Image Processing for Physicists

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Image manipulation in the spatial domain

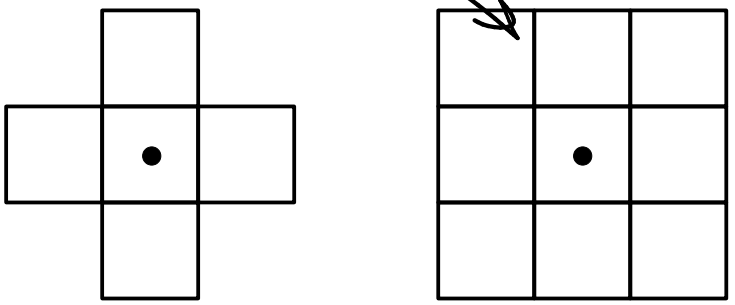
Overview

- coordinate transformations
  - translation, rotation, shear, ...
- intensity transformations
  - normalization, gamma, thresholding, ...
- image analysis using morphological operations
  - dilation, erosion, opening, closing, ...
- image segmentation
  - by morphology, intensity, region, ...

General image transformations

- coordinate transformations → change pixel positions
  - → implies interpolation most of the time
- intensity transformations → change pixel values
- pixel-wise transformations
- neighborhood transformations

} morphology



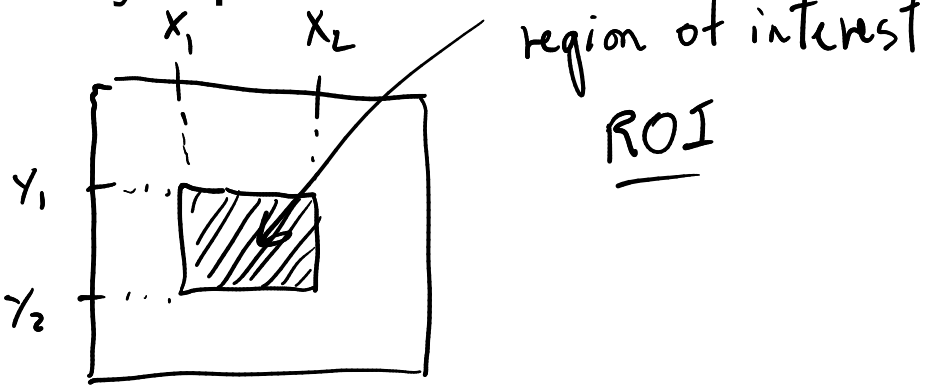
4-neighborhood 8-neighborhood

General image transformations

- images as an array

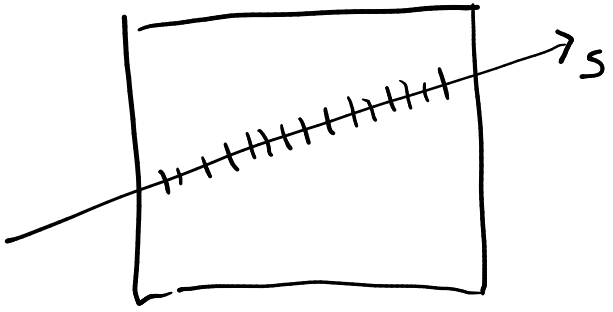
$M \times N$ (x channels)

- sub array operations



- line extractions

= line profile



General image transformations

- element wise addition

$$I = I_1 + I_2$$

$$I(i,j) = I_1(i,j) + I_2(i,j)$$

pixel coordinates

also subtraction of course

- element wise multiplication

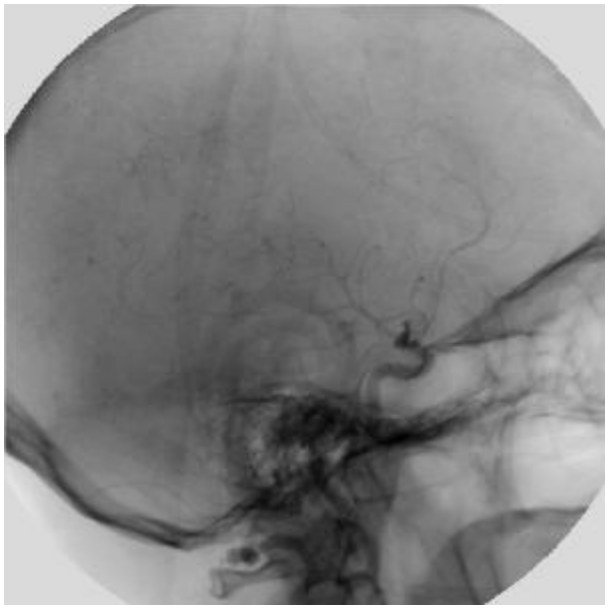
$$I = I_1 \times I_2$$

$$I(i,j) = I_1(i,j) \times I_2(i,j)$$

also division

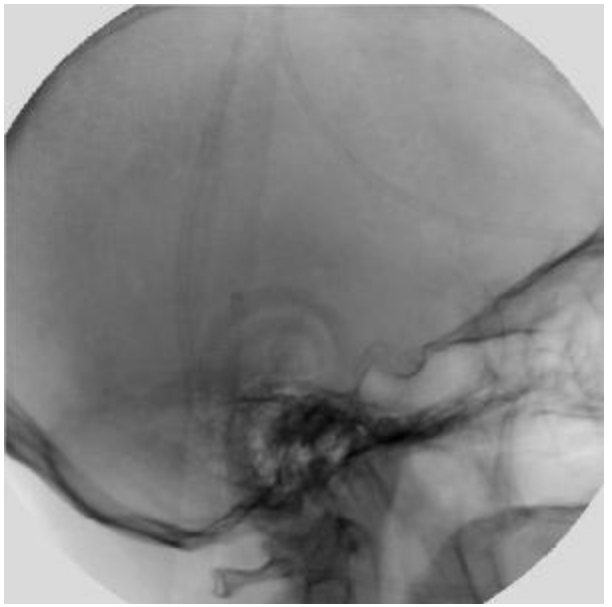
Image Subtraction Example

- Digital Subtraction Angiography
- Xray images before/after contrast agent



Live or contrast image

-



Mask image

=

DSA image

Source: Gonzales, Digital Image Processing

Image Addition Example

- Add multiple noisy images of same object
- (More on noise in later lectures)

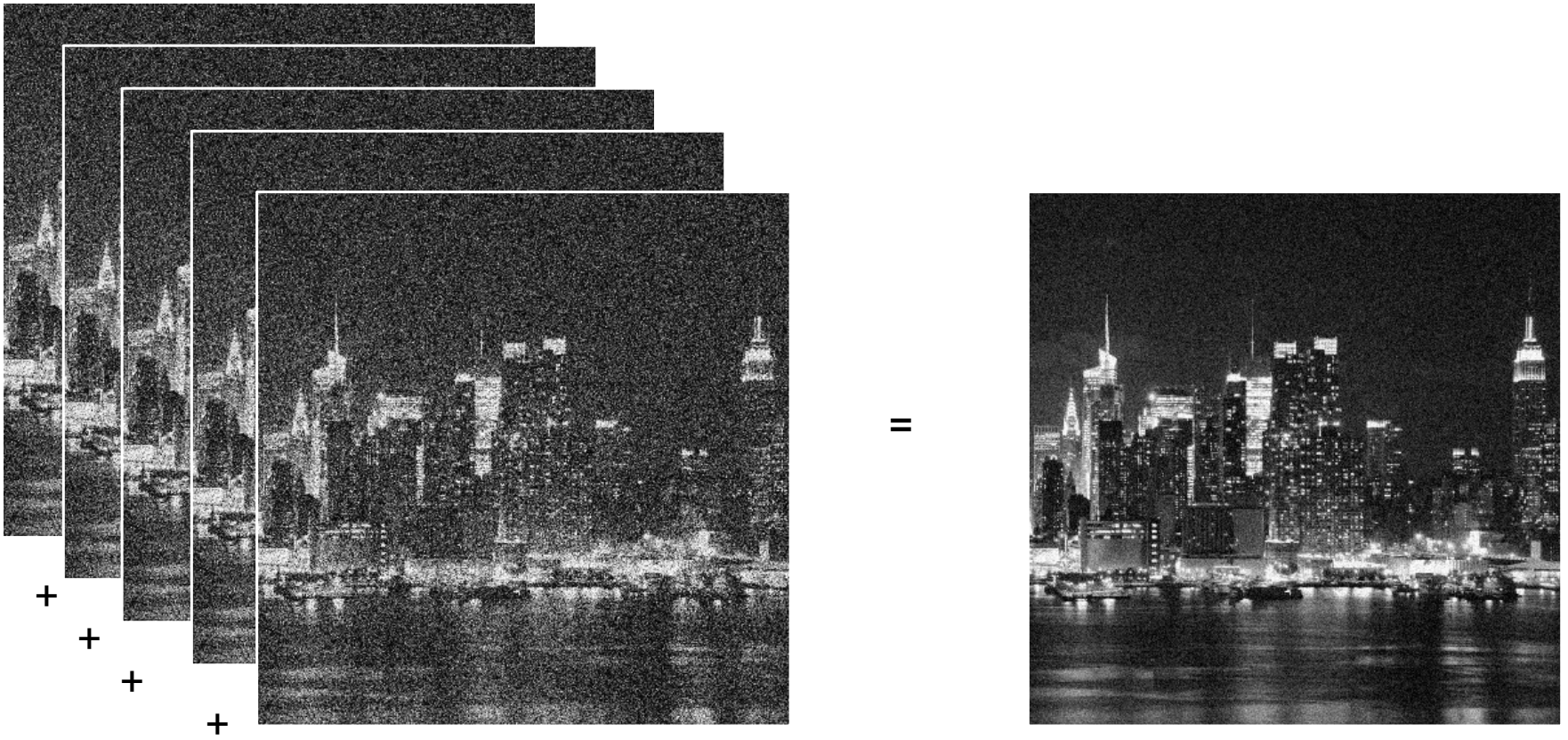
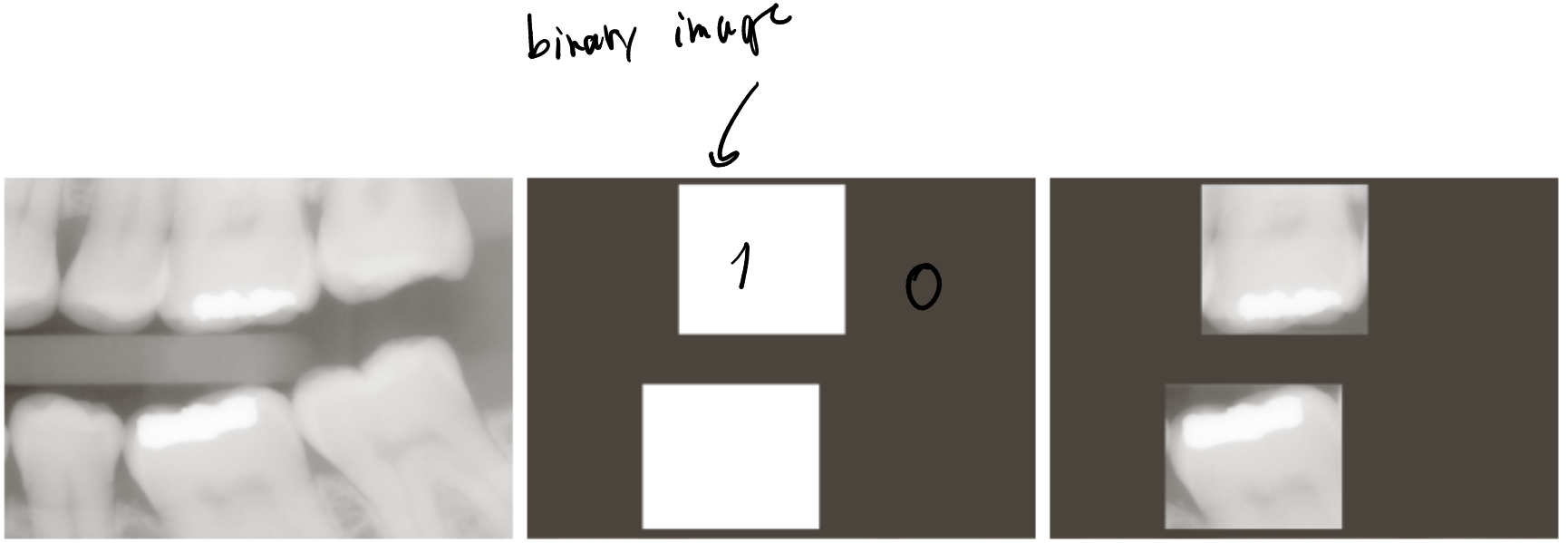


