Chapter 8

Spaced Out: Spatial Encoding

8.1 Introduction

By now you are probably a regular user of the MR scanner and are familiar with the appearance of images (as seen in Chapter 3) produced from various common sequences (Chapter 4). You may have a feel for the digital nature of the images as pixels, voxels and slices (Chapter 5), how parameter changes affect them (Chapter 6) and the artefacts that may sometimes arise (Chapter 7). In a way, that concludes much of your basic hands-on training. In this chapter we begin to provide the theoretical basis for how the scanner produces images from MR signals. Chapter 9 will continue to develop the theory by looking at how the signals are made in the first place.

It should be stressed here that understanding image formation in MRI is neither simple nor obvious and most people struggle to conceptualize it. There are a number of ways of understanding this and what matters is that you find a way that makes sense to you. Persistent students also find that eventually the penny always drops, a light bulb inside their brain suddenly switches on and usually, like the current in a superconducting magnet (or the skill of bicycle riding), it becomes permanent.

An understanding of the image-formation process is helpful for obtaining the optimum diagnostic information from an examination, modifying or creating new protocols, recognizing common image artefacts and taking measures to overcome or avoid them. It will also help as a basis for understanding the pulse sequences considered in Chapters 12, 13 and beyond. It is not just theory. It is the heart and soul of MRI.

In this chapter you will see that:

- magnetic field gradients form the basis of MR signal localization;
- 2D slices are produced by the combination of an excitation RF pulse and simultaneous slice-select gradient;
- the in-plane MR signal is encoded in terms of the spatial frequencies of the object using phase-encoding and frequency-encoding gradients;
- we collect or sample every spatial frequency that can exist within the image before we Fourier transform these data (known as 'k-space') to produce the image directly;
- inadequate or erroneous k-space sampling leads to certain image artefacts.

8.2 Anatomy of a Pulse Sequence

You will have noticed that some scans take a long time to acquire and involve loud banging sounds from the scanner. Each sound is produced by gradient pulses applied to localize the MR signals in the body. In MR the static magnetic field B_0 ('B-nought') is constantly present. The gradients are not. They are applied in a controlled fashion to form a pulse sequence. An MR pulse sequence diagram is a simple means of showing how the RF and gradients are applied. The vertical axis represents pulse amplitude and the horizontal axis is time. Figure 8.1 shows

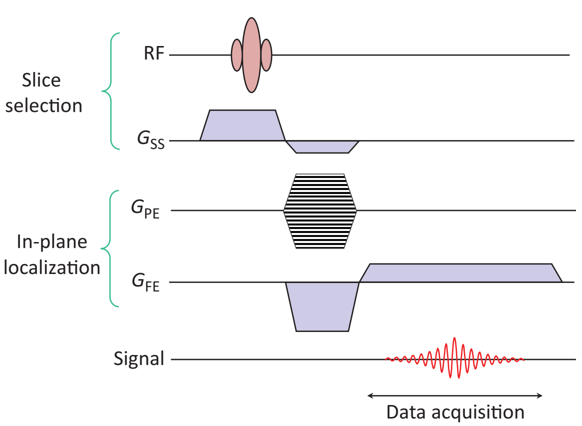

Figure 8.1 Basic gradient-echo MR imaging sequence. Amplitude is shown vertically, time horizontally. G_{SS} is the slice-selective gradient, G_{PE} the phase-encoding gradient and G_{FE} the frequency-encoding gradient.

the basic gradient-echo MR imaging sequence that we will use to illustrate the image-formation process. For the present we will only say what each bit does (how and why will follow).

First (top line), an RF pulse is applied simultaneously with a *slice-selective* gradient G_{SS} (line 2). The RF pulse stimulates the MR interactions in tissue which lead to the MR signal. By combining the RF *excitation* with a gradient the MR interactions are restricted to a two-dimensional plane, slab or slice. Any physical gradient G_x, G_y or G_z or combinations of these can be used for this purpose, allowing us to produce transverse, sagittal or coronal, oblique or double oblique slices.

Next, in line 3, *phase encoding* is applied in a direction orthogonal to the slice selection. This encodes the MR signal in the phase-encode direction. In line 4, the *frequency-encode* or readout gradient is applied in the third direction and finally line 5 shows the time when the MR signal is measured or *acquired*. Note that this is during the frequency-encode gradient but after the phase encoding. The whole sequence pattern has to be repeated for every 'line' of data, corresponding to a different value of phase-encode gradient until the data or *k-space* matrix is filled. A time period, TR, occurs between the application of one RF excitation and the next.

The total scan time is

$$\text{Scan time} = \text{NSA} \times \text{TR} \times N_{PE}$$

where NSA is the number of signal averages and N_{PE} the size of the phase-encoding matrix.

Once all the data are acquired, a two-dimensional Fourier transform is applied. This converts the data, already encoded as *spatial frequencies*, into an image. Reconstruction in MRI is generally simpler than in X-ray CT; most of the hard work has been done during the acquisition by the gradients.

Although this is the simplest possible MR imaging sequence, once you have grasped the purpose of each element, it is relatively easy to make the jump to more complicated sequences as they all have the same basic elements. We will describe three of the steps towards localization – slice selection, phase encoding and frequency encoding – in some detail. First, however, it is important to make sure you understand some underpinning principles.

8.3 From Larmor to Fourier via Gradients

The basic groundwork for this chapter is a knowledge of the Larmor equation to describe the behaviour of excited nuclei (or 'spins'), an understanding of the effect of magnetic field gradients and familiarity with the concept of spatial frequencies. Mathematical skills that would be useful include an understanding of sine waves and an awareness of Fourier transforms (see Appendix). If you already know about sine waves you can skip to Section 8.3.1.

A purely sinusoidal signal or waveform has three basic properties: amplitude, frequency and phase. Amplitude describes how large the signal is, measured in real-world units like volts, or arbitrary 'signal units'. Frequency, measured in hertz (Hz), describes how rapidly in time the instantaneous magnitude of the wave is changing: 1 Hz equals one cycle or rotation per second. Phase describes the instantaneous position within the cyclic variation in terms of an angle. It is measured in degrees or radians, and can vary from 0° to 360° (0–2π radians), thereafter repeating itself. It is sometimes helpful to think of a phase as the angle displayed on a 'clockface'. See Appendix A.2.

8.3.1 Larmor Equation

Sir Joseph Larmor was an Irish physicist who died four years before the discovery of NMR, but who nevertheless predicted the relationship between the *precession* frequency of spins and the magnetic field strength (he also has a crater named after him on the moon). In a simple picture we can think of the spins as rotating at the *Larmor* or *resonance frequency*, which is also the frequency of the MR signal given by the equation

$$\text{Frequency} \approx 42 \times \text{magnetic field}$$

where frequency is in megahertz (MHz) and magnetic field is in tesla (T). The good news is that this is the only equation you need to know to understand spatial localization, although you do need to develop a thorough understanding of its consequences. The number 42 is called the *gyro-magnetic ratio* (which has the symbol γ, pronounced 'gamma') and is a property of the nucleus in question. Its value more exactly is 42.56 MHz T^{-1} for hydrogen (water or fat protons). Other nuclei (e.g. phosphorus) have a different value of gamma, but for most of MRI we only care about hydrogen.

So, if the magnetic field strength of the MR magnet is 1.5 T, the MR signal obtained has a frequency of

$$1.5\ \text{T} \times 42\ \text{MHz T}^{-1} = 63\ \text{MHz}$$

Similarly, at 3 T the Larmor frequency becomes 126 MHz. An RF pulse applied at 63 MHz in a 1.5 T MR system will result in MR signals of a periodic (sinusoidal) nature also at 63 MHz. These can be detected by a coil and receiver tuned, in the same manner as a transistor radio, to this frequency. The proportionality of field and frequency underlies all of the image-formation process.

Where's the Bar?

Conventionally, the Larmor equation is written as

$$\omega_0 = \gamma B_0$$

where ω_0 is the angular frequency of the protons ($\omega = 2\pi f$). Using this scheme gives γ a value of 2.67×10^8 radians s^{-1} T^{-1}. We find this number unmemorable and angular frequencies are not as intuitively understandable as regular (scalar) frequencies. When the use of scalar frequency is helpful or important for understanding, we will use the symbol $\gamma\!\!\!^-$ ('gamma bar'), which is equal to $\gamma/2\pi$ (i.e. 42 MHz T^{-1}). The use of gamma and gamma bar only affects the material in the advanced boxes. Beware, not all authors realize whether or not they are using the bar.

This signal alone is insufficient to produce an image of a patient lying within the magnet bore because we would have no way of assigning parts of the signal to where in the patient they originated. To achieve this localization we now need to introduce the concept of magnetic field gradients, or in short 'gradients'.

8.3.2 Gradients

In MRI the term 'gradient' refers to an additional spatially linear variation in the static field strength in the *z* direction, i.e. along B_0. For example an '*x* gradient' (G_x) will add to or subtract from the magnitude of the static field at different points along the *x* axis or *x* direction. Figure 8.2 shows representations of the main field (a) and the field plus an *x* gradient (b) with the total field represented by the spacing of the 'field lines'. Gradient field strength is measured in milli-tesla per metre (mT m^{-1}).

In Figure 8.2a all the protons (spins) experience the same field and have the same frequency. When a gradient is added (b) the magnetic field produced by the gradient adds to the main field B_0. At the centre ($x = 0$) the total field experienced by the nuclei is simply B_0, so these spins resonate at the Larmor frequency. As we move along the *x* direction, however, the total field either increases or decreases linearly and thus these protons resonate faster or slower depending upon their position. Faster or slower

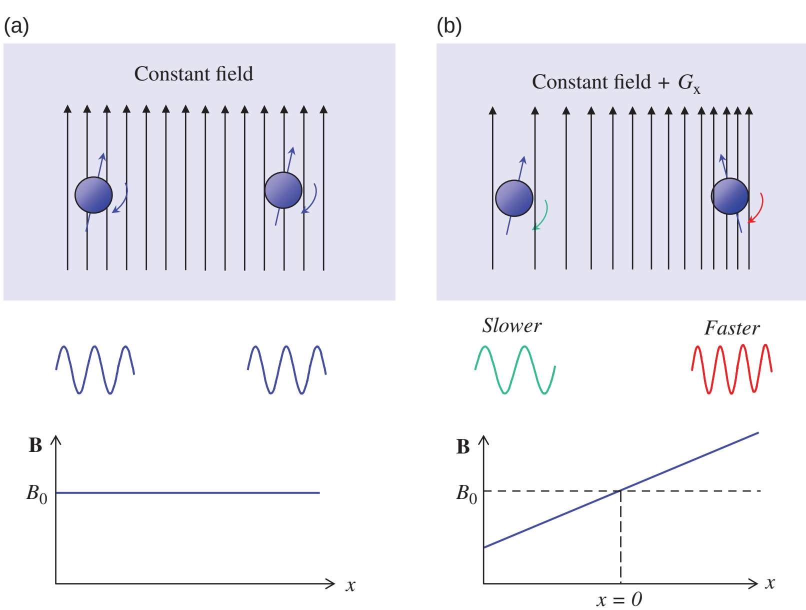

Figure 8.2 Effect of field gradient on nuclei. (**a**) B_0 only, all nuclei precess at the same frequency. (**b**) B_0 plus gradient G_x – precession frequency now depends upon position.

precession is detected as higher or lower frequencies in the MR signal, and so frequency measurements may be used to distinguish between MR signals at different positions in space. Gradients can be applied in any direction or orientation. Three sets of gradient coils – G_x, G_y and G_z – are included in the MR system. They are normally applied only for a short time as pulses. It is these three sets of gradients that give MR its three-dimensional capability.

