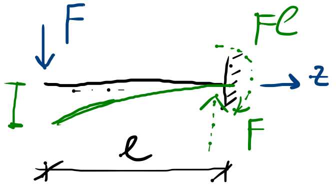
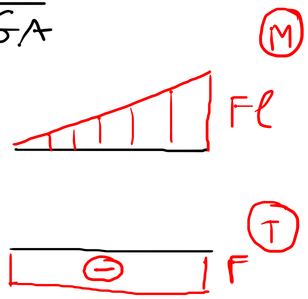
APPROFONDIM. TRAVE DI TIMOSHENKO

SDC 1371N, 23/12/23

ULTIMA LEZIONE

$$\begin{cases} \varphi'(z) = \dfrac{M(z)}{EI} \\ v'(z) = -\varphi(z) + \dfrac{\kappa T(z)}{GA} \end{cases}$$





$M(z) = -Fz$

$T(z) = -F$

$$\begin{cases} \varphi'(z) = -\dfrac{Fz}{EI} \Rightarrow \varphi(z) = -\dfrac{Fz^2}{2EI} + C_1 \\ \varphi(\ell) = 0 \Rightarrow C_1 = \dfrac{F\ell^2}{2EI} \end{cases}$$

$$\varphi(z) = -\frac{Fz^2}{2EI} + \frac{F\ell^2}{2EI}$$

$$\begin{cases} v'(z) = \dfrac{Fz^2}{2EI} - \dfrac{F\ell^2}{2EI} - \dfrac{\kappa F}{GA} \\ v(\ell) = 0 \end{cases}$$

$$v(z) = \frac{Fz^3}{6EI} - \frac{F\ell^2}{2EI} z - \frac{\kappa F}{GA} z + C_2$$

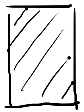
$$v(\ell) = 0 \; ; \quad C_2 = \frac{F\ell^3}{3EI} + \frac{\kappa F\ell}{GA}$$

OSSERVO IL VALORE DI $v(0)$ E DETERMINO IL CONTRIBUTO DEL TERMINE κ/GA -

$$v(0) = \frac{F\ell^3}{3EI} + \kappa\frac{F\ell}{GA} = \frac{F\ell^3}{3EI}\left(1 + \underbrace{\frac{\kappa}{\ell^2}\frac{3EI}{GA}}_{\beta}\right)$$

$\beta \to 0$; $v(0) = \dfrac{F\ell^3}{3EI}$: TH. DI EULERO-BERNOULLI

$$\beta = \frac{3 \kappa E I}{\ell^2 G A}$$

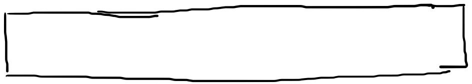


h

b

$$I = \frac{b h^3}{12} \; ; \; A = b h \, , \; \kappa = \frac{6}{5} \; ; \; G = \frac{E}{2(1+\nu)}$$

$$\beta = 3 \, \frac{\cancel{6}}{5} \, \cancel{E} \, \frac{\cancel{2}(1+\nu)}{\cancel{E}} \, \frac{\cancel{b} h^{\cancel{3}\,2}}{\cancel{12}} \, \frac{1}{\cancel{b}\cancel{h}} \, \frac{1}{\ell^2} = \underbrace{\frac{3}{5}(1+\nu)}_{1.3 \; \simeq 1} \, \frac{h^2}{\ell^2} \simeq \left(\frac{h}{\ell}\right)^2$$



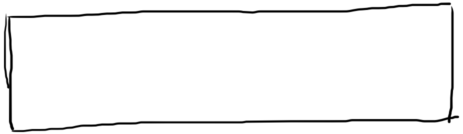
$h = 10$ cm

$\ell = 400$ cm

TRAVE SNELLA

$$\beta \simeq \left(\frac{1}{40}\right)^2 = \frac{1}{1600} \simeq 0.06\% \lll 1$$