Image Multiplication Example



a b c

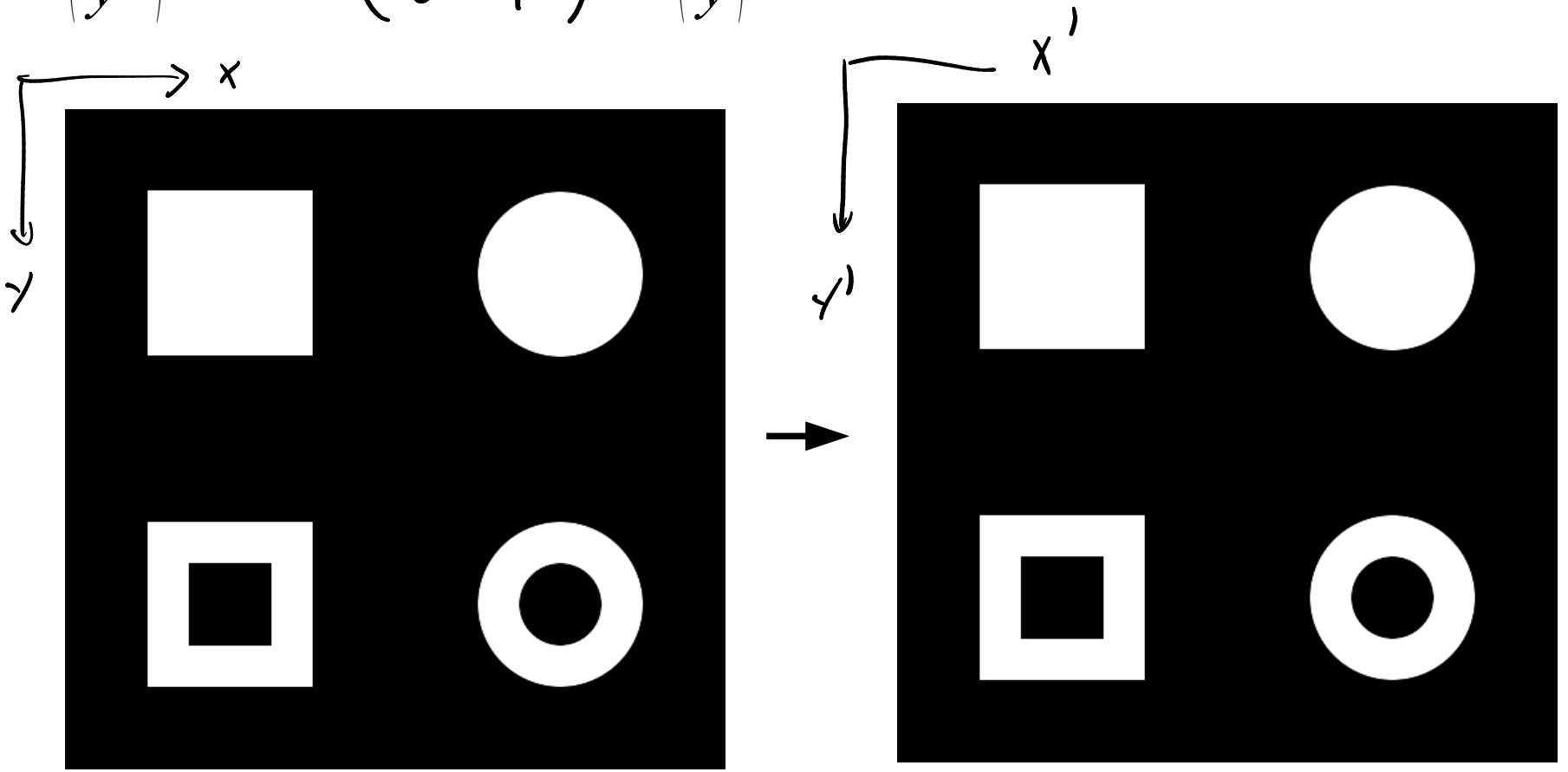
FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Source: Gonzales, Digital Image Processing

Affine transformations

- identity

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Affine transformations

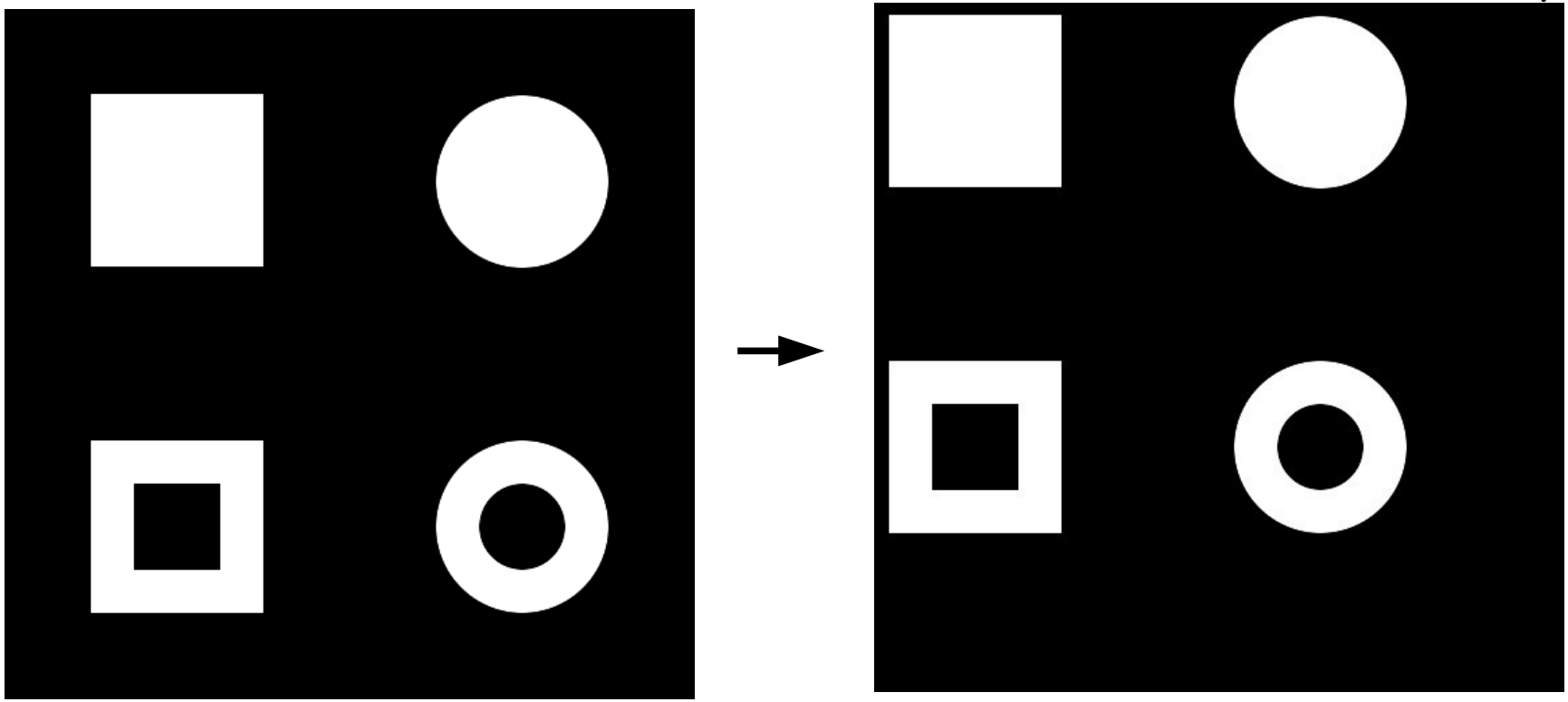
- translation

linear transformation

offset

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

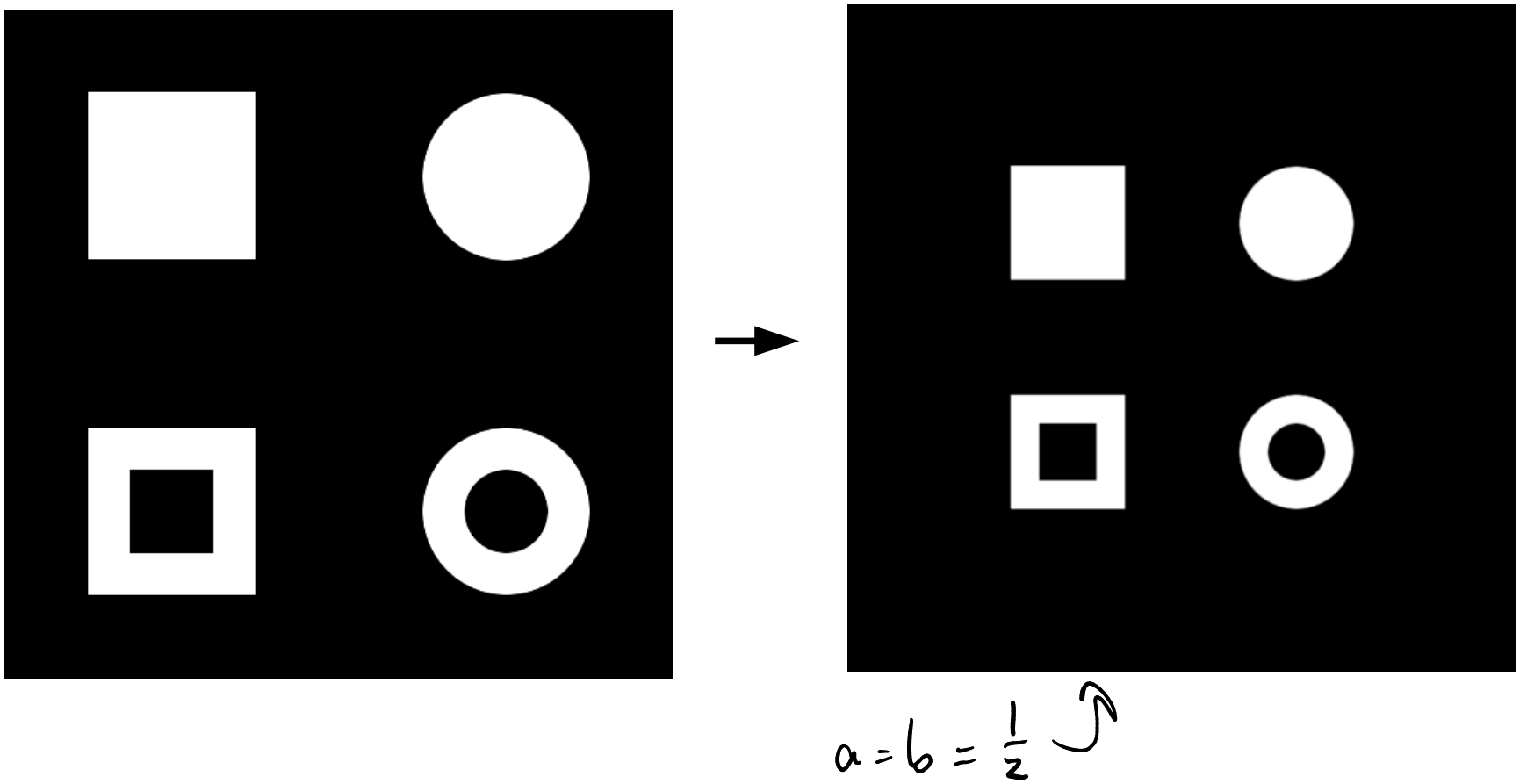
(Affine transforms defined by 6 parameters)



Affine transformations

- scaling

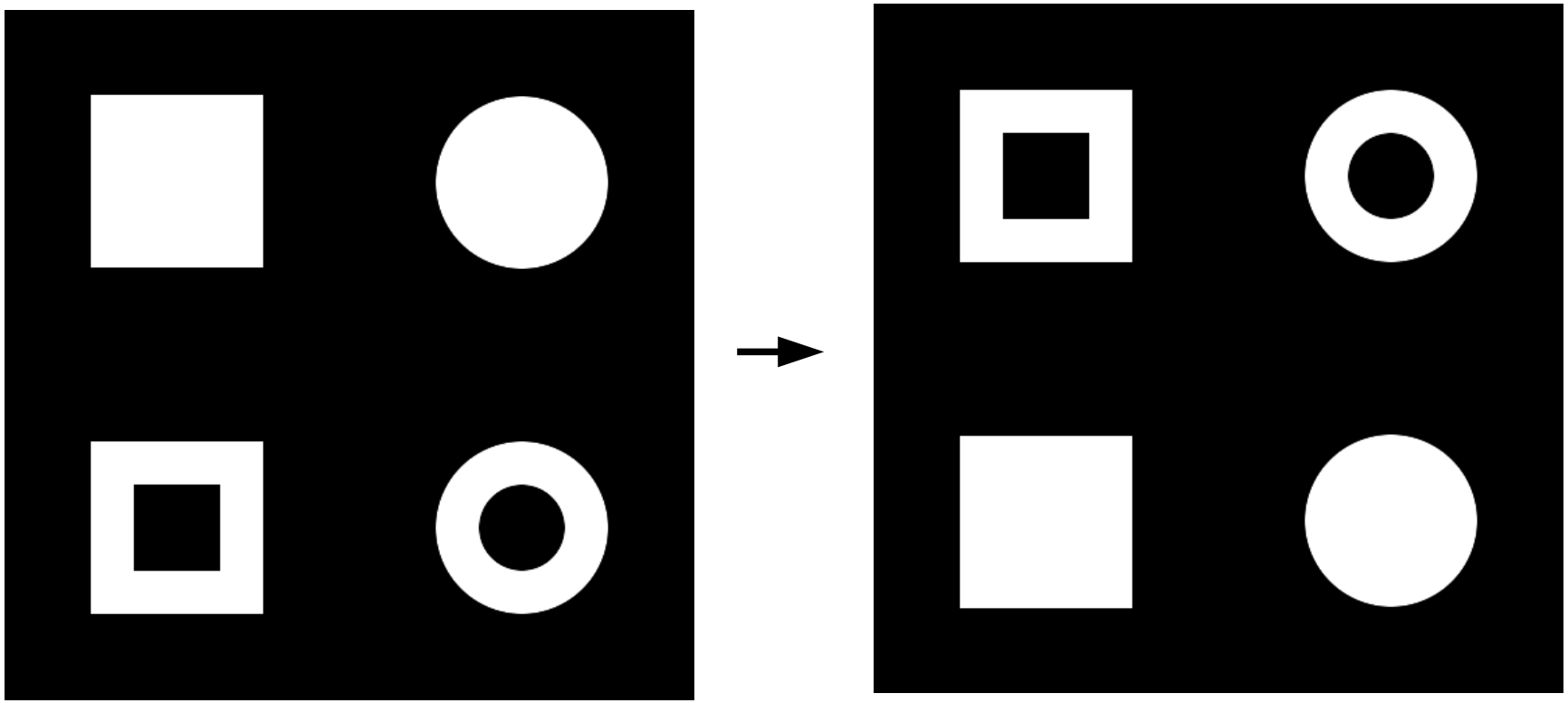
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Affine transformations