The Effect of Gradients

Mathematically the three orthogonal spatial gradients of B_z are defined as

$$G_x = \frac{\partial B_z}{\partial x} \qquad G_y = \frac{\partial B_z}{\partial y} \qquad G_z = \frac{\partial B_z}{\partial z}$$

When a gradient (e.g. G_x) is applied, the total field in the *z* direction experienced by nuclei will be dependent upon the position in space, e.g.

$$B(x) = B_0 + x \cdot G_x$$

When a gradient is applied the Larmor frequency will depend upon the total *z* component of the magnetic field and thus becomes spatially dependent, e.g. for the *x* gradient.

$$f(x) = \gamma\!\!\!-(B_0 + x \cdot G_x)$$

where we are using $\gamma\!\!\!- \approx 42$ MHz T^{-1}.

8.3.3 Dephasing and Rephasing

If we have a uniform distribution of water producing an MR signal and we apply a gradient *G* for a given time, what will happen to the MR signal? There will be variations in frequency of the MR signal, either faster or slower, depending upon position, as in Figure 8.2. The spins which are precessing faster, because of the action of a gradient, appear to move apart or dephase (see Figure 8.3), and those which are precessing slower dephase in the opposite direction. The combined effect is often thought of as a 'fanning out' due to dephasing. The speed at which this happens depends upon the gradient amplitude or strength. The total angle of dephasing depends upon the product of the gradient strength and its duration, also known as the gradient moment.

If we now apply another gradient with a reversed sign or polarity (i.e. negative amplitude) as shown in Figure 8.4, the signals which sped up before now will start to precess slower and the ones which had travelled

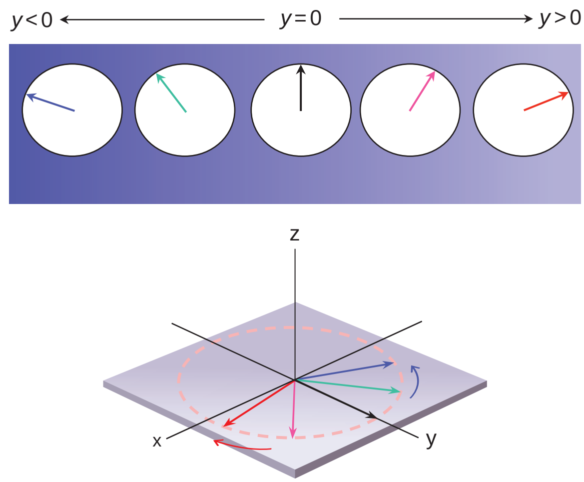

Figure 8.3 Effect of gradient on MR signal (transverse magnetization). Signal originating from different positions along the *y*-gradient axis will have a position-dependent phase change. These are shown as clock-face diagrams in the upper part of the figure. It is usual in the MRI literature to 'collapse' or superimpose these all on the same *xyz*-axes as in the lower portion.

with a slower rotation will now speed up. The spins will start to rephase until, when the gradient moments are equal, the components of the MR signal will all be pointing in the same, original direction. At this point in time we get a measurable MR signal, known as a gradient echo. Each gradient pulse is known as a lobe and is described as dephasing if it occurs first, or as rephasing if it corrects for an earlier dephasing.

Gradient Dephasing

In the rotating frame (see Box 'My Head's in a Spin!') we can view the action of the gradient as a dephasing of components of transverse magnetization in the *xy* plane. The phase change at any time and place is

$$\phi(x,t) = \exp(i\gamma \cdot x \cdot G_x \cdot t)$$

and it evolves for as long as the gradient is applied. Once the gradient is switched off, the accumulated phase changes remain encoded until the transverse magnetization decays to zero or further gradients are applied. A gradient echo results from the application of gradient of equal moment but with the opposite polarity. The echo time TE occurs for

$$\int_0^{TE} G^-(t) \cdot dt = -\int_0^{TE} G^+(t) \cdot dt$$

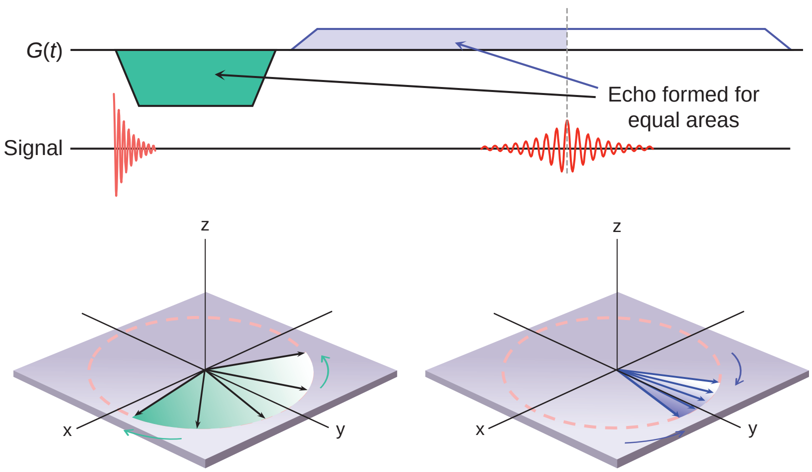

Figure 8.4 Rephasing of signal by a bipolar gradient to form a gradient echo.

where the plus and minus signs refer to the positive and negative lobes of the gradient waveform. Signal loss due to main B_0 field inhomogeneity is not restored. The MR signal decays exponentially with time constant T_2^*.

My Head's in a Spin!

What's the rotating frame? It's a set of *xyz* axes that rotates around the *z* axis at the Larmor frequency. In this frame of reference, a proton at exactly the Larmor frequency is static, which makes all the maths a bit more straightforward. We're not going to say any more than that for now, because it's easy to get confused. Once you have come to the end of this chapter and read the next (Chapter 9), you will realize that all along in this chapter we have been subversively operating in the 'rotating frame of reference'. You don't need to worry about this at all; in MR we tend to naturally adopt this frame of reference; after all, we live on one! The rotating frame is fully explained in Box 'The Rotating Frame of Reference' in Chapter 9.

8.3.4 Fourier Transforms

Joseph Fourier was a French mathematician who enjoyed a colourful life spanning science, politics and high society during the time of Emperor Napoleon Bonaparte. His lasting achievement was the invention of the *Fourier transform*, which entirely underpins the theory of MR imaging. Fourier's great idea was that any signal or waveform in time could be split up into a series of 'Fourier components', each at a different frequency. For example, the sound of a musical instrument could be described either by the actual pressure waveforms it produced in the time domain, or by the appropriate magnitude of its constituent frequencies or its spectrum in the frequency domain. An acoustic signal, such as that produced by a musical instrument, is an example of a one-dimensional waveform, and when Fourier transformed gives a one-dimensional spectrum. In MR we use two- or three-dimensional Fourier transforms. Variables which relate to each other in their respective domains are called Fourier transform pairs. Examples are shown in Figure 8.5. One of the key features of the Fourier transform is that 'less is more': if a shape is small in one domain, its transform will be large in the other.

8.4 Something to get Excited About: The Image Slice

Slice selection or selective excitation is the process whereby MR signals are restricted to a two-dimensional plane or slab within the patient. The position, width and orientation of the slice can all be controlled by the operator.

8.4.1 Selective Excitation

In selective excitation we apply a specially designed RF excitation pulse at the same time as a gradient (the slice-selective gradient) as shown in the top two lines of the pulse sequence diagram in Figure 8.1. The designer RF pulse contains a narrow range of frequencies of RF, centred about the Larmor frequency. In technical terms we say it has a 'narrow bandwidth'. (Note that this is different from the receive bandwidth: the transmit bandwidth is not operator-controlled.)

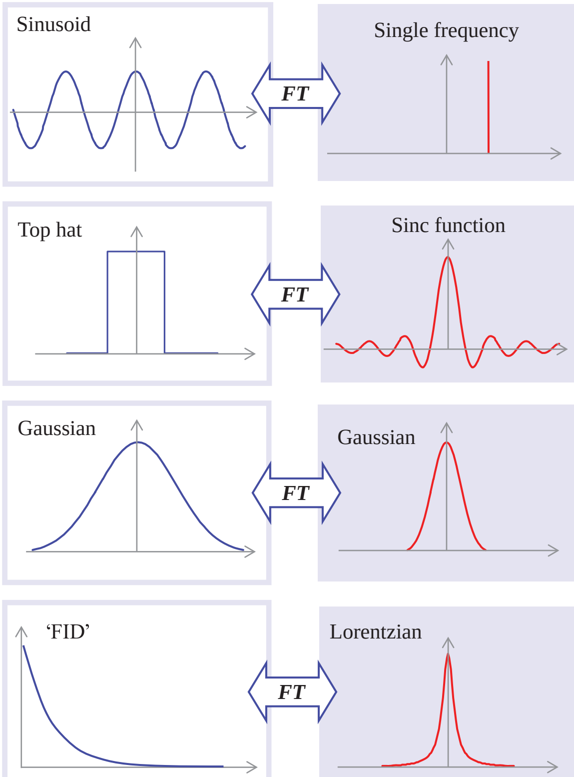

Figure 8.5 Spectra and waveforms (Fourier transform pairs). A narrow extent in one domain is equivalent to a wide extent in the other. FID stands for free induction decay.

In contrast, a simple block RF pulse which is simply switched on and then off again has a wide bandwidth because it is a sinc function in the frequency domain (see Figure 8.5).

The principle of slice selection is illustrated in Figure 8.6. The presence of the gradient causes the resonant frequency (required for producing MR signals) to vary with position in the gradient direction. At the isocentre where the additional value of the gradient is zero, the normal Larmor frequency will apply. Further away along the selection axis, either a higher or lower RF frequency will be needed. If the required frequency is present within the RF pulse's bandwidth then resonance will happen, i.e. protons will be excited. If the required frequency is not present within the RF pulse's bandwidth then nothing will happen. Thus, excitation for the production of signal can only take place at or close to the isocentre. If the slice-select gradient is applied along the *z* axis, the resultant slab of excited nuclei or slice will form a transverse plane.

In the advanced Box 'Slice Selection Maths' (which is for those who wish to know more about the maths), we show that the shape of the physical slice is related to the shape of the spectrum of the RF pulse. We could use Figure 8.5 to give an indication of the distribution of flip angle in the slice-select direction (the slice profile) for various RF waveforms. Commonly, a version of a 'sinc' or 'sin*x*/*x*' pulse (an apodized or truncated sinc) is used for the RF, which gives an approximately rectangular slice profile.