DEF. TAGLIANTE TRASCURABILE



$h = 100$ cm

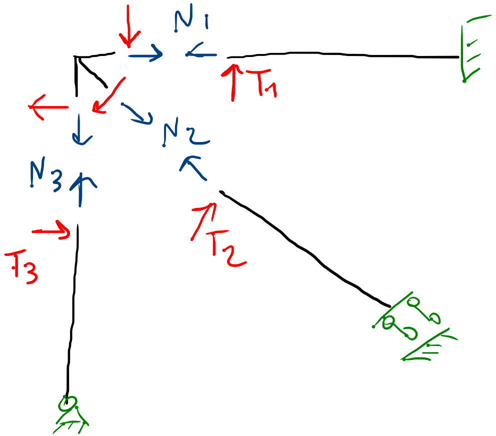
$\ell = 400$ cm

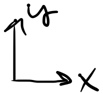
TRAVE TOZZA

$$\beta \simeq \left(\frac{1}{4}\right)^2 = \frac{1}{16} \simeq 6\% \ll 1$$

DEF. TAGLIANTE NON TRASCURABILE

SUL CALCOLO DI FORZE NORMALI E TAGLI NEI NODI DI TELAI



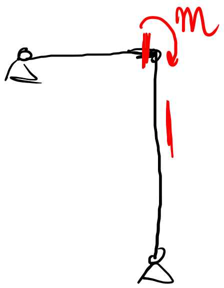


$$\xrightarrow{+} : \sum_i T_{ix} + \sum_i N_{ix} = 0$$

$$+\uparrow : \sum_i T_{iy} + \sum_i N_{iy} = 0$$

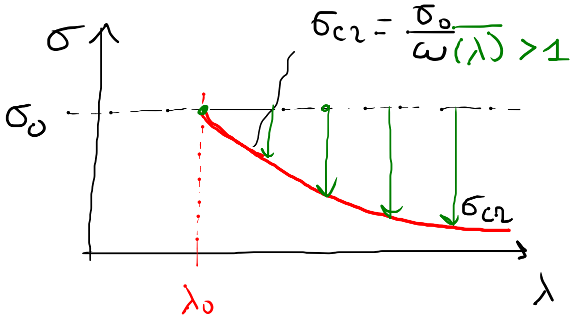
RELAZ. UTILI PER DETERMINARE GLI SFORZI N_i NELLE ASTE.

SE IL N° DI INCOGNITE N_i SUPERA IL VALORE 2 ALLORA È NECESSARIO CONSIDERARE G.D.L. NODALI RELATIVI ALLA TRASLAZ. DEL NODO E CONSIDERARE LA DEFORMABILITÀ ASSIALE.



OSSERVAZ. SULL'ASTA DI EULERO: CENNO AL METODO ω.

$$|\sigma_{cr}| = \frac{P_{cr}}{A} \rightarrow |\sigma_{cr}| = \pi^2 \frac{E}{\lambda^2}$$



Per le aste snelle la VERIFICA DI RESISTENZA diventa ($N<0$)

$$\frac{N}{A} \leq \sigma_{cr} \Rightarrow \frac{N}{A} \leq \frac{\sigma_0}{\omega(\lambda)}$$

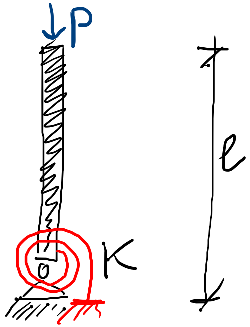
$$\Rightarrow \omega(\lambda)\frac{N}{A} \leq \sigma_0$$

VERIFICA DI RESISTENZA PER ELEMENTI COMPRESSI (METODO ω)

AMPLIFICAZIONE DELLA CDS N

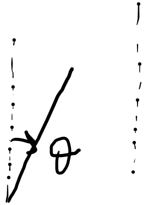
$\omega(\lambda)$: COEFFICIENTE TABELLATO