- reflections

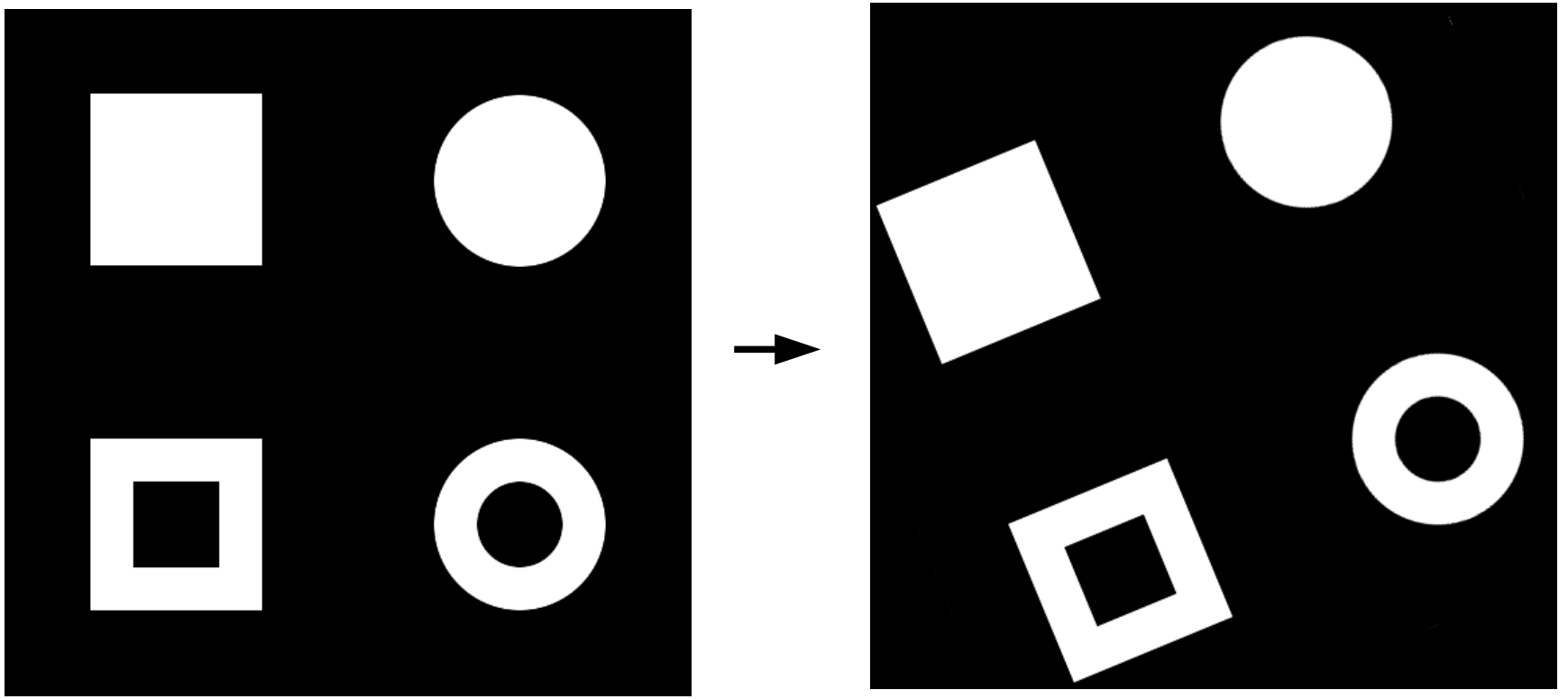
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Affine transformations

- rotation

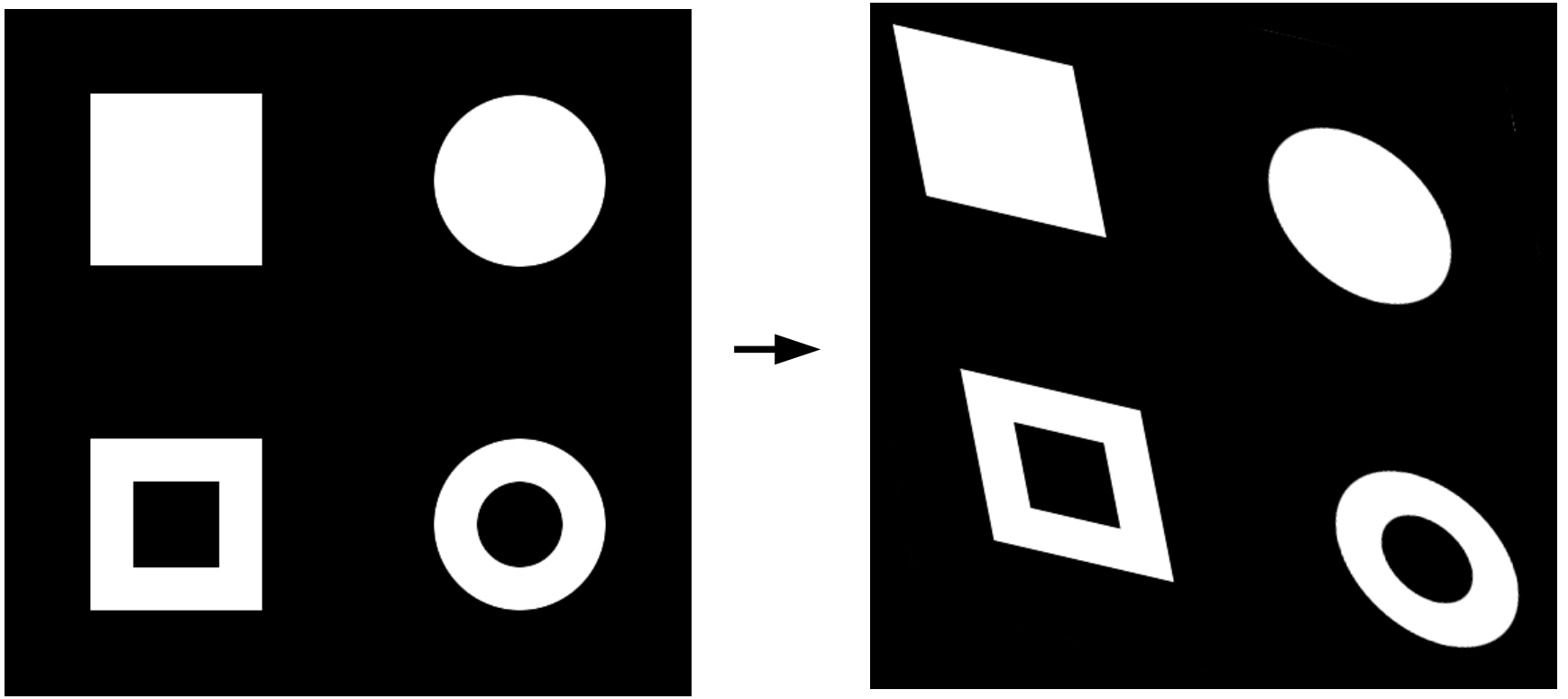
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Affine transformations

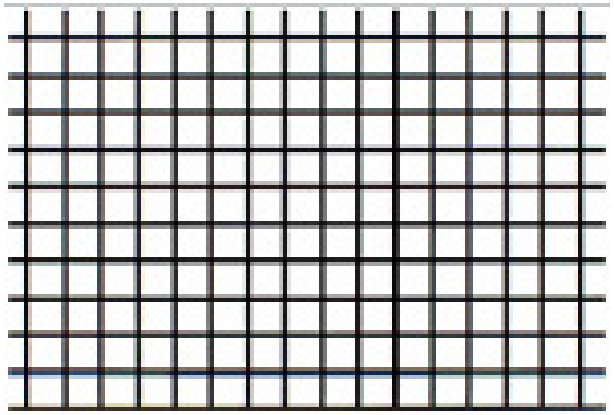
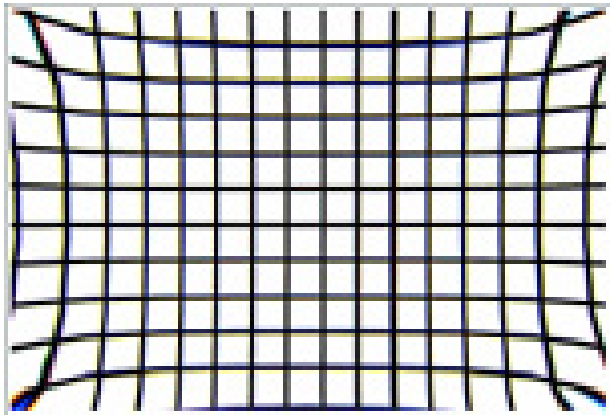
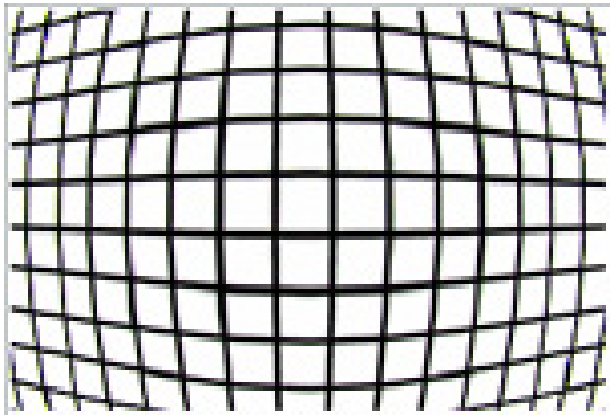
- shear

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Nonlinear coordinate transformation