Slice Selection Maths

For a selective pulse the magnetic field gradient introduces a position-dependent spread Δf in the Larmor frequency about the carrier frequency f_0 such that

$$\Delta f(z) = \bar{\gamma} \cdot z \cdot G_z$$

using the *z* gradient for excitation (to produce a transverse slice). Let us apply an amplitude-modulated RF 90° pulse with a form

$$B_1(t) = A(t)\cos(2\pi \cdot f_0 \cdot t)$$

where *A* is the pulse envelope or shape and f_0 is the 'carrier' frequency. Applying a result which will be derived in Section 9.3, the resultant flip angle will be (approximately)

$$\begin{aligned}\alpha(z) &= \gamma \int A(t) \cdot \exp(i\gamma \cdot z \cdot G_z \cdot t)dt \\ &= \gamma \int A(t) \cdot \exp(i2\pi \cdot \Delta f \cdot t)dt\end{aligned}$$

The integral is the Fourier transform of $A(t)$, i.e. $\alpha(z) = \gamma A(f)$. So the shape of the RF pulse's spectrum determines the shape of the slice with regard to the selection direction (in this case *z*).

The position of the slice is given by

$$z = \frac{f_1 - f_0}{\bar{\gamma} \cdot G_z}$$

where f_1 is the shifted carrier frequency. Thus for a slice shift of 100 mm, using a 5 mT m^{-1} gradient an RF carrier frequency shift of about 20 kHz is required. The slice width or thickness is given by

$$\text{slice width} = \frac{\text{RF bandwidth}}{\bar{\gamma} \cdot G_z}$$

So for a 5 mm thickness with a 5 mT m^{-1} gradient the RF bandwidth needs to be approximately 1 kHz. This implies an RF pulse duration of the order of 1 ms.

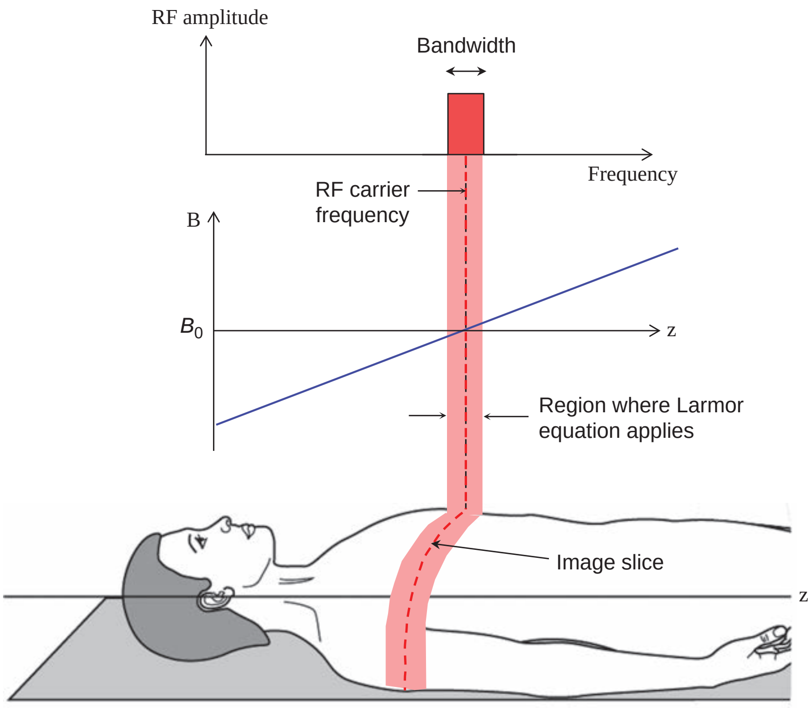

Figure 8.6 Selective excitation of an image slice by applying a shaped RF pulse and a field gradient at the same time.

8.4.2 What's Your Orientation? Manipulating the Slice

All features of the slice can be manipulated by adjusting the gradient or RF waveform properties, i.e. electronically, rather than having to move the patient as required in X-ray CT. These include position, orientation and thickness. Box 'Slice Selection Maths' contains a more mathematical description of these features.

First, the position of the slice can be varied simply by changing the basic (or carrier) frequency of the RF pulse but using the same gradient strength. The region which now fulfils the MR resonant condition will have moved. Second, the thickness of the slice can be controlled by changing either the shape of the designer RF pulse (changing its bandwidth) or the strength of the gradient. A stronger gradient will result in a thinner slice (Figure 8.7a). Alternatively, we can use a narrower RF pulse bandwidth. According to Fourier theory, this means using a longer duration RF pulse. Notice the 'less is more' principle again: you can have a thinner slice but it will take longer to excite (Figure 8.7b).

Third, the orientation of the slice can be varied by using a physically different gradient axis. The selected slice is always orthogonal (perpendicular) to the gradient applied. So far we have assumed the application of G_{SS} in the z–axis, along the patient, giving a transaxial or transverse slice. If we use G_x as a slice-select gradient we get a sagittal slice. For a coronal slice we use G_y (as was shown in Figure 5.8). Oblique and double oblique slices can be created using combinations of G_x, G_y and G_z. See Box 'An Oblique View'.

An Oblique View

Oblique slices may be obtained by driving two orthogonal gradients in proportion to the sine and cosine of the angle required, e.g. to obtain a transverse slice rotated through an angle ϕ from the x axis requires the simultaneous application of

$$G_x = G_{SS}\cos\phi \quad G_y = G_{SS}\sin\phi$$

while the generation of a double oblique angulation of ϕ from x and θ from z requires the application of

$$G_x = G\sin\theta\cos\phi, \quad G_y = G\sin\theta\sin\phi, \quad G_z = G\cos\theta$$

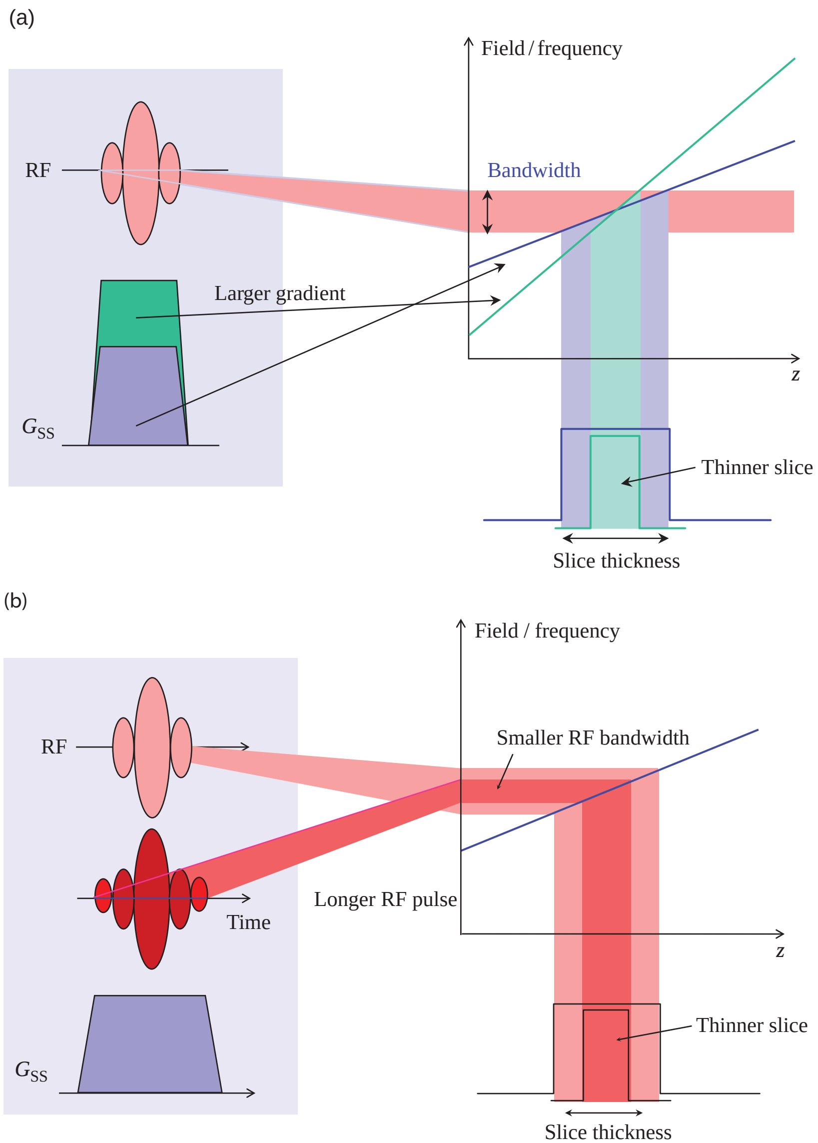

Figure 8.7 Dependence of slice thickness on (**a**) gradient strength and (**b**) RF transmit bandwidth. Larger gradient amplitude and longer RF pulses both result in thinner slices.

8.4.3 Multiple Slices

It doesn't take very long to excite a slice and collect its echo, typically much less than the TR needed to control the image contrast. We can use the 'dead time' within the TR period to acquire multiple interleaved slices (see Section 5.7). By applying a slice-select gradient and changing the central frequency of the RF pulse we can move the position of the slice (Figure 8.8). This is the standard means of image acquisition. A multi-slice interleaving scheme is shown in Figure 8.9. It is possible to acquire the slices in any order. Normally an 'interleaved' slice order is used, e.g. for an eight-slice sequence acquiring the following positions in this order: 1, 3, 5, 7, 2, 4, 6, 8 (see Section 5.7).

8.4.4 Rephasing

In Figure 8.1 you will have noticed a negative portion of the slice-select gradient. This is necessary to rephase the MR signal in order to get the maximum possible signal. While the selective excitation process is occurring, the signal being generated is also being dephased by the gradient. We normally consider the action of the RF pulse to occur at its centre in time. In

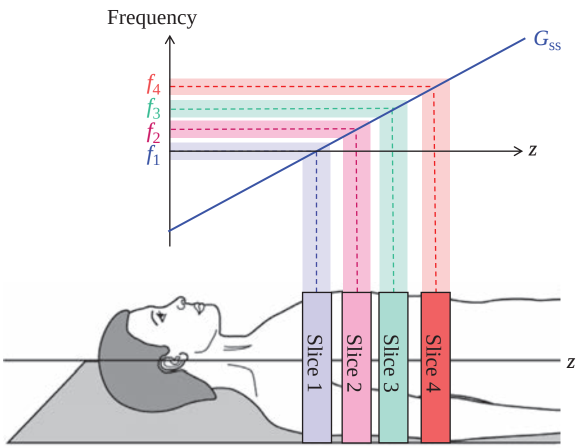

Figure 8.8 By changing the RF centre frequency, multiple slices may be acquired at different locations quite independently of each other.