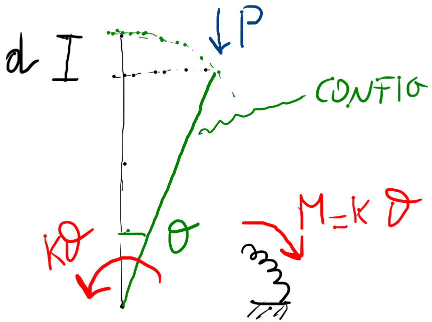
CENNI ALLA STABILITÀ DI STRUTTURE RIGIDE AD ELASTICITÀ CONCENTRATA



STUDIO CONFIG. DI EQUIL. AL VARIARE DEL G.D.L. ϑ

(ϑ MODERATAMENTE GRANDE : $M = K\vartheta$)





$$d = \ell - \ell\cos\vartheta$$

STUDIO L'EQUIL. CON L'EN POT. DEL SIST.

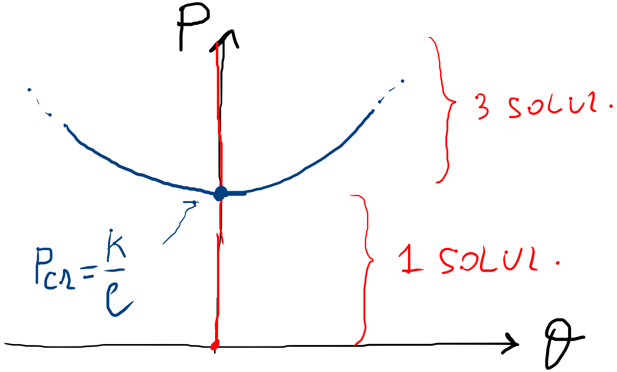
$$\Pi(\vartheta) = \frac{1}{2}K\vartheta^2 - Pd = \frac{1}{2}K\vartheta^2 - P\ell(1-\cos\vartheta)$$

$$\frac{d\Pi}{d\vartheta} = 0 \; : \; \boxed{K\vartheta - P\ell\sin\vartheta = 0}$$

1) $\vartheta = 0$

2) $(\vartheta \neq 0)$; $P = \frac{K}{\ell}\,\frac{\vartheta}{\sin\vartheta}$

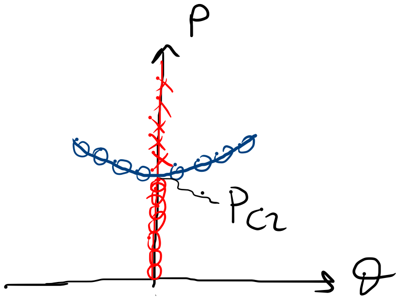
DIAGR. DI BIFORCAZ. (PERCORSI DI EQUIL.)

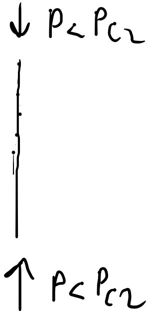


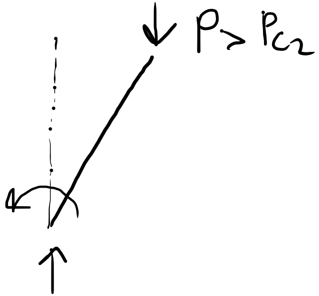
STABILITA' $\frac{d^2\Pi}{d\theta^2} = K - Pl\cos\theta$; si valuta il segno della $\Pi''(\theta)$ nei percorsi di equilibrio ottenuti in precedenza

$\Pi''(\theta) > 0$ CONFIG. STABILE (MINIMO DELL'EN POT.)