- pincushion and barrel distortion

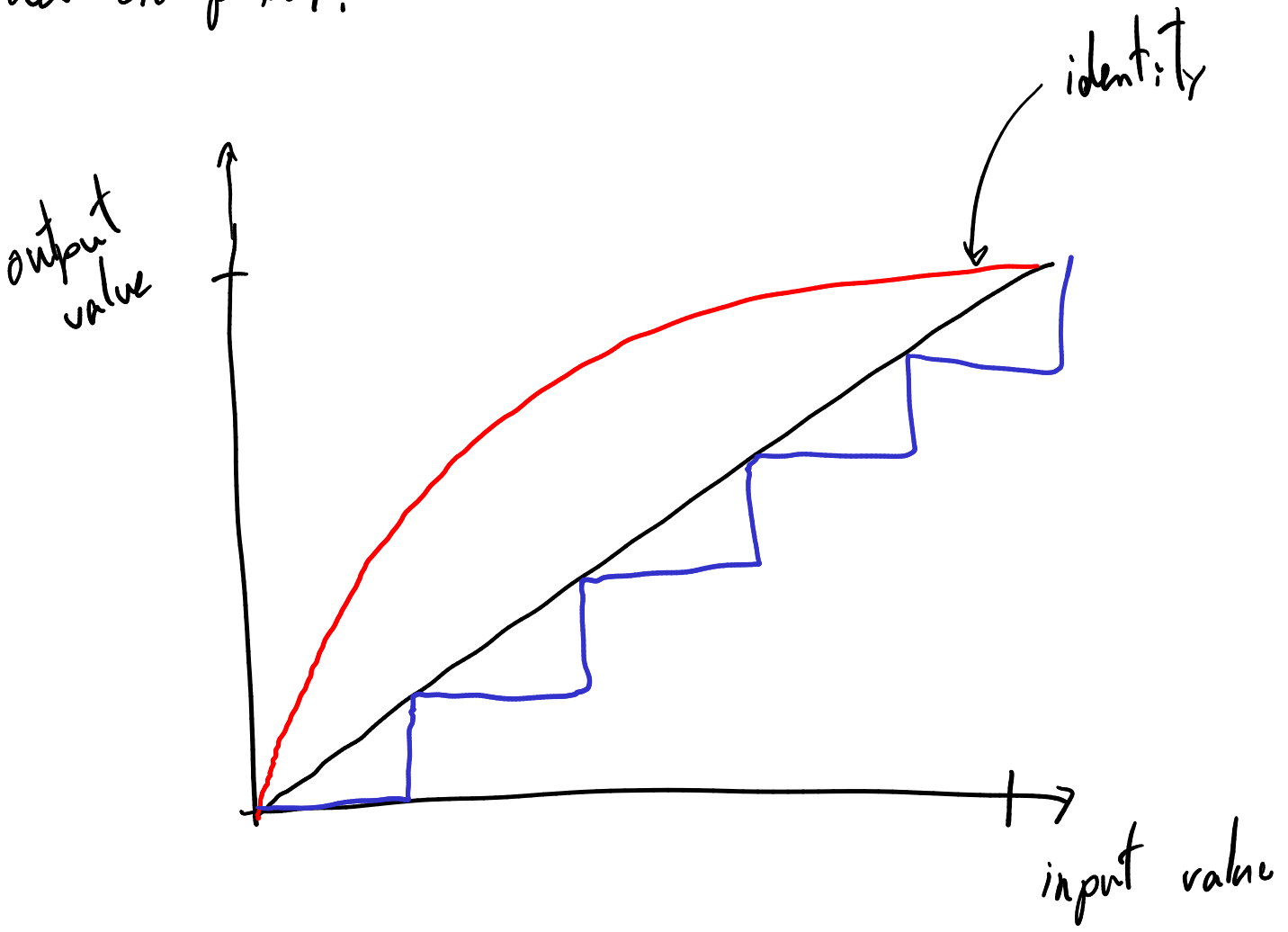

- mapping depends on radial distance from centre

$$x' = x_0 + ax + by + cx^2 + dxy + ey^2 + \dots$$

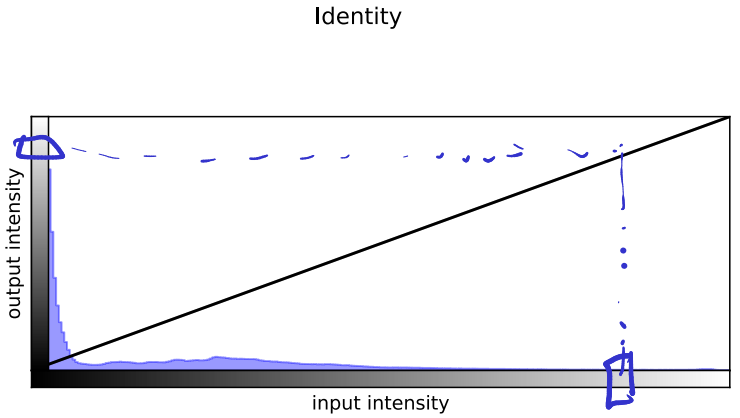
$$y' = y_0 + fx + gy + hx^2 + ixy + jy^2 + \dots$$

Intensity mapping

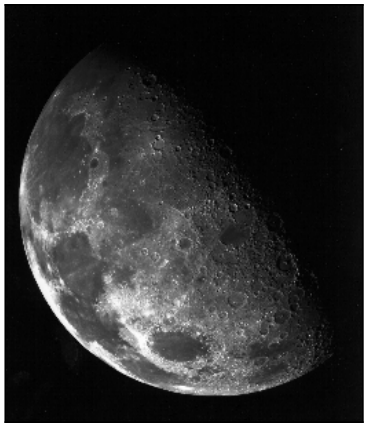
Consider on pixel:



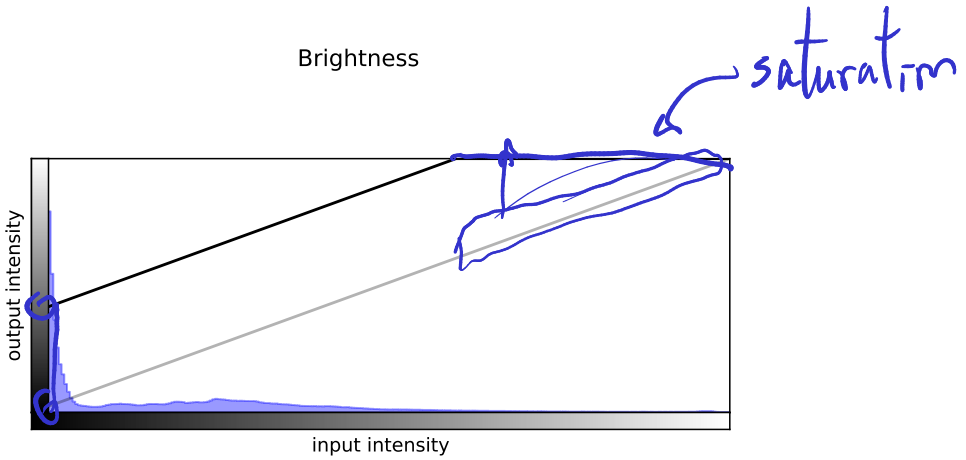
Intensity mappings



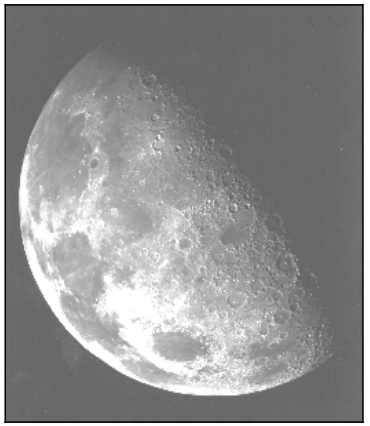

original


remapped

Intensity mappings

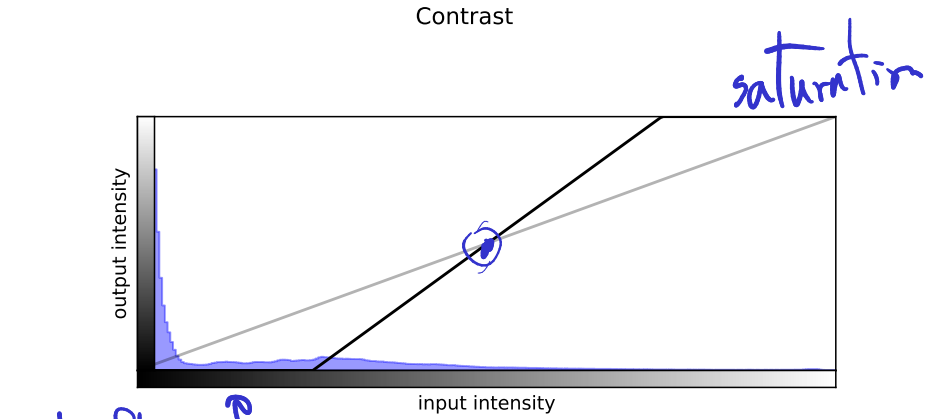


original



remapped

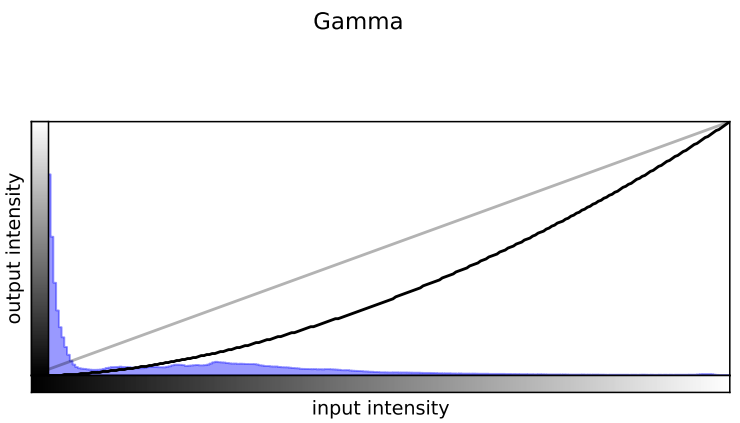
Intensity mappings



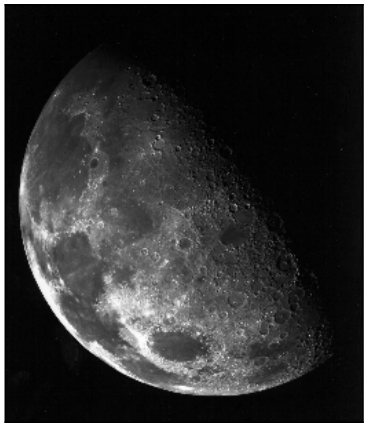
original

remapped

Intensity mappings



$$output = max \left(\frac{input}{max}\right)^{\gamma}$$

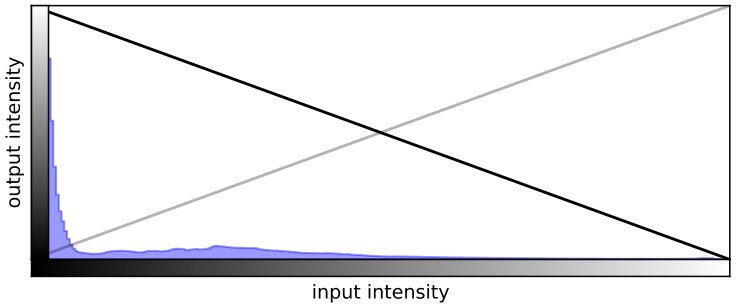

original


remapped

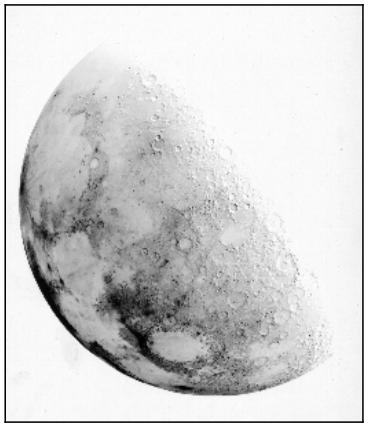
$$I_{max} \left(\frac{I}{I_{max}}\right)^{\gamma}$$