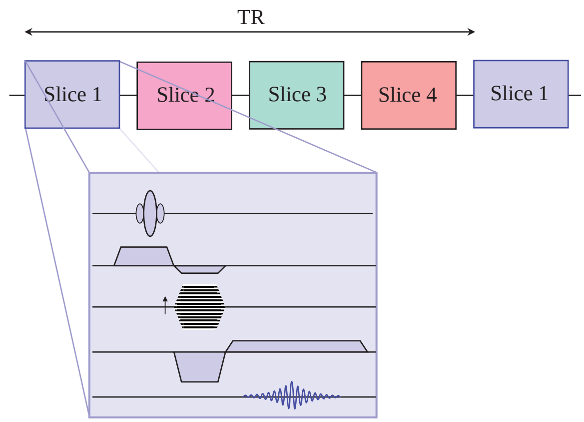

Figure 8.9 Multi-slice sequence. Different slices can be selected and lines of data acquired within each TR period. The expanded section is repeated for each slice position using a different RF carrier frequency.

this case, a rephasing gradient moment of half the slice-selection gradient is required to leave all the spins in phase throughout the slice.

8.5 In-Plane Localization

In MRI we use the gradients to measure the two- (or three-) dimensional spectrum of the object being imaged. This spectrum is what we call k-space and consists of an array or matrix of individual spatial frequencies. The next sections will explain the process conceptually. If you want (or need) the maths, check out Box 'Encoding for 2D FT Imaging', but you don't need to in order to understand image formation.

Encoding for 2D FT Imaging

Following the excitation of a localized slice, frequency- and phase-encoding gradients are applied to manipulate the MR signal to encode spatial frequencies. The effect of a gradient G_{FE} applied along the x direction following the initial excitation on a discrete signal element ∂s is

$$\partial s(t) = \rho(x) \cdot \exp\left(\frac{-t}{T_2^*}\right) \cdot \exp(i\gamma \cdot x \cdot G_{FE} \cdot t) \cdot \mathrm{d}x$$

where $\rho(x)$ is the proton density along x, and i is the square-root of –1, denoting complex notation (see Appendix). This gradient is applied continuously during the signal acquisition (sampling). A dephasing gradient is usually applied prior to sampling in order to generate a symmetrical echo.

The phase encoding is applied (along the y direction for our example) through a gradient G_{PE} with a duration of τ prior to the signal measurement (sampling). The signal from a small element following the application of both gradients is

$$\partial s(t) = \rho(x,y) \cdot \exp\left(\frac{-t}{T_2^*}\right) \cdot \exp(i\gamma \cdot x \cdot G_{FE} \cdot t) \cdot \exp(i\gamma \cdot y \cdot G_{PE} \cdot \tau) \cdot \mathrm{d}x\mathrm{d}y$$

In words, this is

signal = spin density × T_2^* decay × phase change due to G_{FE} × phase change due to G_{PE}

The total MR signal is the integral of this with respect to x and y. In a complete MR acquisition the signal is sampled M times at intervals Δt, and the pulse sequence repeated N times, each time incrementing the PE gradient amplitude such that

$$G_{PE}(n) = \Delta G \cdot n \qquad \text{for } n = \left(\frac{N}{2}\right) \text{ to } \left(\frac{N}{2} - 1\right)$$

Now define quantities k_{FE} and k_{PE} such that

$$k_{FE} = \gamma \cdot G_{FE} \cdot \Delta t \cdot m$$
$$k_{PE} = \gamma \cdot \Delta G \cdot n \cdot \tau$$

The total signal S acquired in two dimensions time t and 'pseudo-time' $n \cdot \tau$ is found by integrating over x and y

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$$S(m,n) = \iint \rho(x,y) \cdot \exp\left(\frac{-t}{T_2^*}\right) \cdot \exp(i2\pi \cdot x \cdot k_{FE}) \cdot \exp(i2\pi \cdot y \cdot k_{PE}) \cdot dxdy$$

which (except for the T_2^* term) is the form of an inverse Fourier transform of the spin density $\rho(x, y)$, i.e.

$$S(m,n) = \rho(k_{FE}, k_{PE})$$

Thus the 2D FT of the encoded signal results in a representation of the spin density distribution in two dimensions. An alternative way of viewing this is that the spatial frequency components are given by the discrete signal elements $S(m, n)$, the raw k-space data. Position (x, y) and spatial frequency (k_{FE}, k_{PE}) constitute a Fourier transform pair.

We have seen how the gradient-encoded MR signal represents the matrix of spatial frequencies. However, a glance at the maths shows that this equivalence is not exact – there is a term which depends upon T_2^*. This affects some spatial frequencies more than others and can lead to loss of resolution and blurring of the image. This is explored in Section 13.4.

8.5.1 Spatial Frequencies Demystified

The concept of spatial frequencies is not just a theoretical abstraction dreamt up to torment students of MRI. In real life the brain makes use of spatial frequencies to construct the visual images that you see. Spatial frequencies may be hard to conceptualize but they are very natural and we'd all be in the dark without them!

One of the easiest ways of understanding spatial frequencies is to think of a line-pair test object, such as those used for testing X-ray imaging systems. These consist of alternate light and dark bands or *line-pairs* of differing spacings (Figure 8.10). Suppose we have five line-pairs per centimetre. This means that five dark–light patterns are contained within a centimetre. The pattern of image brightness produced by this line-pair pattern is like a spatial frequency. In MR a spatial frequency is a periodic variation in signal spatial distribution or image brightness, measured not as line-pairs per centimetre but as 'cycles per centimetre' (which are very similar).

Applying the theory of Fourier, any image (not just MRI) may be decomposed into a spectrum of periodic (sinusoidal) brightness variations or spatial frequencies. In a digital image with a matrix of 256 × 256 pixels there are 256 × 256 possible spatial frequencies, allowing for positive and negative values. If we know the spatial frequencies we can calculate an image of the object that formed them. The purpose of MR localization by gradients is to manipulate the MR signal so that it gives all the spatial frequencies necessary to form an image. Each point of data or k-space is a spatial frequency component.

Figure 8.11 shows an image and its constituent spatial frequencies (k-space). If we remove the high

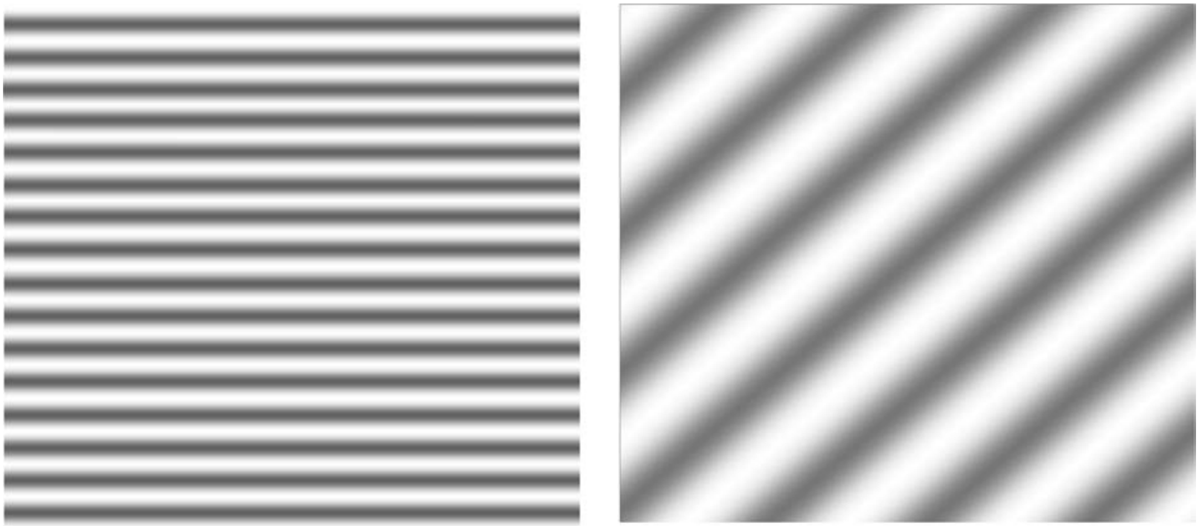

Figure 8.10 Spatial frequencies.

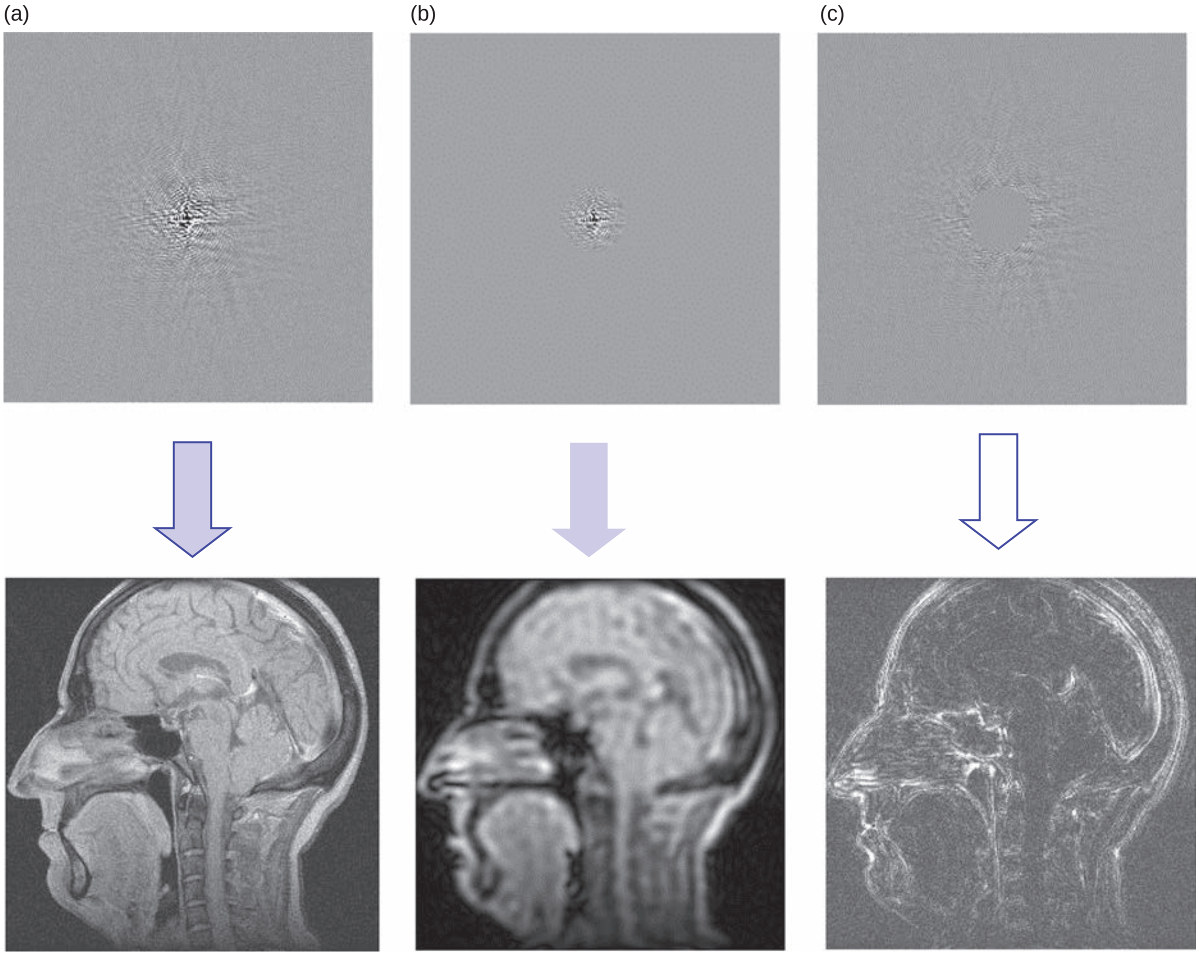

Figure 8.11 Images and their 2D spectra (k-space) showing: (a) reconstruction from all spatial frequencies; (b) low spatial frequencies, i.e. the centre of k-space only; and (c) high spatial frequencies, i.e. the edges of k-space only.

spatial frequencies we are left with an image which has the right brightness but no detail. Removing the low spatial frequencies leaves the image with details of edges and sharp features but low intensity elsewhere. So big objects have low spatial frequencies. Small objects or sharp edges have high spatial frequencies.

