cost

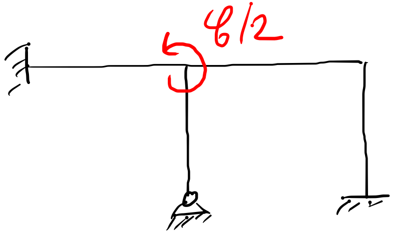


SCH I :

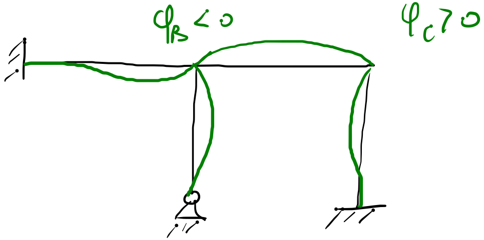


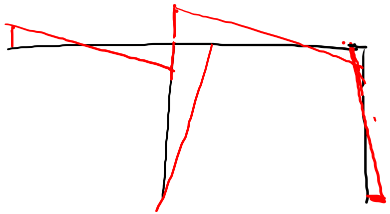


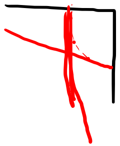
SCH II



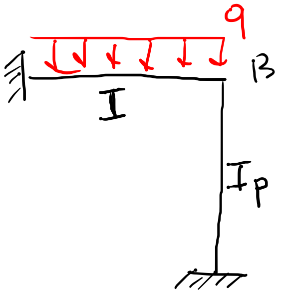
$$\begin{bmatrix} 11R & 2R \\ 2R & 8R \end{bmatrix} \begin{bmatrix} \varphi_B \\ \varphi_C \end{bmatrix} = \begin{bmatrix} -\mathcal{C}/2 \\ 0 \end{bmatrix}$$



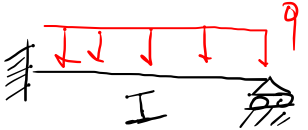




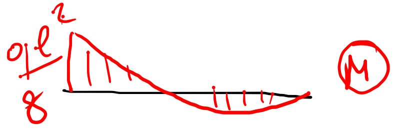
CONSIDERAZ. SUI CASI LIMITE IN UN TELAIO



$I_p << I$



Il nodo B non sente la rigidezza flessionale del pilastro



$I_p >> I$