Intensity mappings

Inversion

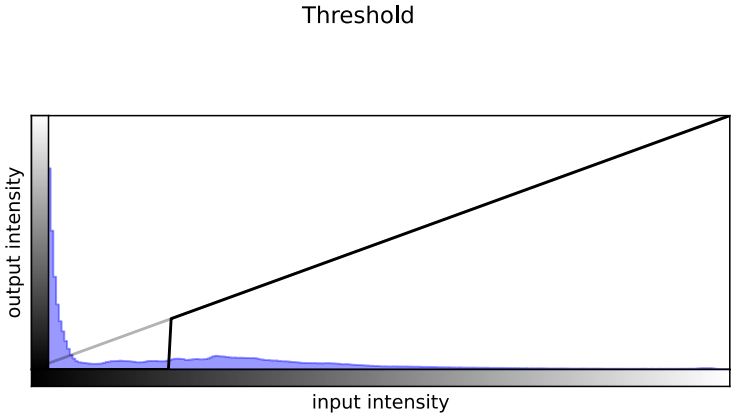


original



remapped

Intensity mappings

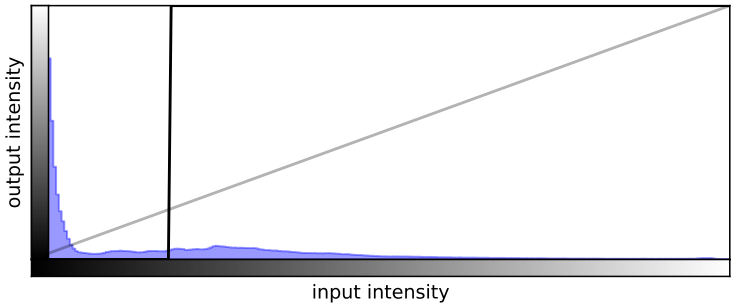



original


remapped

Intensity mappings

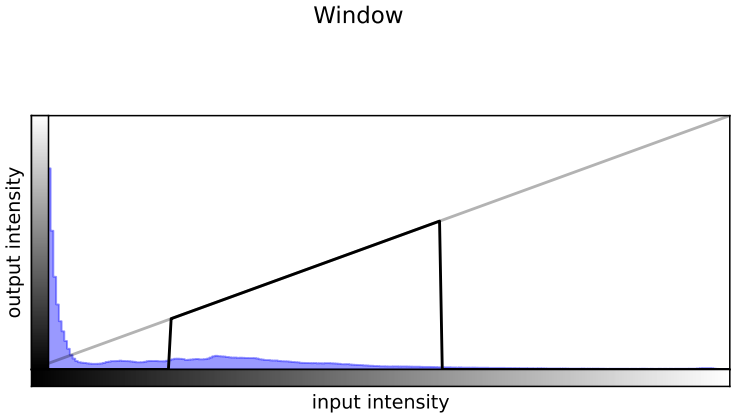
Binary threshold



original

remapped

Intensity mappings

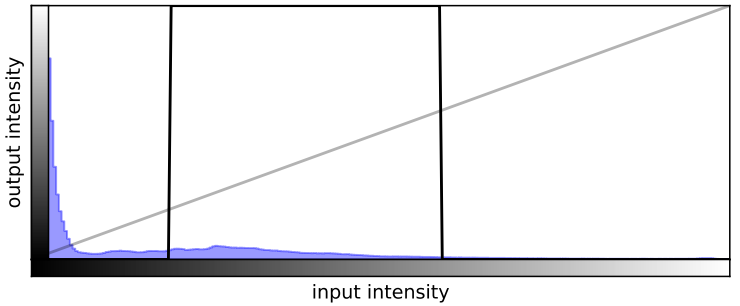



original

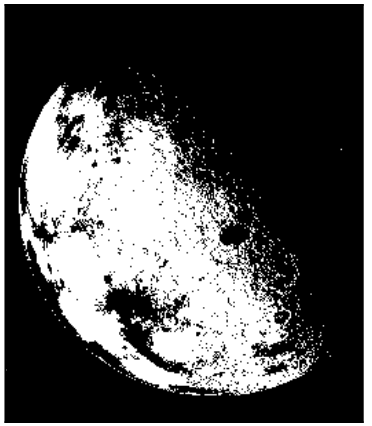

remapped

Intensity mappings

Binary window



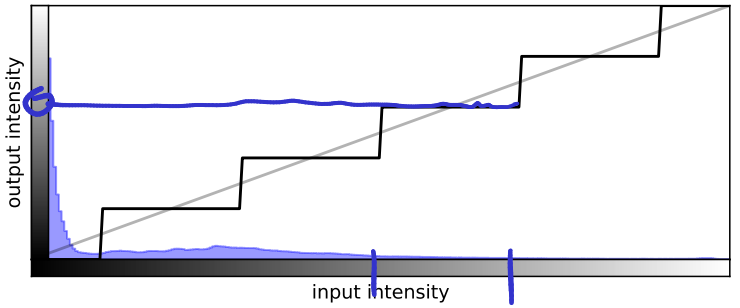
original



remapped

Intensity mappings

Posterization



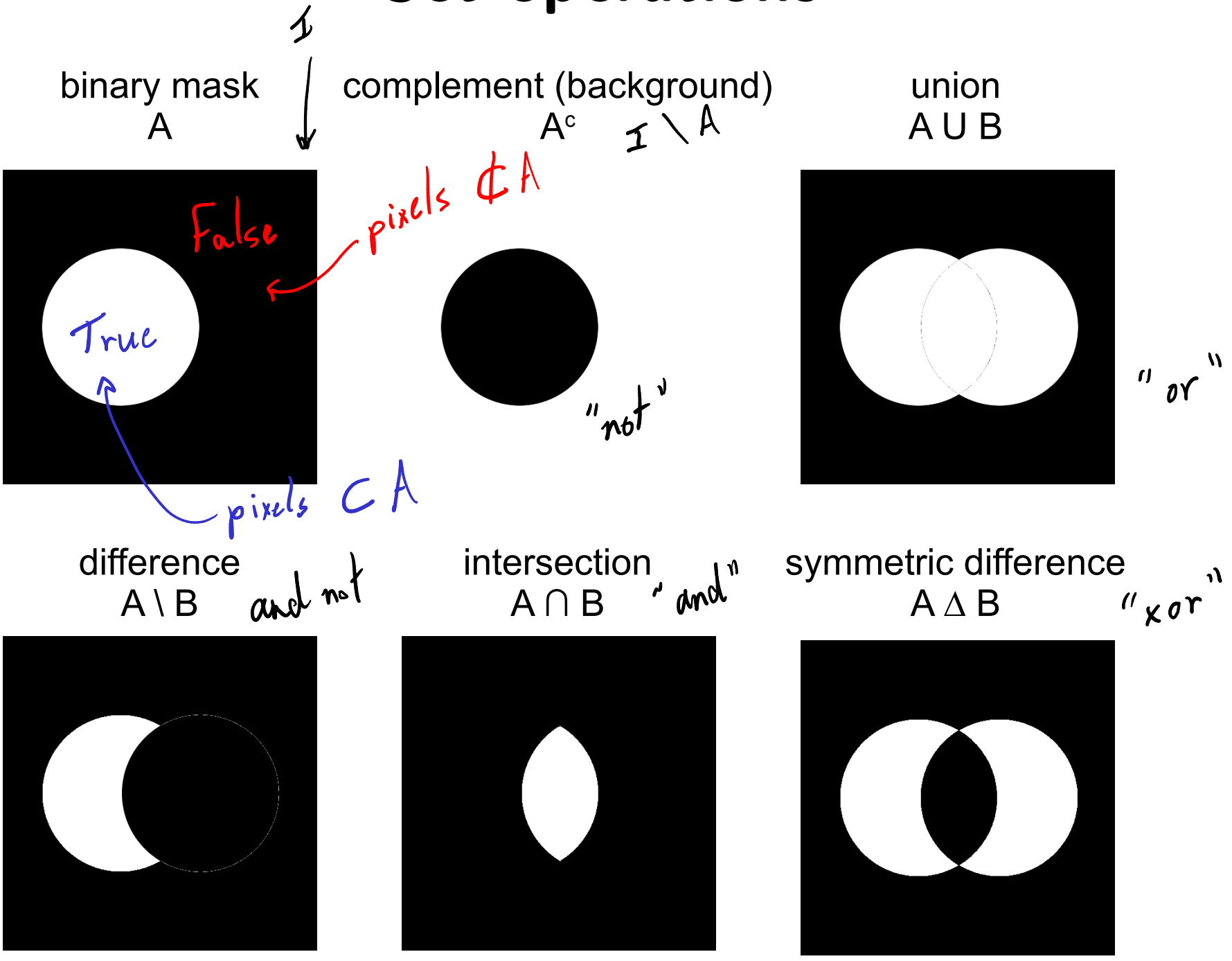
original

remapped

Morphological operations

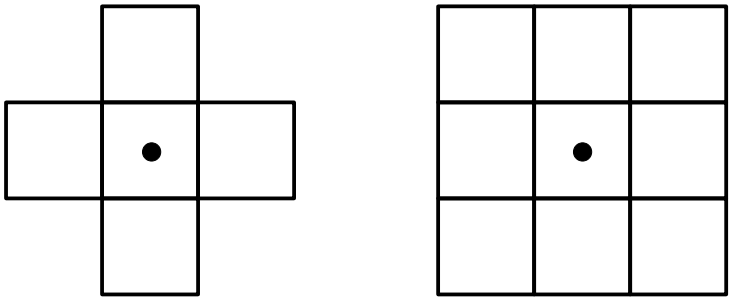
- analyze morphology of image structures
  - based on set theory and topology
- extract image information
  - shape
  - size
  - connectivity
  - number
  - boundary
- mostly on binary images

Set operations



Structuring elements

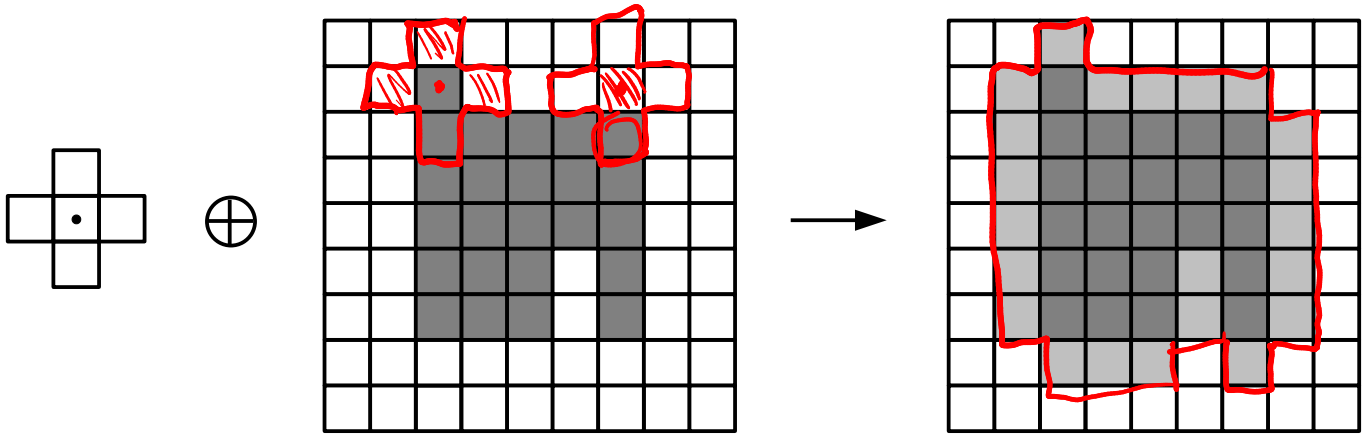
- small bit mask to probe the image
- scan origin of SE over image
- check overlap between SE and image
- set pixel(s) to zero (or one)



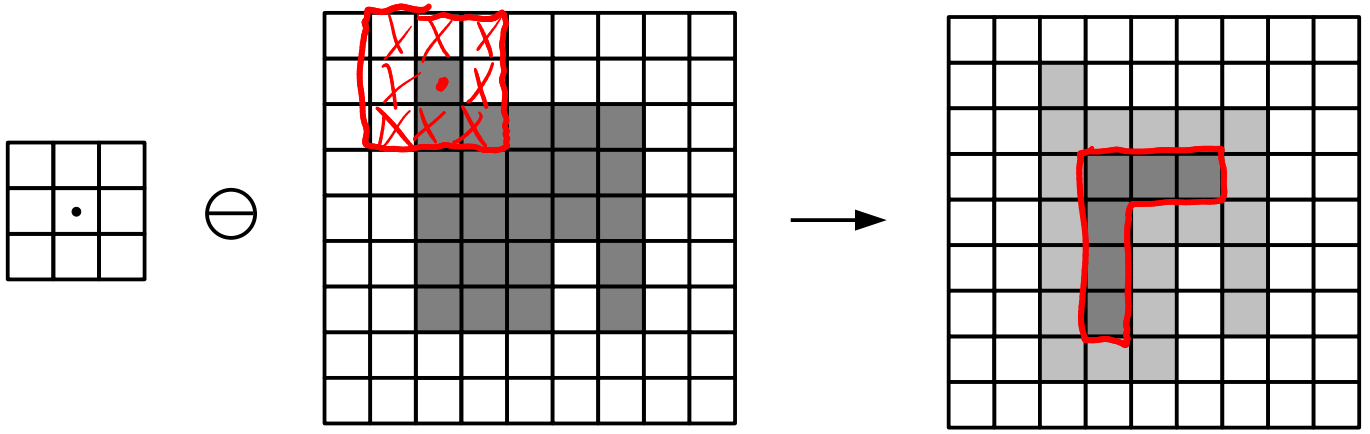
also called "footprint"