8.5.2 Totally Fazed: Phase Encoding

Most people find phase encoding the hardest part of MR image formation to understand, but gaining a conceptual grasp of it will pay dividends in terms of your overall understanding. Consider the following in conjunction with Figure 8.12, which shows the effect of the phase-encoding gradient on the transverse magnetization at three different locations and times.

Suppose we already have an MR signal with all the spins in phase. If we apply the phase-encode gradient (G_{PE}) at time A in the y direction, then the precession of the nuclei will speed up or slow down according to their position along the y axis. As we saw in Section 8.3.3, this causes the spins to dephase or fan out to a progressively greater degree for as long as the gradient is applied. When we switch off the gradient at time B, all the nuclei will revert to their original frequency or speed, but will keep their different phase angles. They are said to be *phase encoded*. The relative phase differences between signals in different locations remain until either another gradient is applied or the MR signal decays due to T_2 relaxation.

Figure 8.13 shows the phase encoding generated by three different gradient amplitudes on a column of

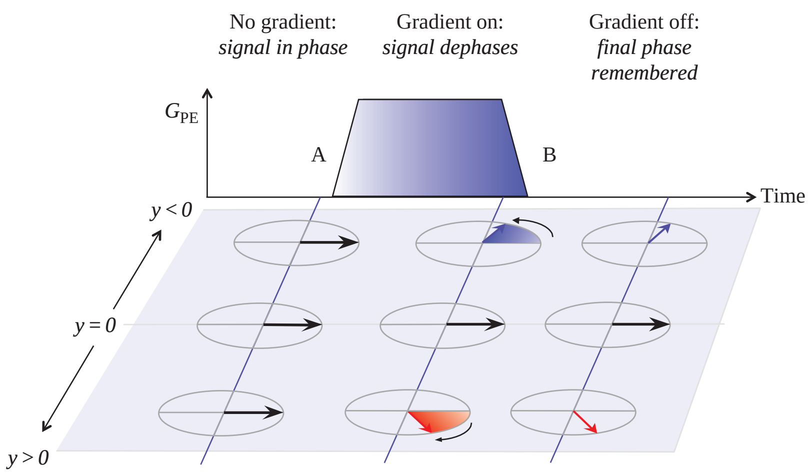

Figure 8.12 Phase encoding returns the signal to the Larmor frequency but with position-dependent phase changes.

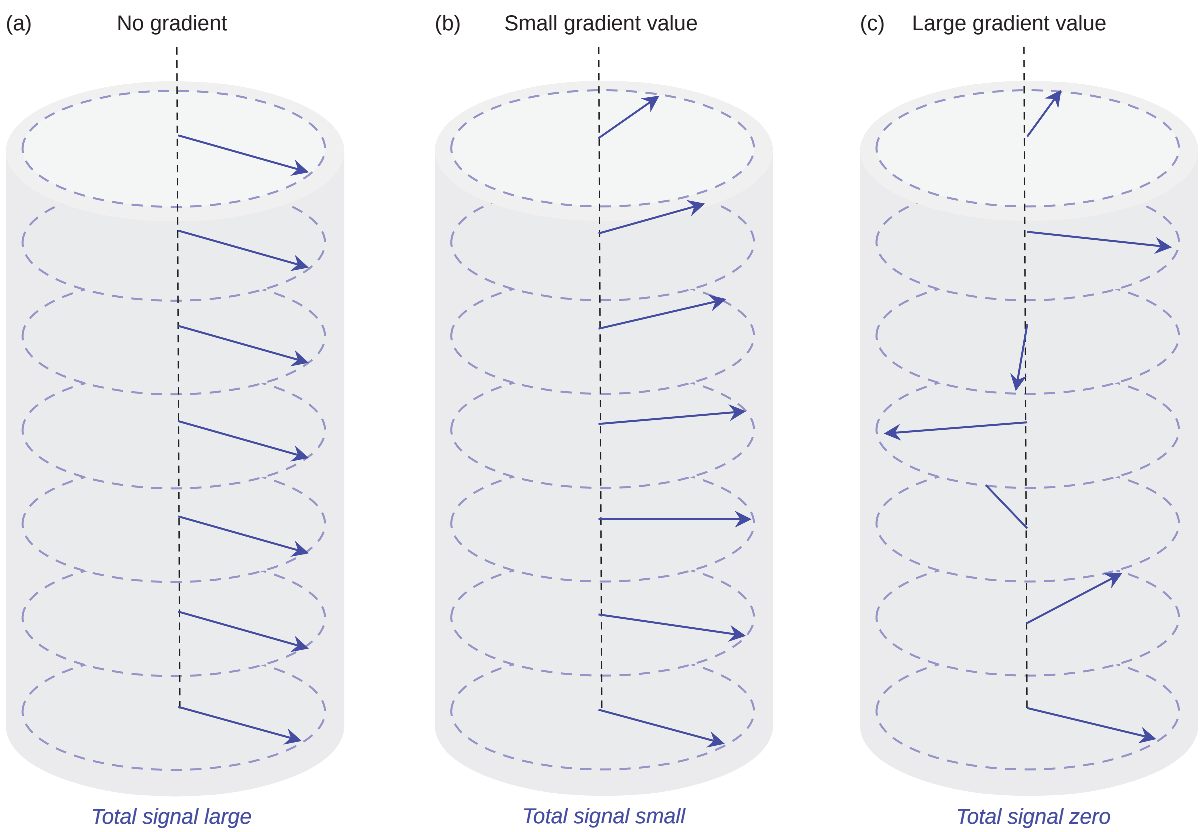

Figure 8.13 Effect of three different strengths of phase encoding on a uniform distribution of signal-producing material. The MR signal detected is given by the sum of all the vectors. (**a**) No gradient, (**b**) small gradient, (**c**) large gradient.

protons in the phase-encode axis. We see that without any applied gradient, the spins are all in phase and a large signal is obtained, but that the dephasing or twisting of the spins increases with gradient strength until the dephasing is large enough for all the spins to cancel each other out and no signal is obtained.

How is this measuring spatial frequencies? Referring to Figure 8.13, suppose we have a uniform

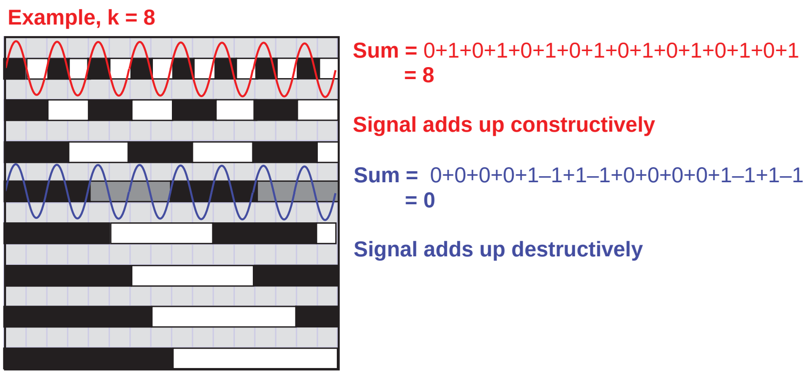

Figure 8.14 Picking out a single spatial frequency with phase encoding step 8. Black represents an area which produces no signal (i.e. with zero proton density). Only one pattern results in any signal. This is spatial frequency or k value 8.

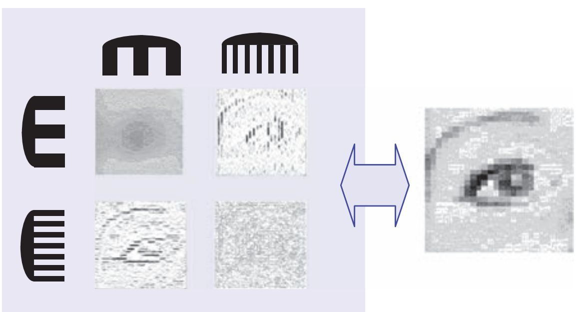

Figure 8.15 The 'MR eye': picking out spatial frequencies in two dimensions by applying spatial frequency 'combs' or filters. The thumbnails show the parts of the overall image selected by each comb. Like understanding MRI, the eye is not always obvious at first.

distribution of protons and we apply a sufficient phase-encode gradient to cause the phase of the spins to vary by 360° (2π radians or multiples of 2π). When we add up the MR signal from this column we get zero as the spins are evenly distributed throughout each direction. We can say that this object contains no information at the spatial frequency of one cycle per unit length.

Consider now a series of line-pair structures as in Figure 8.14 with alternating sections containing protons or nothing. Obviously only the sections containing protons can contribute to the signal. In the instance where k = 8 due to their distribution in space they are in phase and add up to give a net positive signal. That is to say, this particular value of phase-encode gradient is sensitive to objects containing the spatial frequency eight cycles per FOV. Looking at the other patterns in Figure 8.15, none of the lines add up to anything other than zero.

In general, however, an object (i.e. the patient) will have a range of spatial frequencies. Each value of phase encoding can be considered as a template or a comb (technically, a filter) that only responds to one spatial distribution of MR signal or spatial frequency. To build up a whole picture, the entire range of possible spatial frequencies has to be interrogated. This is achieved by stepping through all the values of phase encoding (the 'ladder' in Figure 8.1), once per TR period. When no gradients are applied, we get a signal from the whole object, and this is referred to as the zero spatial frequency or zero k.

So the MR sequence consists of multiple repetitions of the excitation process followed by a different phase-encode gradient until all possible spatial frequencies are collected. You can think of each phase-encode step as being a filter or a comb, as in Figure 8.15. Once all these signals are collected, the application of a Fourier transform converts the spatial frequency distribution into a spatial distribution of the excited nuclei, i.e. an image of the patient.

8.5.3 Frequency Encoding

There is no reason why this phase-encode process should not be re-applied to obtain the full image in the other directions. The only practical difficulty is that for every value of G_{PE} (or spatial frequency in the PE direction, k_{PE}) we have to collect ALL the values of k_{FE} (apply all the G_{FE} gradient steps). This would take a long time, but it is possible, and three-dimensional imaging or three-dimensional Fourier transform (3D FT) does something similar. Fortunately there is a quicker, more convenient and conceptually simpler method of encoding the second in-plane direction: *frequency encoding*.