< 0 " INSTABILE

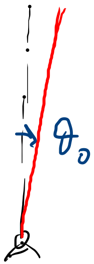


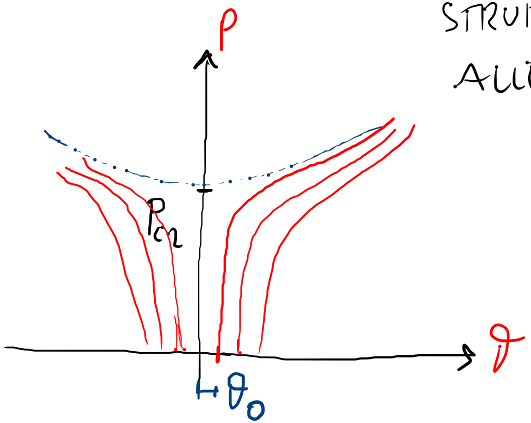




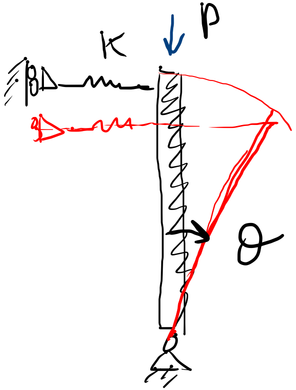
SE CI SONO PICCOLE IMPERFEZ NEL SISTEMA, COME SONO I PERCORSI DI EQUILIBRIO?

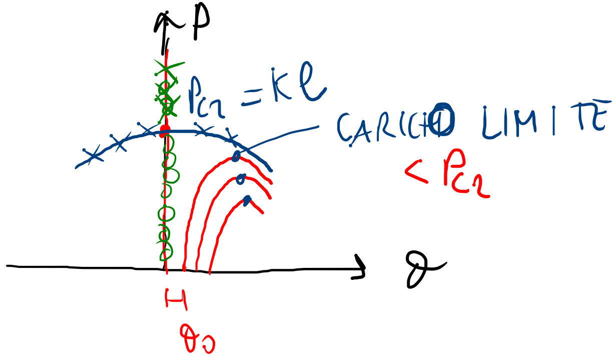
($\theta = \theta_0$ QUANDO $P = 0$)





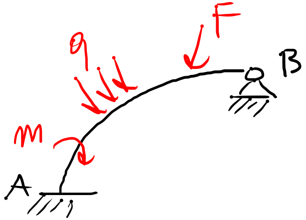
STRUTTURA POCO SENSIBILE ALLE IMPERFEZIONI



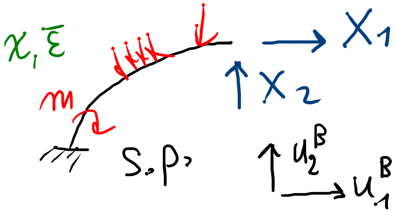


STRUTTURA SENSIBILE ALLE IMPERFEZIONI

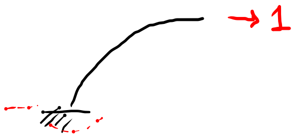
EQ. DI MÜLLER-BRESLAU (1908): FORMALIZZAZ. DEL METODO DELLE FORZE UTILIZZANDO IL P.L.V.



NO AT 2V. IPERST.



u_1^B ?



SCH. FITTIZIO (*)

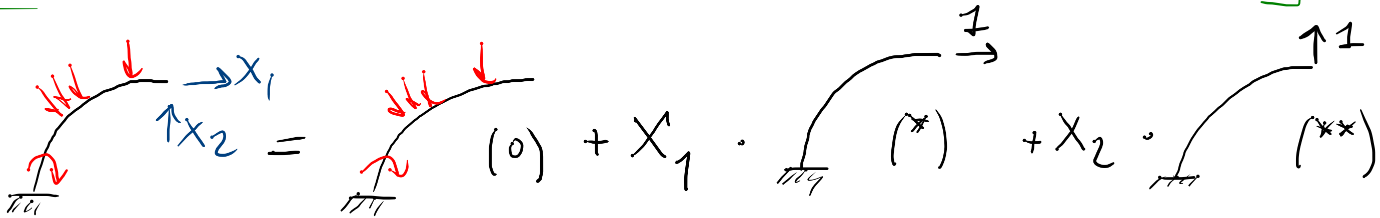
M^*, N^*

$$1 \cdot u_1^B = \int_{STR} M^* \chi\, ds = \int_{STR} M^* \frac{M}{EI}\, ds = \int_{STR} M^* \frac{1}{EI}\left(M^0 + X_1 M^* + X_2 M^{**}\right) ds \Longrightarrow$$