Basic operations

- Dilation: expand region

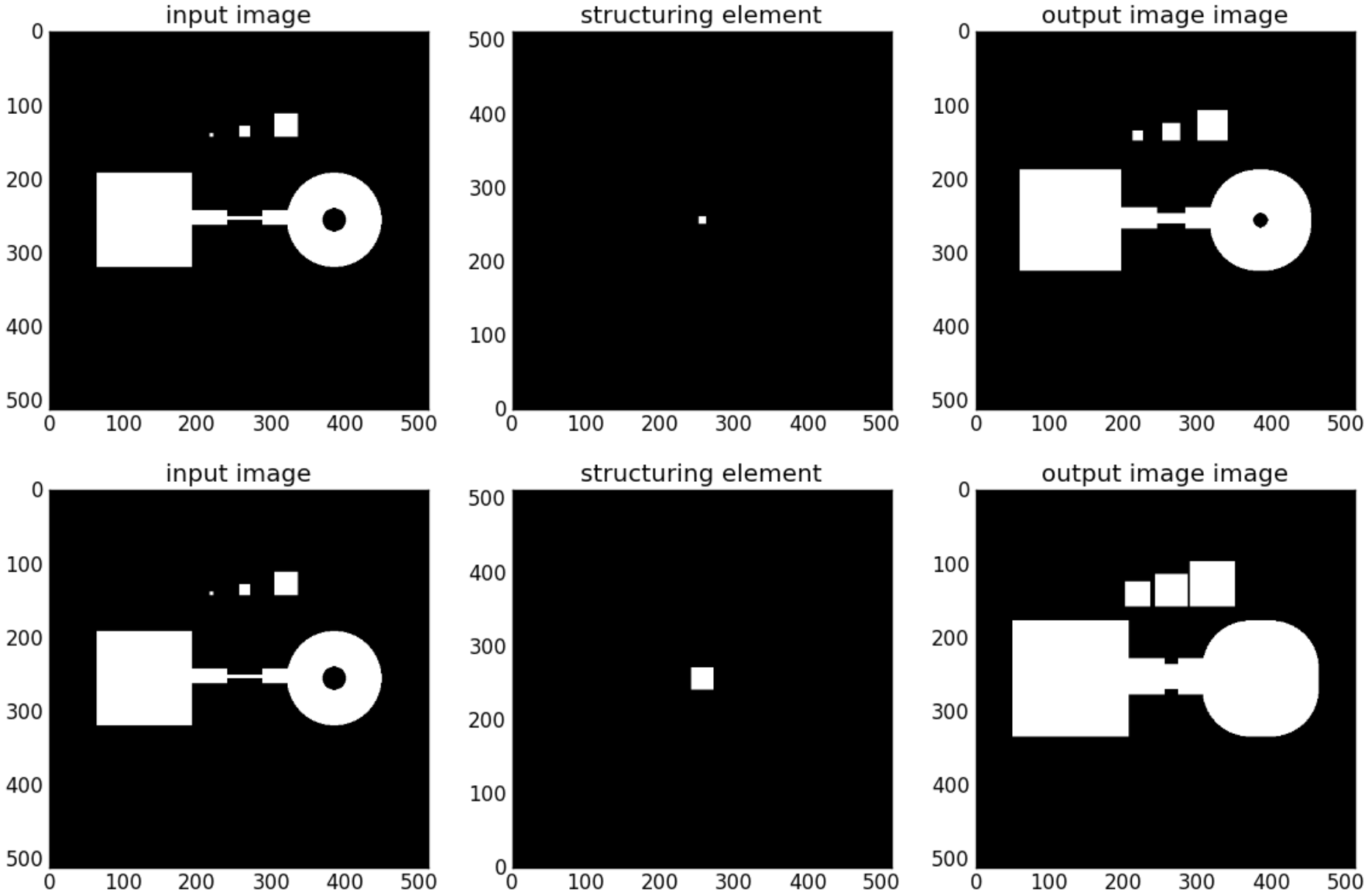


- Erosion: shrink region



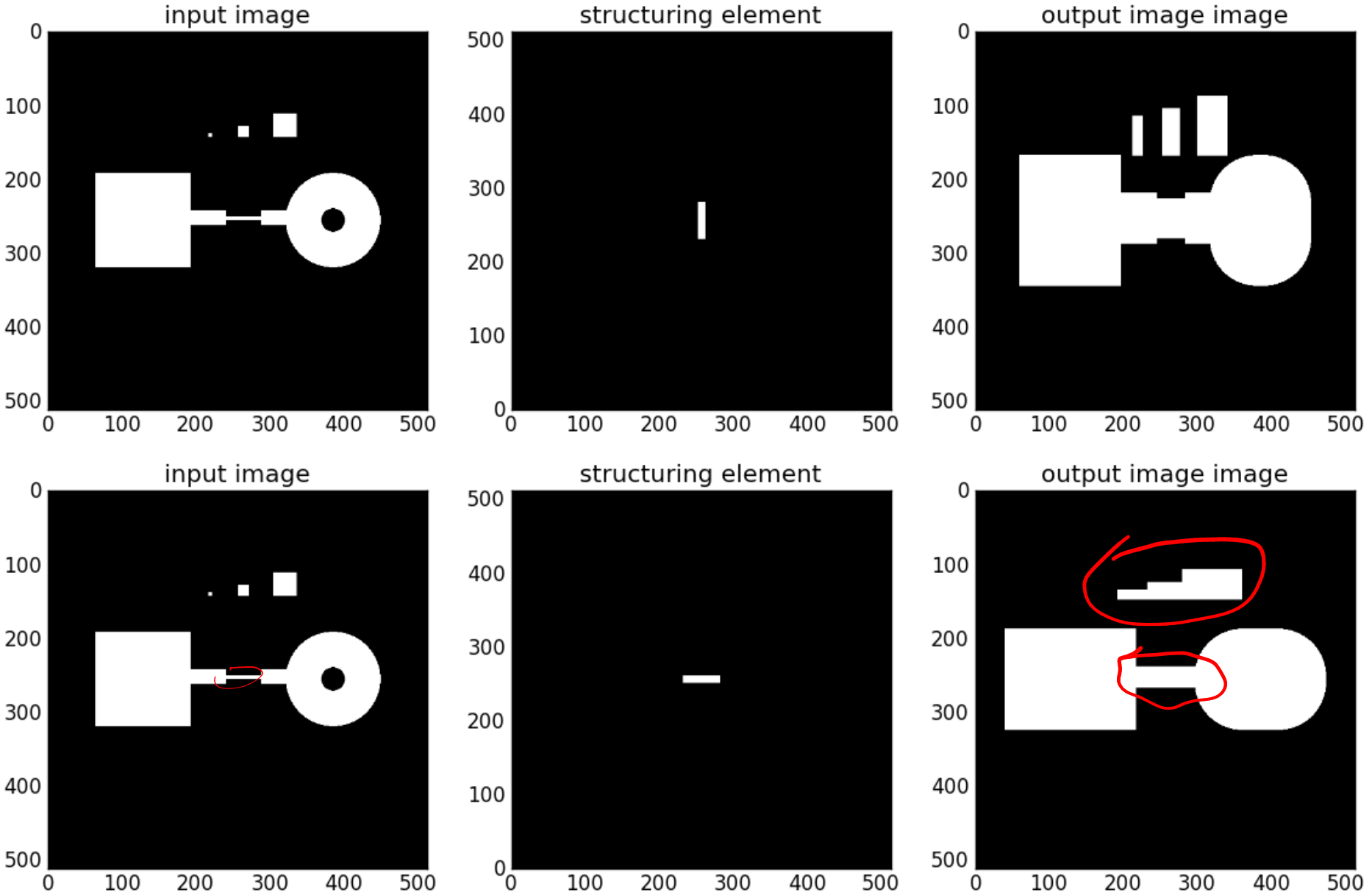
Morphological operations

- dilation



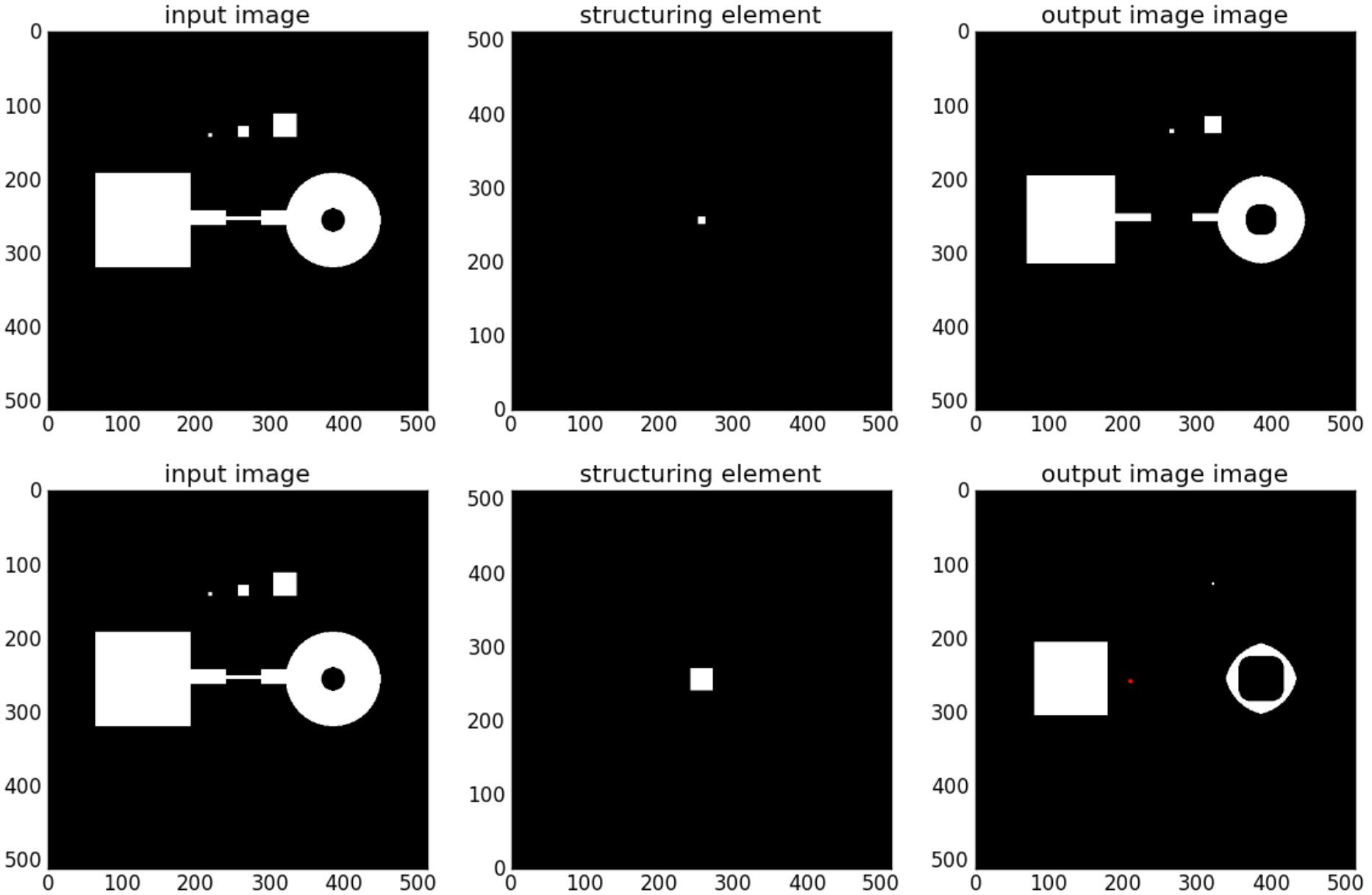
Morphological operations

- dilation



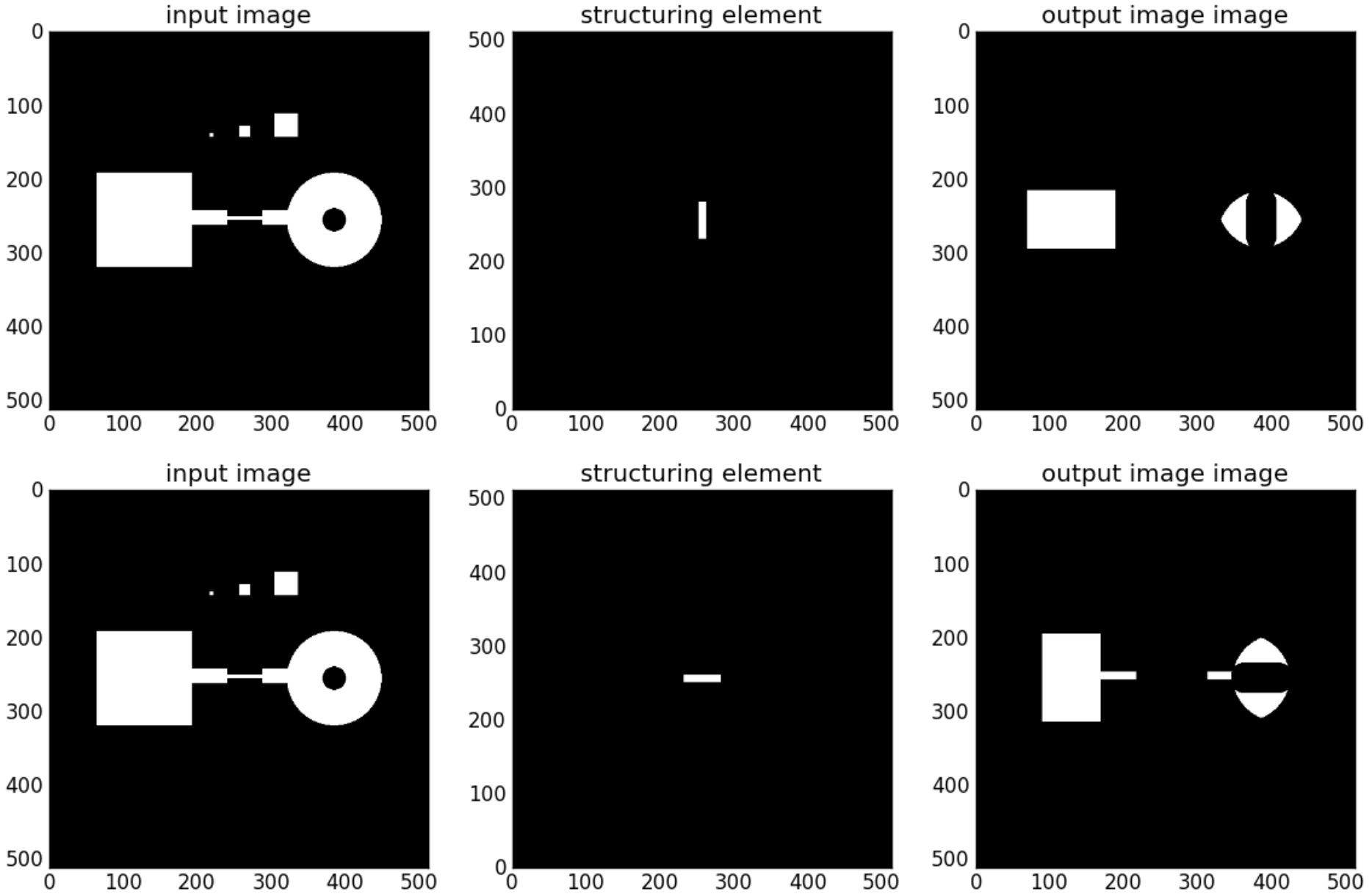
Morphological operations

- erosion



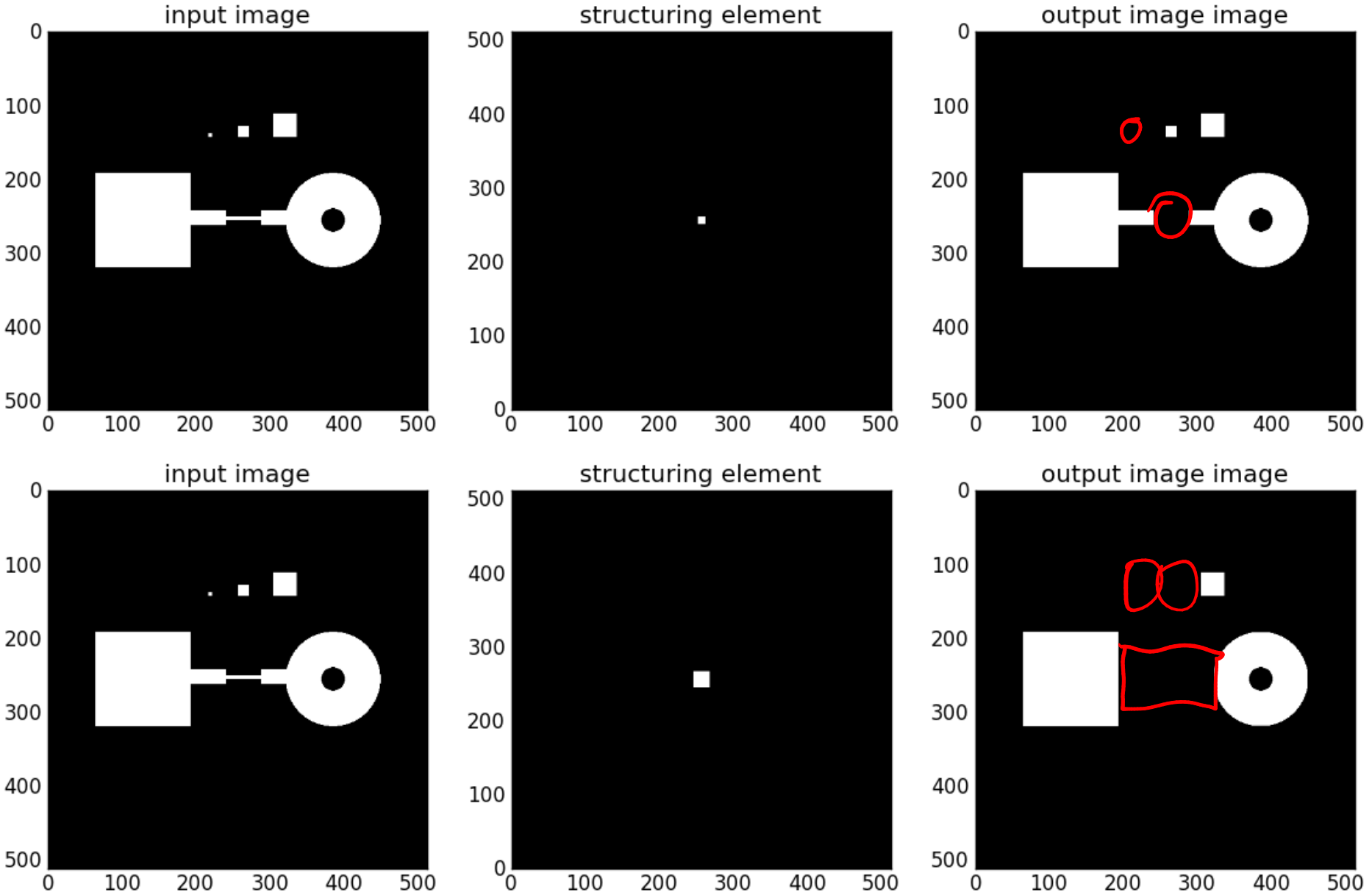
Morphological operations

- erosion



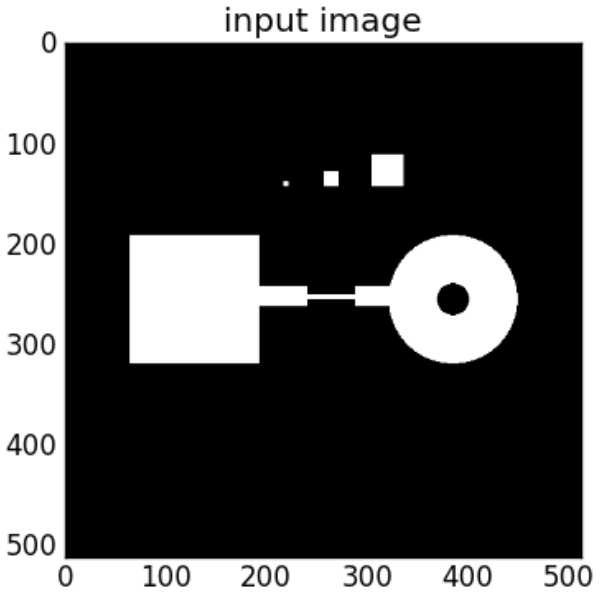