In frequency encoding we can acquire all the spatial frequency information needed from one MR signal following one RF excitation. In phase encoding we required one MR excitation (RF pulse) for every

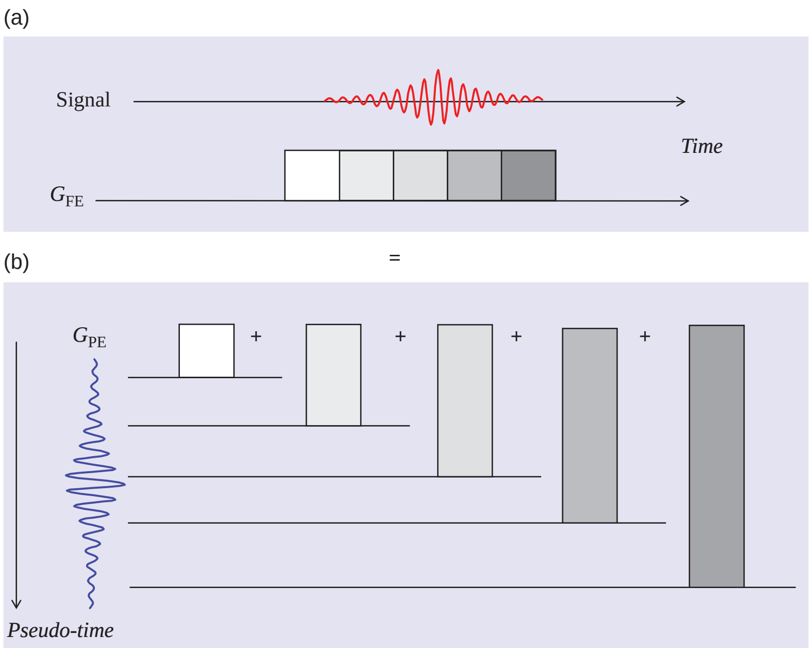

Figure 8.16 Equivalence of frequency encode acquired continuously in real time (a), and phase encode acquired step-wise in 'pseudo-time' (b).

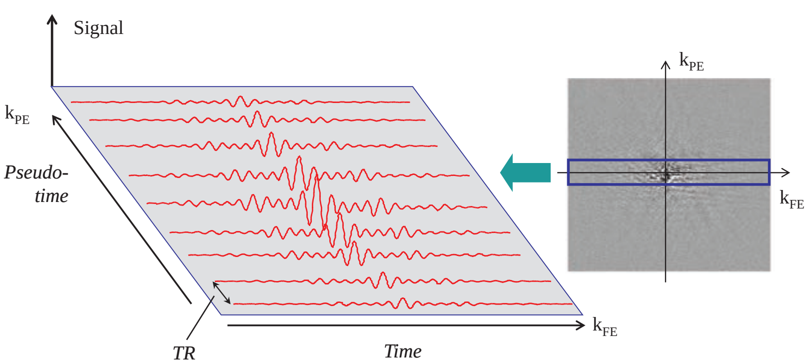

Figure 8.17 Central lines of k-space (magnified) showing the equivalence of phase and frequency axes. Signal strength is shown in the vertical direction in the magnified portion and as a greyscale in the thumbnail. Each PE line is separated by time TR.

line of data (i.e. every value of k_{PE}). For a 256-pixel image we thus required 256 MR excitations, and this will take 256 × TR ms. Figure 8.16 shows a gradient being applied for a certain time, giving a certain gradient moment and phase change, after which the signal strength is measured. The next data point is measured after a different gradient step (and gradient moment and phase change). We then have data points corresponding to the strength of the MR signal after a whole range of gradient moments.

Suppose, however, that we apply a gradient continuously and measure or sample the MR signal at different time-points during the application of that gradient. At each point, the MR signal is affected by a different amount of gradient moment and has a different amount of phase change. Each data point therefore reflects a different amount of 'phase encoding' and thus corresponds to a different spatial frequency. We can therefore collect all the spatial frequencies for that direction from the evolving MR signal in real time following a single RF excitation. This is analogous to the phase-encode acquisition which works in 'pseudo-time', with a sampling separated by TR, as shown in Figure 8.17. The resulting raw data matrix is sometimes referred to as k-space.

So if we can do frequency encoding all at once, why waste all that time with multiple excitations and phase encoding? The answer is that frequency is a scalar parameter, i.e. it is described by a single number. If we applied frequency-encoding gradients in two directions at the same time we would have no way of knowing whether a particular frequency in the signal originated from one or the other (or both) of the applied gradients. By combining phase encoding and frequency encoding in two orthogonal directions we can collect all the spatial frequencies unambiguously that we need to make the image.

Another way of understanding frequency encoding, more usually considered in texts, is to consider the effect of G_{FE} on the frequency of the MR signal, illustrated in Figure 8.18. Because the frequency-encode gradient is present at the same time as the MR signal is being measured, the signal's frequency will now depend upon the position of the material from which it originated within the gradient field. It will not be a single sinusoid wave but a mixture of many frequency components. We then have a signal which is frequency encoded. It is easy to determine the frequency components; we simply perform another Fourier transform. Applied in one dimension this produces a spectrum which represents a one-dimensional projection of the object.

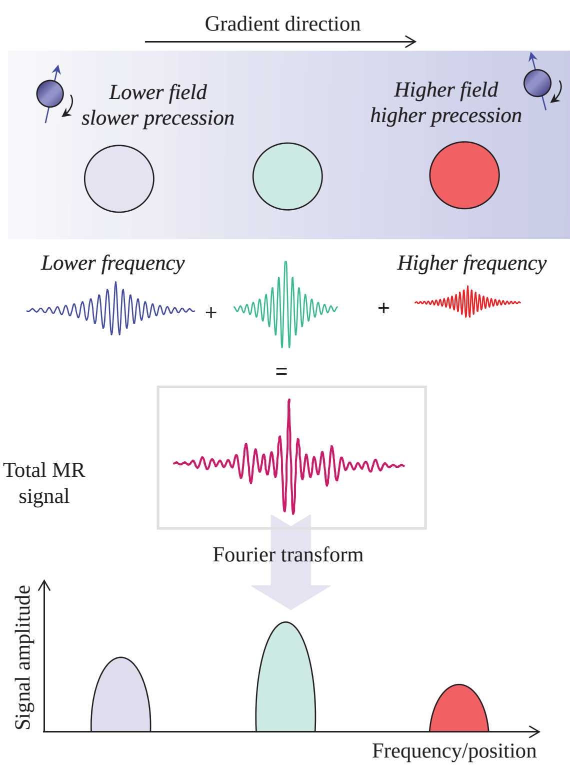

Figure 8.18 Alternative description of frequency encoding in terms of position-dependent frequency changes.

8.5.4 Spatial Encoding: A Musical Analogy

You can think of MRI in-plane localization in terms of playing a multi-stringed musical instrument such as a guitar. Suppose you wish to play every possible note (or frequency) distinctly, you first have to pluck a string. This is like the RF excitation. The sound it makes is like the MR signal. It is then relatively easy to play every note available on that string by running a finger up the fretboard. As one does this the length of the string is in effect shortened (this is like a gradient) and the pitch of the note (or frequency) changes. All the while the sound can be heard. This is like frequency encoding.

However, there are other strings. To hear them we have to pluck one of them (perform another RF excitation) and repeat the pitch-changing action. The action of changing to a different string is analogous to the operation of phase encoding in MRI.

8.5.5 2D FT Reconstruction

To reconstruct the image we do a 2D FT on the raw data matrix or k-space. Usually the image is then displayed as a 'magnitude' image. The result of the 2D FT is actually a complex image with 'real' and 'imaginary' parts, as shown in Figure 8.19, or with amplitude and phase (see the Appendix for a reminder about complex numbers). We usually combine these as a complex magnitude (this gets round some problems with the B_0 field) and the images only contain positive values. Sometimes a phase image can be optionally produced too.

Certain types of scan may require 'real' reconstruction, which allows the image to have positive and negative values. In this case the image background is mid-grey. Real-valued inversion recovery is an example of this (see Section 12.4.1).

8.5.6 Resolution and Field of View

For a 2D FT MR acquisition the resolution is normally pixel limited provided the signal-to-noise ratio (SNR) is adequate. So if the pixel size is 1 mm, then you should be able to see details of this size clearly. To increase the resolution for a fixed field of view (decrease the pixel size) you can do one of three things: increase the gradient strengths, increase the matrix or increase the sampling time (for the FE direction only). In practice, you cannot get arbitrarily small pixels as sooner or later you will run out of gradient power and SNR.

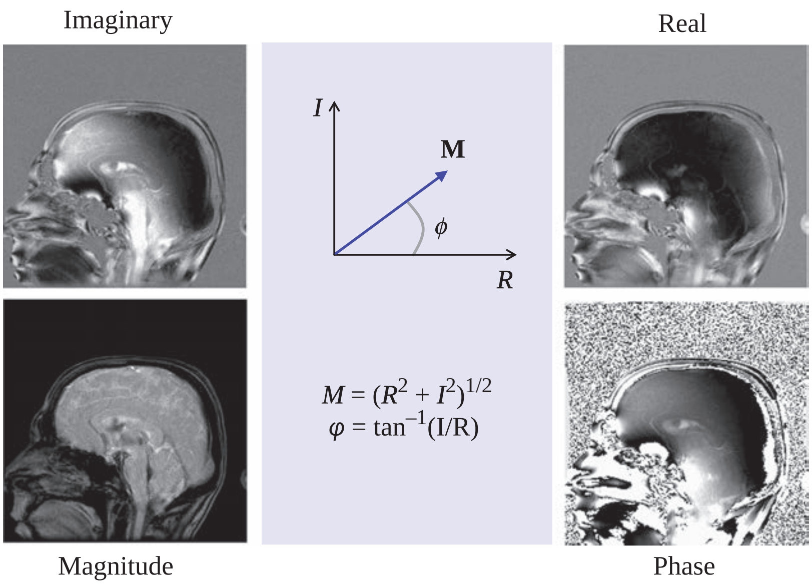

Figure 8.19 Reconstruction of a magnitude (M) image from real and imaginary parts following Fourier transformation. A phase (ϕ) image can also be calculated.

To decrease the field of view (zoom in) while maintaining the matrix size you can either increase the gradients or increase the sampling time (in the FE direction only).

Resolution and Field-of-View Maths

The maximum resolution is given by the pixel size

$$\Delta x = \frac{1}{\gamma \cdot G_{FE} \cdot M \cdot \Delta t} \qquad \Delta y = \frac{1}{\gamma \cdot \Delta G \cdot N \cdot \tau}$$

The size of the image or the field of view (FOV) is given by the inverse of the minimum spatial frequency step

$$FOV_{FE} = \frac{1}{\gamma \cdot G_{FE} \cdot \Delta t} \qquad FOV_{PE} = \frac{1}{\gamma \cdot \Delta G_0 \cdot \tau}$$

Notice that the Fourier transform principle of ‘less is more’ applies: it is the maximum size of the gradient which controls the pixel size, while the time between samples or phase-encode step size controls the FOV.

8.6 Consequences of Fourier Imaging

The consequences of Fourier imaging relate to the properties of k-space, the determination of resolution and field of view and the generation of typical artefacts.