χ^{el}

OSSERVO CHE PER LA STR REALE.....

$$\boxed{M = M^0 + X_1 M^* + X_2 M^{**}}$$



CARICHI ESTERNI

EQ. DI CONGRUENZA

$$\begin{cases} u_1^B = 0 \\ u_2^B = 0 \end{cases}$$

$$u_1^B, u_2^B = f(q, m, F; X_1, X_2)$$

$$L_{ve} = L_{vi}$$

$$u_1^B = \underbrace{\int_{STR} M^* \frac{M^0}{EI} ds}_{u_{10}} + X_1 \underbrace{\int_{STR} \frac{M^{*2}}{EI} ds}_{\eta_{11}} + X_2 \underbrace{\int_{STR} M^* \frac{M^{**}}{EI} ds}_{\eta_{12}}$$

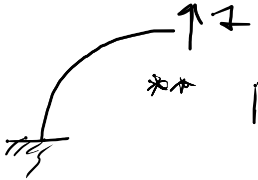
$\Downarrow$

$= 0$

(eq di congr.)

$$0 = u_{10} + X_1 \eta_{11} + X_2 \eta_{12}$$

SVILUPPO LO STESSO RAG. CON u_2^B



** M^{**}

$$u_2^B = \underbrace{\int_{STR} M^{**} \frac{M^0}{EI} ds}_{u_{20}} + X_1 \underbrace{\int_{STR} M^{**} \frac{M^*}{EI} ds}_{\eta_{21}} + X_2 \underbrace{\int_{STR} \frac{M^{**2}}{EI} ds}_{\eta_{22}}$$

$\Downarrow$

CONGR.

$$0 = u_{20} + X_1 \eta_{21} + X_2 \eta_{22}$$

$$\begin{cases} 0 = u_{10} + \eta_{11} X_1 + \eta_{12} X_2 \\ 0 = u_{20} + \eta_{21} X_1 + \eta_{22} X_2 \end{cases}$$

$\Rightarrow X_1, X_2$

$$\boxed{0 = u_{i0} + \sum_{j=1}^{m} \eta_{ij} X_j \quad (i = 1, \dots, m)}$$

m EQUAZ.

EQ. DI MÜLLER-BRESLAU

PROPRIETÀ DEI COEFF η_{ij}

$\Downarrow$

COEFF. DI INFLUENZA

$$\eta_{ii} > 0$$

$$\eta_{ij} = \eta_{ji}$$

NEL CASO FOSSE IMPORT. CONSIDERARE LA DEF. ASSIALE ALLORA:

$$\eta_{12} = \int_{str} \frac{M^* M^{**}}{EI} + N^* \frac{N^{**}}{EA} \, ds$$

$$\eta_{22} = \int_{str} \frac{M^{**2}}{EI} + \frac{N^{**2}}{EA} \, ds$$

IN TERMINI MATRICIALI

$$\begin{bmatrix} 0 \end{bmatrix} = \underbrace{\begin{bmatrix} u_{10} \\ u_{20} \\ \vdots \end{bmatrix}}_{\underline{u}_0} + \underbrace{\begin{bmatrix} \eta_{11} & & \eta_{1n} \\ & & \\ \eta_{n1} & & \eta_{nn} \end{bmatrix}}_{\underset{\sim}{H}} \underbrace{\begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix}}_{\underline{X}}$$

TERMINI NOTI (SPOST. INDOTTI DALLE FORZE ATTIVE)

$\underset{\sim}{H}$ < SIMMETRICA, DEF. POSITIVA ; MATRICE DI CEDEVOLEZZA o FLESSIBILITA'

$$\underset{\sim}{H}^{-1} = \underset{\sim}{K}$$