Morphological operations

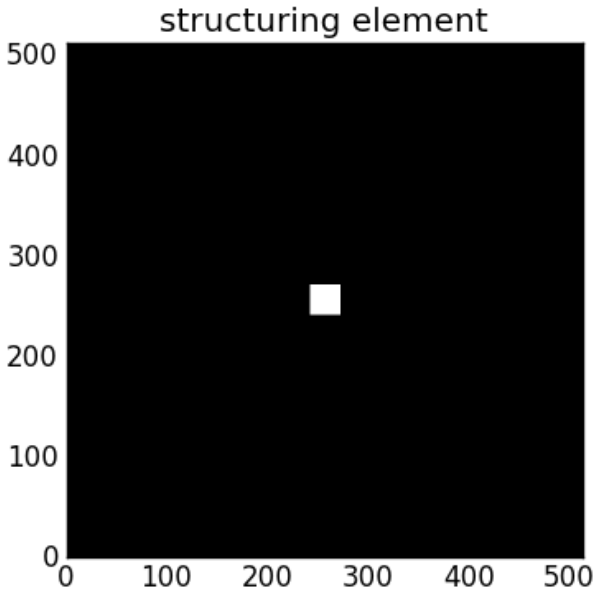
- opening: first erosion, then dilation

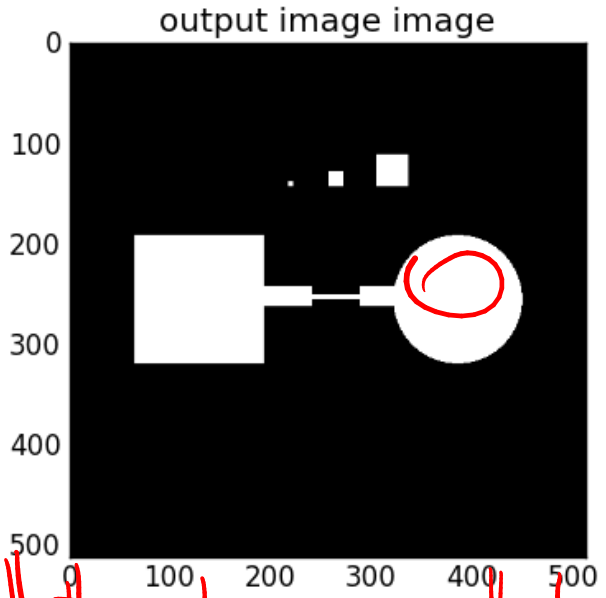


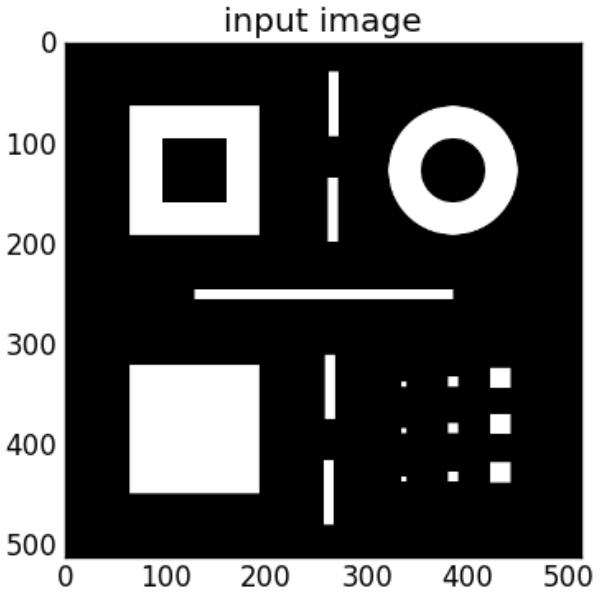
Morphological operations

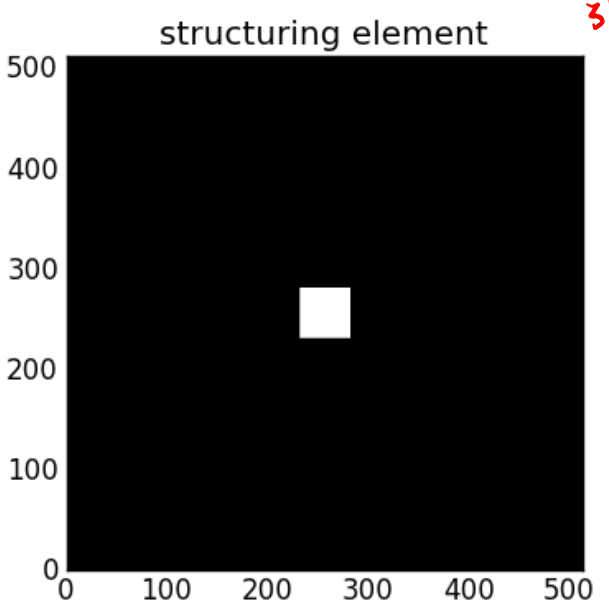
- closing: first dilation, then erosion

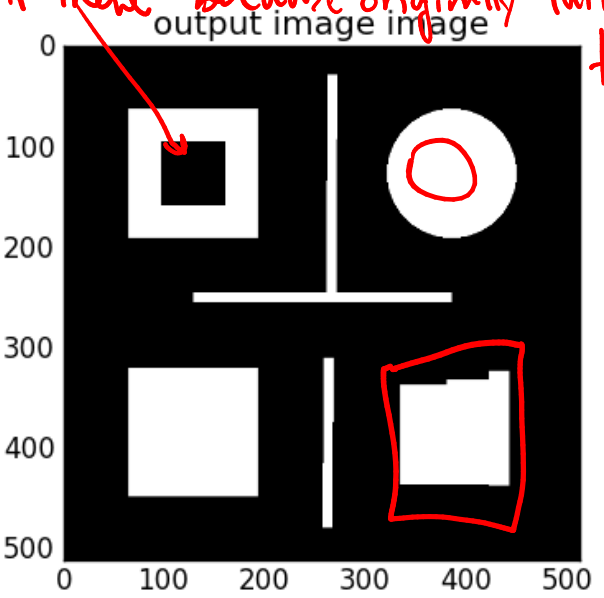








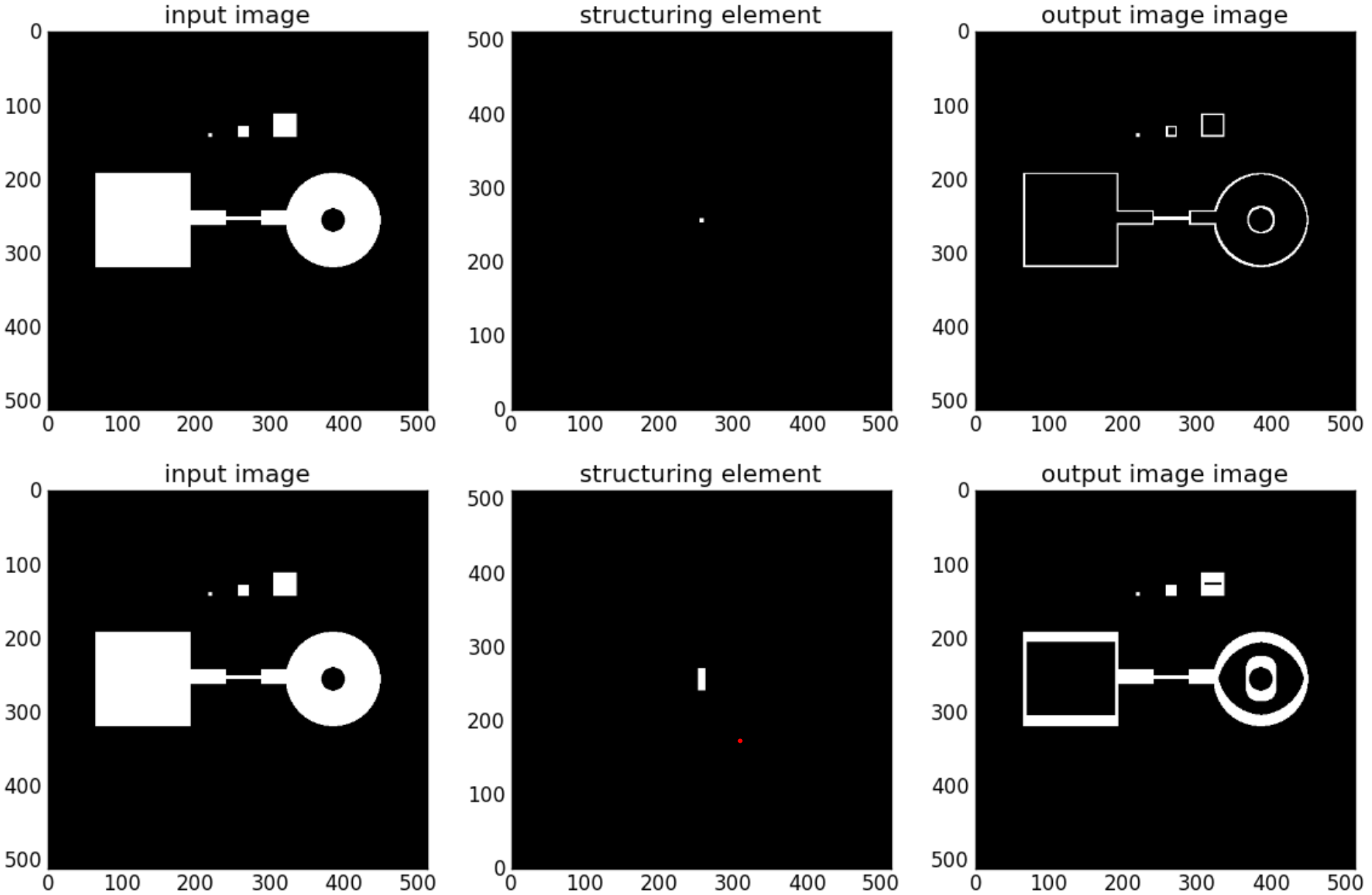




still there because originally larger than S.E.