8.6.1 Adventures in k-Space

In simplest terms k-space is the raw data matrix which stores the already-encoded MR signals (Figure 8.17). We can think of the application of the gradients as defining a path or a trajectory through k-space, as shown in Figure 8.20. At time A, the application of the frequency- and phase-encoding gradients, the signal is at the centre of k-space (corresponding to a summation of the total MR signal from the object). The dephase portion of G_{FE} gradient combined with the maximal negative G_{PE} step moves it to the bottom left corner at time B. The read-out part of G_{FE} moves it along a line of k_{FE} from left to right. The peak of the gradient echo occurs on crossing the k_{PE} axis (time C). Provided the MR signal has totally decayed before the next excitation we will start at the centre again. This time G_{PE} moves us to the second bottom line of k-space. By the end of the scan we will have acquired N_{PE} gradient echoes, each corresponding to a different k_{PE} position. This gives us a full sample of the spatial frequencies in the image.

The pixel size is defined by the total length of the k-space axes. The FOV is determined by the separation of the k_{PE} lines and of the samples along each k_{FE} line as shown in Figure 8.21. We have already seen that the central portion of k-space determines the overall brightness or contrast of the image while the outer regions determine the fine detail (Figure 8.11).

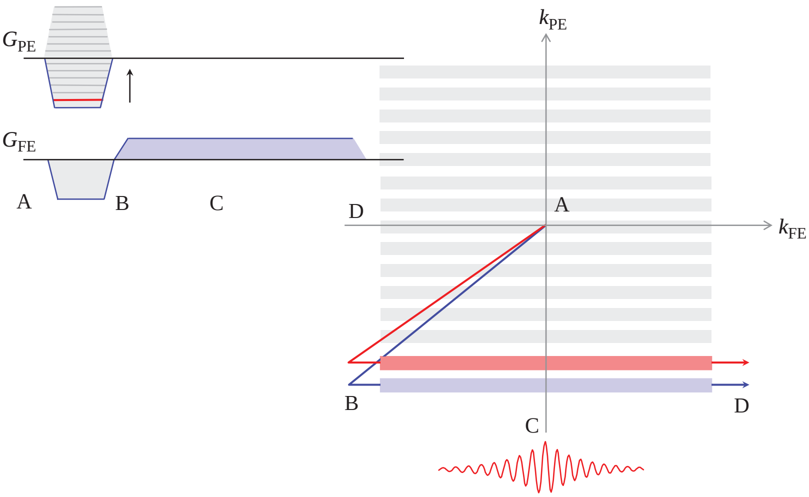

Figure 8.20 k-space path for a pair of frequency- and phase-encode gradients, showing echo formation. Following the RF pulse, and before the gradients are applied, the signal is at the centre of k-space. This means that it represents the total image brightness irrespective of spatial localization.

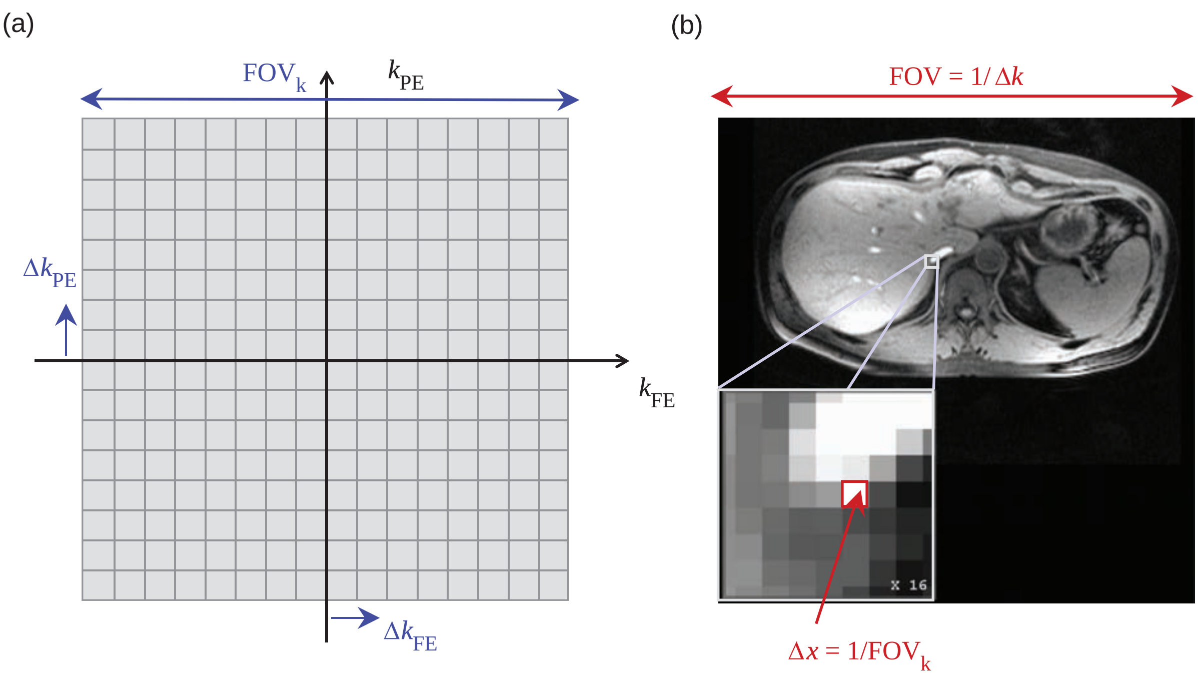

Figure 8.21 Relationship between (a) k-space and (b) image resolution and FOV.

k-Space Exploration

For a 2D object represented by the function $f(x, y)$, the spatial frequencies k_x, k_y are given by

$$F(k_x, k_y) = \iint f(x, y) \cdot \exp\left(i2\pi \cdot \left(x \cdot k_x + y \cdot k_y\right)\right) \cdot \mathrm{d}x\mathrm{d}y$$

and the object expressed in terms of the spatial frequencies is

$$f(x,y) = \iint F(k_x, k_y) \cdot \exp\left(-i2\pi \cdot \left(x \cdot k_x + y \cdot k_y\right)\right) \cdot \mathrm{d}k_x\mathrm{d}k_y$$

The functions f and F are a Fourier transform pair. The quantities x and k_x bear a reciprocal arrangement, i.e. small spatial objects (little x) have big k values and vice versa. Less is more again! Thus the highest spatial frequency represents the smallest object detectable (i.e. the pixel size)

$$\Delta x = \frac{1}{N\Delta k} = \frac{1}{FOV_k}$$

and the largest object (i.e. the field of view)

$$FOV_x = \frac{1}{\Delta k_x}$$

For example, in the phase-encode axis, if the maximum gradient strength is 8.6 mT m^{-1} applied for 0.7 ms with 256 steps

$$\Delta k = \frac{8.6}{128} \times 0.7 \times 42.56 = 2 \text{ m}^{-1}$$

(remember we step through G_{PE} from $-N/2$ to $N/2$–1) so the FOV is 0.5 m or 500 mm. (Hint: if you use units of mT m^{-1}, ms and use $\gamma\!\!\!^{-}$ in MHz T^{-1}, you get k in m^{-1}.) Similarly,

$$FOV_k = 2 \times 8.6 \times 0.7 \times 42.56 = 512 \text{ m}^{-1}$$

The pixel size in mm is therefore

$$\Delta y = \frac{1000}{512} = 1.95 \text{ mm}$$

For a readout gradient of amplitude 5.87 mT m^{-1} and a total data sampling time of 2.048 ms

$$FOV_k = 42.56 \times 5.87 \times 2.048 = 512 \text{ m}^{-1}$$

giving the same pixel size Δx = 1.95 mm. If 256 samples are taken (a sampling time of 8 μs) Δk will be 2 m^{-1} and the frequency-encode FOV will be 500 mm again.

The MRI image-formation process can be thought of as a sampling of k-space. If sampling conventions (i.e. the Nyquist criterion) are not fulfilled with regard to the spatial frequencies in the object, then artefacts will occur as shown in Section 8.6.2.

In a generalized form for an arbitrarily shaped gradient we can write

$$k = \gamma\!\!\!^{-} \cdot \int_0^t G(t)\mathrm{d}t$$

which defines any k-space trajectory.

8.6.2 Artefacts

Fourier imaging is susceptible to specific artefacts, e.g. phase wrap and Gibbs' ringing (see Section 7.4). Aliasing occurs because the anatomy being scanned exceeds the FOV in the PE direction causing image wrap-around. In Fourier terms this means that the sampling interval Δk is insufficient. Phase encoding will occur as in Section 8.5.2 and Figure 8.13, but as the gradient field is physically larger than the selected FOV, the twisting of the columns of vectors outside the FOV will be too tight. This will exceed the Nyquist criterion and is therefore interpreted erroneously. Phase-encode oversampling reduces the problem. Aliasing is not commonly a problem in the frequency-encode direction as the signal from outside the FOV is encoded as a real frequency which can be removed by electronic filtering.

Haunted by Fourier

It can be argued that the fundamental weakness of Fourier transform encoding and reconstruction is its susceptibility to modulation artefacts which produce 'ghost' replications of the image displaced relative to the true image and often aliased in the phase-encode direction. Any interaction which results in a modulation of either the frequency (FM) or amplitude (AM) of the MR signal will result in ghosting artefacts.

Consider a one-dimensional example of a signal $s_0(t)$ giving an image $i_0(\omega)$ by its FT

$$i_0(\omega) = \text{FT}\{s_0(t)\}$$

Suppose we have a temporal modulation of this signal such that

$$s = s_0(1 - m \cdot \cos\omega_m t)$$

where m is the modulation amplitude and ω_m is the modulation frequency (i.e. this is amplitude modulation or AM). We can then apply the Fourier modulation theory to predict the image in terms of modulation frequency ω

$$i(\omega) = i_0(\omega) + \frac{m}{2} i_0(\omega - \omega_m) + \frac{m}{2} i_0(\omega + \omega_m)$$

The encoding produces an equivalence between frequency (or 'pseudo-frequency') and position, giving a resultant image

$$i(y) = i_0(y) + \frac{m}{2} i_0(y - \Delta y) + \frac{m}{2} i_0(y + \Delta y)$$

This is a combination of the true image plus two shifted ghost images with intensity $m/2$. Converting the modulation frequency in terms appropriate to k-space (PE) we get a distance shift Δy given by

$$\Delta y = \frac{1}{p\Delta k}$$

$$\Delta y = \frac{FOV_{PE}}{p}$$

where p is the periodicity of the modulation in numbers of PE lines. In real time this becomes

$$\Delta y = \frac{FOV_{PE} \cdot TR}{T_m}$$

where T_m is the real time period of the modulation. The effect of signal averaging is to change the periodicity of the modulation relative to the phase-encode time scale. If $TR = T_m/n$ there will be n ghosts, separated by FOV/n.

Gibbs' artefact is a ringing of signal on sharp edges in the image. In k-space terms it is a truncation effect; that is, there are not enough k-values to represent the detail. In basic Fourier theory the transform of a sharp (high-frequency) detail will involve spectral components at all frequencies, theoretically extending infinitely over k-space. Obviously an infinite k-space is impossible. The ringing occurs because of the abrupt ending, or truncation, of k-space. Filtering the data by multiplying it by a smooth function prior to Fourier transformation helps to reduce ringing, at the cost of spatial resolution. The best remedy is more k-space samples, i.e. to increase the PE matrix.