Morphological operations

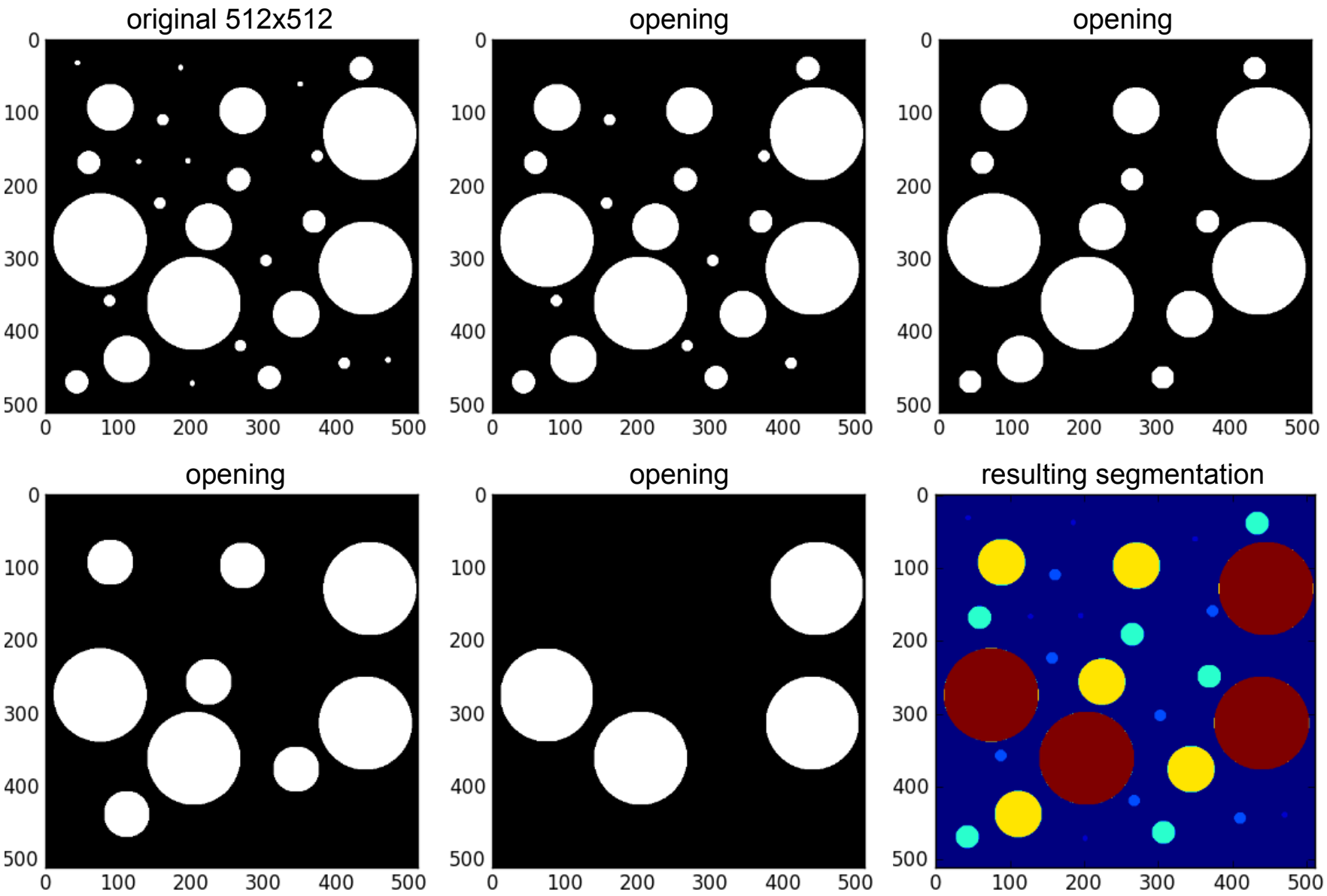
- boundary: original - erosion



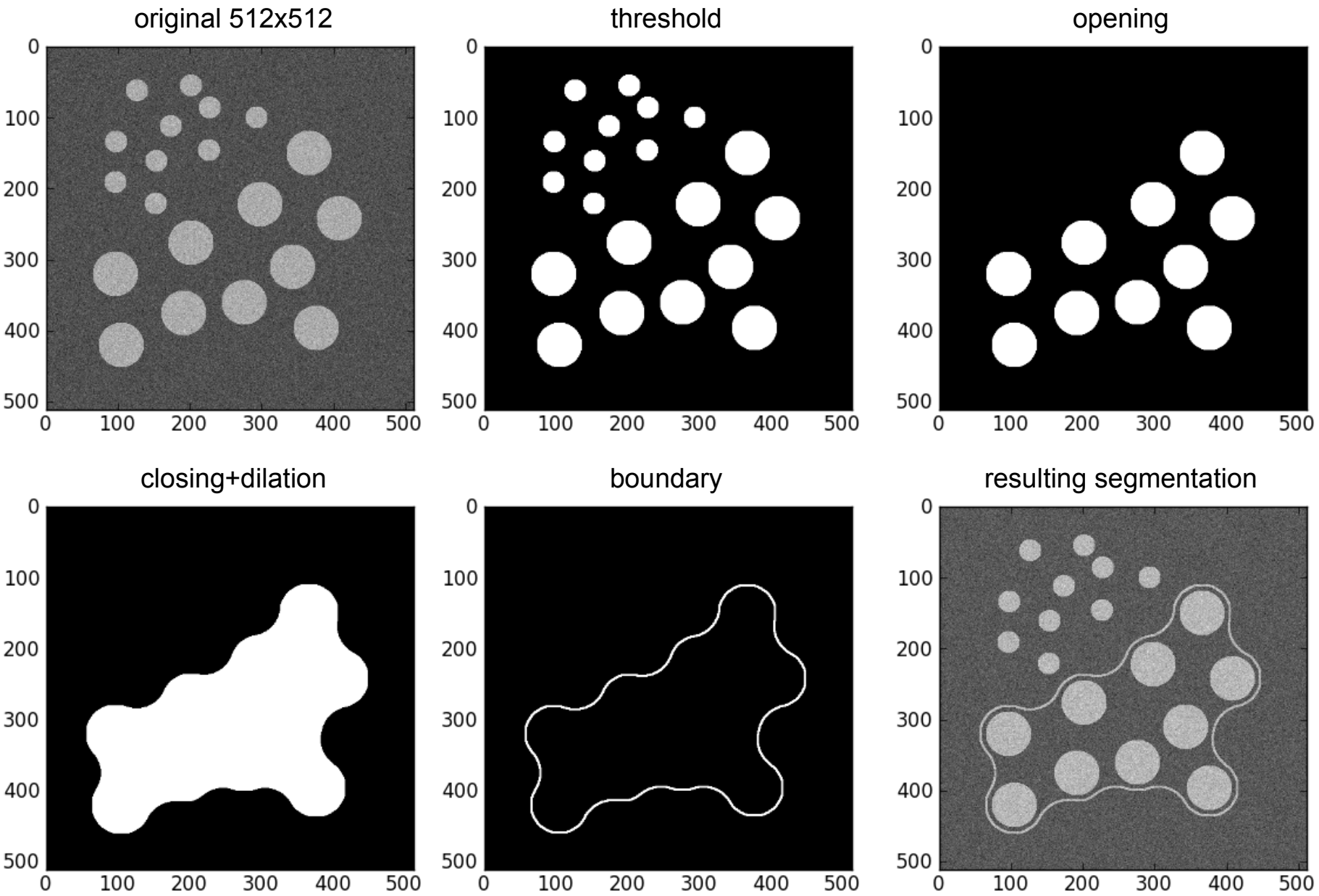
Segmentation: Motivation

- Partitioning of image by regions-of-interest ROI
- various methods available
  - by morphology
  - by intensity
  - by region
  - by boundary
  - ...

Segmentation by morphology



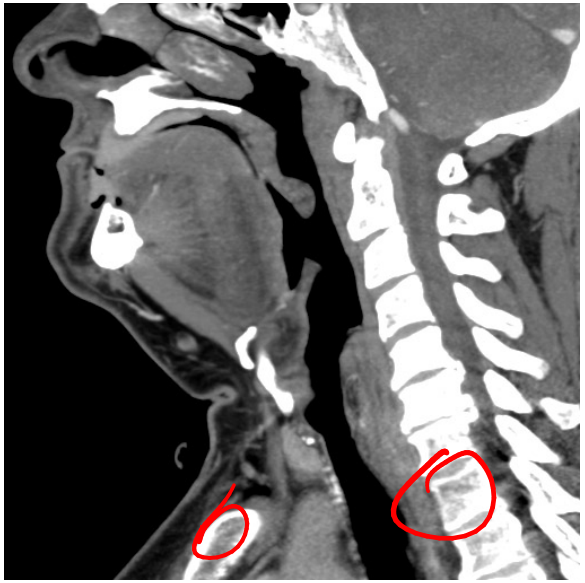
Segmentation by morphology



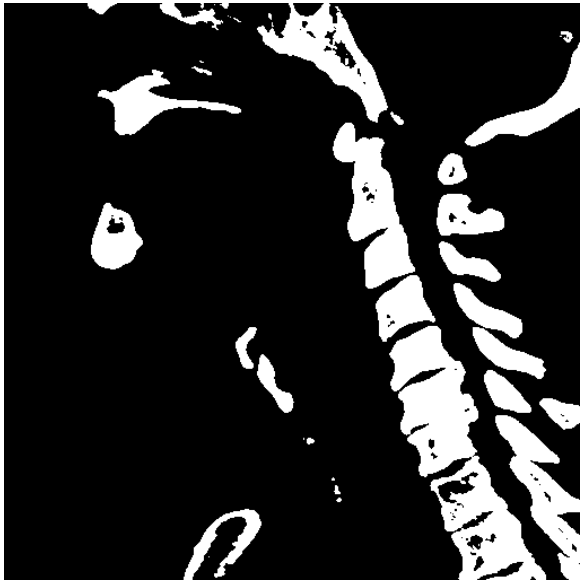
Segmentation by intensity

- easy
- widely used

original



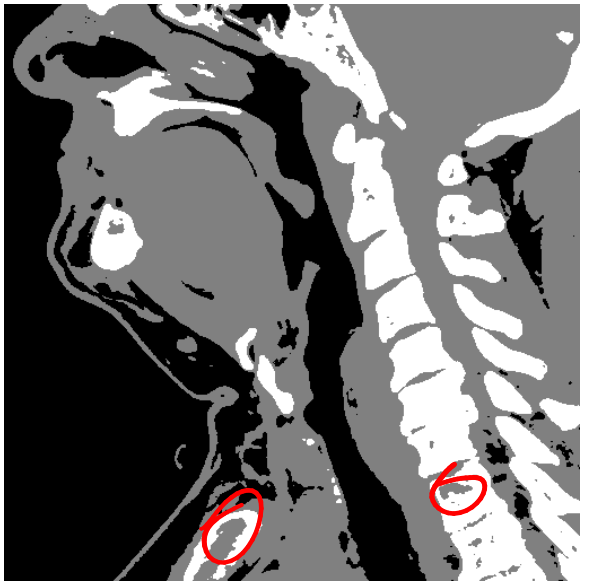
high window



mid window

low window

segmented

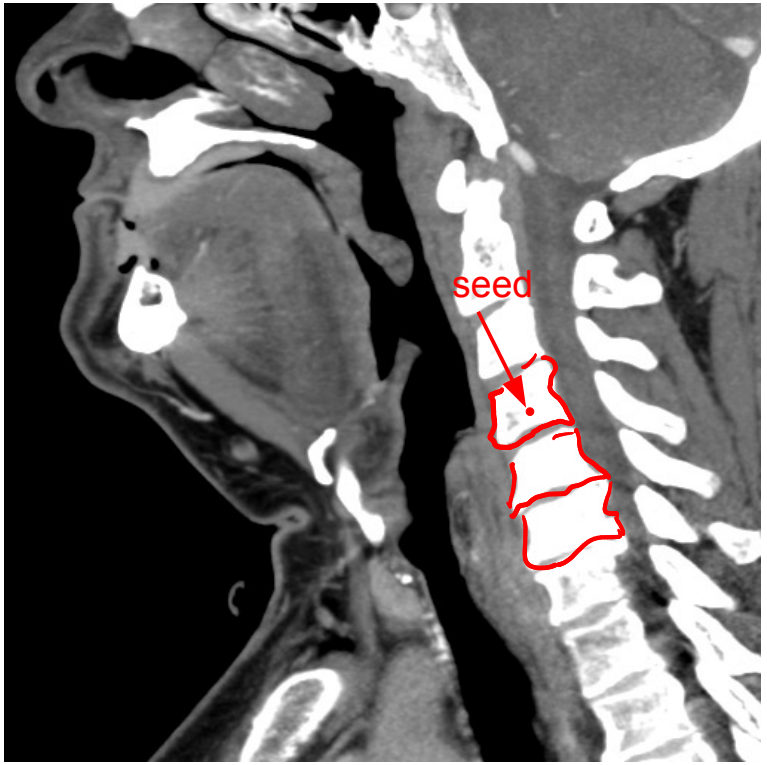


- noise prone
- no connectivity

Segmentation by region growth

- start with seed
- check intensity in neighborhood
- if intensity within window, set to 1
- iterate until no change

original



Segmentation by boundaries

- look for sharp changes in intensity
- more next week...

original

laplace