Ghosting is slightly different in that it arises from a modulation, i.e. a variation in amplitude or time, of the MR signal over the lines of k-space (k_{PE}) possibly arising from physiological motion or equipment imperfections. The shift of the ghost images is inversely proportional to the period of the unwanted modulation. The fastest perceptible modulation is from one line to the next (period = $\Delta k/2$). This gives a ghost separation of half the FOV. Slower modulations, covering several TR intervals, will result in less shifted ghosts. The size (or depth) of the modulation determines the amplitude of the ghost images.

8.7 Speeding It Up

There are three things we can do to speed up the data acquisition which involve tricks in k-space: they are half Fourier, reduced matrix and rectangular field of view (RFOV). We also encountered these in Chapter 6. Some are illustrated in Figure 8.22, along with a fourth option, partial echo. Partial echo is like half Fourier but applied to frequency encoding (see Box 'Real or Imaginary').

8.7.1 Half Fourier

The most radical trick we can do is called 'half Fourier', 'halfscan' or 'half NEX', in which we only acquire slightly more than half the data, i.e. we omit half the phase-encoding gradient steps (either the positive or negative ones). In terms of k-space we acquire just the lower (or upper) half (Figure 8.22a) and then estimate the other half of the data using a mathematical trick called complex conjugate synthesis. This is a property of Fourier transform of 'real' functions. This produces a time saving of approximately 50%, does not significantly affect spatial resolution but loses about 30% in signal-to-noise ratio (SNR).

8.7.2 Reduced Matrix

Secondly, we can apply a reduced matrix, or reduced acquisition, i.e. just not bother to acquire the largest phase-encoded lines of data (k-space) (Figure 8.22c). Instead, we replace the omitted k-space data with zeros (zero-filling). The time saving will be proportional to the number of PE lines missed out. The

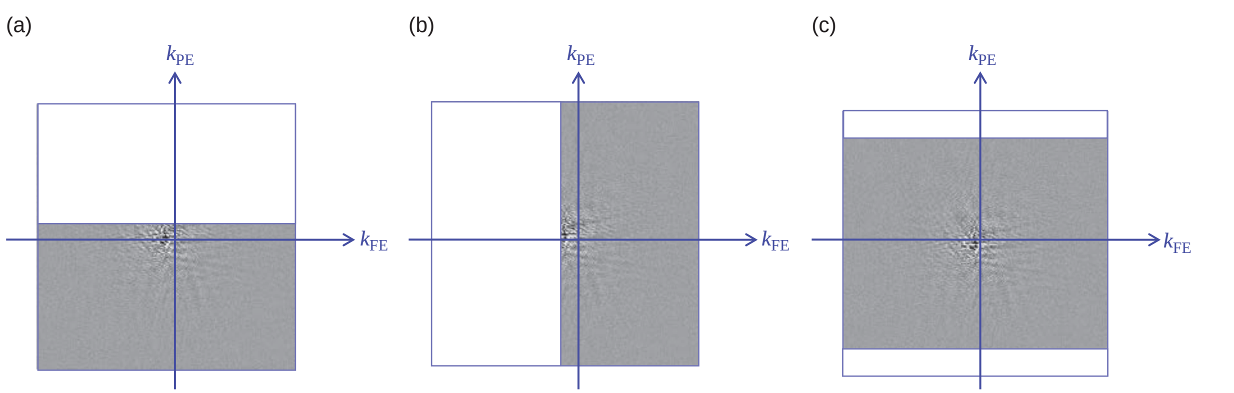

Figure 8.22 Fourier transform speed tricks: (**a**) half Fourier, (**b**) partial echo, which can be used to reduce TE and TR but does not reduce the number of PE lines, and (**c**) reduced matrix.

downside of this technique is a loss of spatial resolution in the phase-encode axis. Small improvements in SNR are made.

Real or Imaginary

A property of 'real' functions is that their Fourier transforms possess complex conjugate symmetry. In a perfect magnetic field we would expect the encoded MR signal to be real, i.e.

$$S(k_{FE}, k_{PE}) = S^{*}(-k_{FE}, -k_{PE})$$

where S^{*} denotes the complex conjugate of S (see Appendix A.4). We can say that signals in k-space have 180° rotational symmetry about the origin (zero), so that values in the top right-hand corner of k-space should be equal to those in the bottom left-hand corner.

This means we can synthesize one-half of k-space from the other, simply by making a copy of the acquired data and swinging it through 180° to fill the other half. In half-Fourier scanning we use this property to reduce the number of phase-encode steps. However, since the magnetic field is never perfect we actually have to collect slightly more than half the data in order to apply phase corrections to the synthesized part. Another application of complex conjugate symmetry is in partial or fractional echo techniques where up to about 40% of the k_{FE} data are not acquired directly, but synthesized (Figure 8.22b). Once again field imperfections prevent a reduction of exactly 50% in the data. Fractional echo is used in rapid imaging to reduce TE and TR.

A 'k-space shutter' is similar, except that it chops off the corners of k-space and thus affects resolution in both frequency and phase-encode directions. This offers no time saving in 2D scanning, but it can help in 3D acquisitions.

8.7.3 Rectangular Field of View

Finally, sometimes we can acquire a rectangular field of view. In the explanations above we have assumed a square field of view (FOV) with an equal matrix size in both directions, e.g. 256 × 256. Often the anatomical region to be scanned is not of similar dimensions in either axis of the image plane, so we can reduce both the phase-encode FOV and the PE matrix. Typical examples are for sagittal or coronal scanning of the knee or spine, where an unequal number of phase-encode to frequency-encode points gives a more efficient coverage of space. To save time (and avoid phase wrap) we always choose the phase-encode axis to be the smaller of the two physical dimensions, i.e. for a sagittal spine we would choose posterior–anterior for phase encoding, while for a transverse head we would commonly choose left–right for the phase-encode direction.

In k-space the effect of a rectangular field of view is to 'space out' the lines in the PE axis (Figure 8.23). Only the field of view is affected as the value of k_{max} remains the same. A time saving of up to 50% is achievable with no resolution loss and only a slight loss of SNR.

It Looks Good, but is it Real?

'Zero-filling' is a very common feature of MRI. Since computers like to work in powers of 2, any non-square matrix (where $N_{PE} \neq N_{FE}$) must be filled up with zeroes before it can be Fourier-transformed. The zeroes are added at the edge of k-space, which corresponds to the high spatial frequencies. This means a smaller pixel size, which is equivalent to interpolating the pixels in the image. However, the zeroes do not contain any signal information about the high spatial frequencies, so it is not real data.

It is also possible for the user to decide to add extra zeroes to improve the apparent resolution of the final image. Remembering that the edge of k-space contains information about high spatial resolution, adding zeroes all around the edge of the acquired k-space can artificially reduce the pixel size. On GE Healthcare scanners these are the 'zip512' and 'zip1024' options, on Siemens it is the 'interpolate' box, and on Philips you set the reconstruction matrix independently from the scan matrix.

The extra zeroes contain no signal, but also no noise, so they have no effect on the SNR of the image. The acquired pixel resolution is unchanged but the displayed pixels are smaller. As with reducing the phase-encode matrix, the rule of thumb is that you shouldn't zero-fill by more than a factor of two.

8.8 3D FT

A fully three-dimensional technique may sometimes prove advantageous over two-dimensional multi-slice acquisition. The principle is simply to apply a second

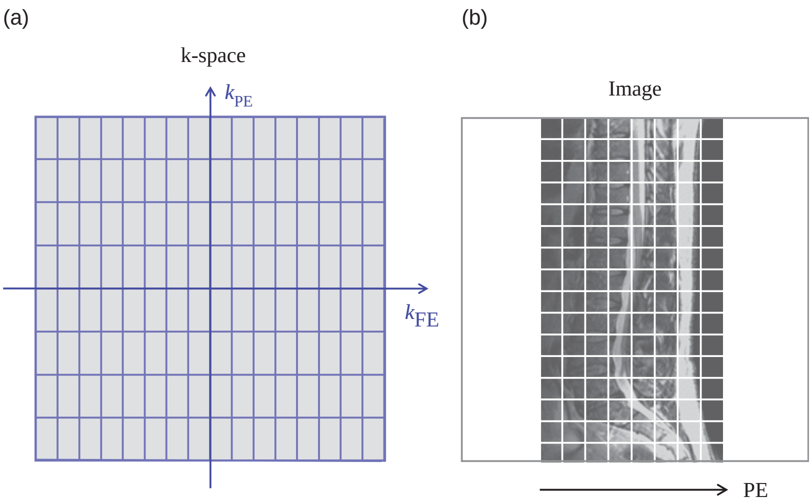

Figure 8.23 (a) k-space sampling for (b) rectangular FOV. Pixel size is maintained by keeping the same maximum and minimum k. The PE field of view is changed by altering the spacing between k_{PE} lines.

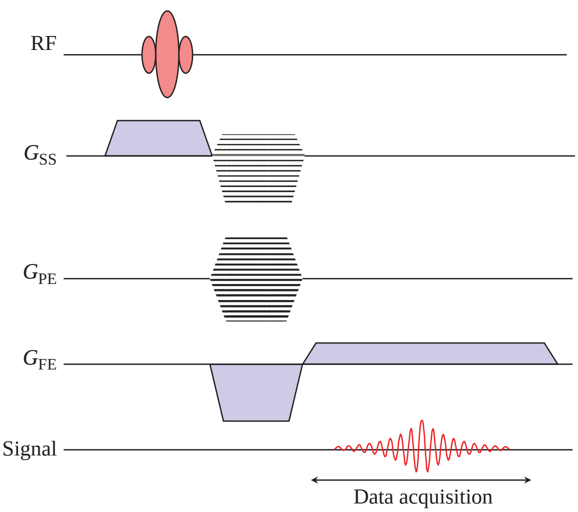

Figure 8.24 Simple 3D FT pulse sequence.

phase-encode axis, ensuring that for every gradient step in the new axis we apply the whole set of other axes steps: i.e. for a $L \times M \times N$ 3D matrix, we acquire $L \times M$ MR signals. The scan time for basic 3D acquisitions is then

$$\text{Scan time} = \text{NSA} \times \text{TR} \times N_{PE} \times \text{number of slices}$$

where, as before, NSA is number of signal averages and N_{PE} the size of the phase-encode matrix.

In practice, multiple slabs are often acquired, with excitation being restricted to a slab or group of slabs by selective excitation and within-slab resolution being produced by stepped rephase gradients (Figure 8.24). In multi-slab or multi-chunk 3D FT we combine interleaved slab or thick-slice selection with additional slice phase encoding to obtain multiple 3D volumes in a single scan. This is a popular technique for producing high axial resolution images of the spine through the vertebrae.

See also:

- Image artefacts: Chapter 7
- Optimizing SNR and resolution: Chapter 6
- Acronyms anonymous: Chapters 12 and 13

Further Reading

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Mosby-Yearbook, chapter 4. Also on the web at http://mri-q.com [accessed 23 March 2015].

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A good explanation of spatial frequencies can be found at http://7e.biopsychology.com/av10.04.html [accessed 22 March 2015].