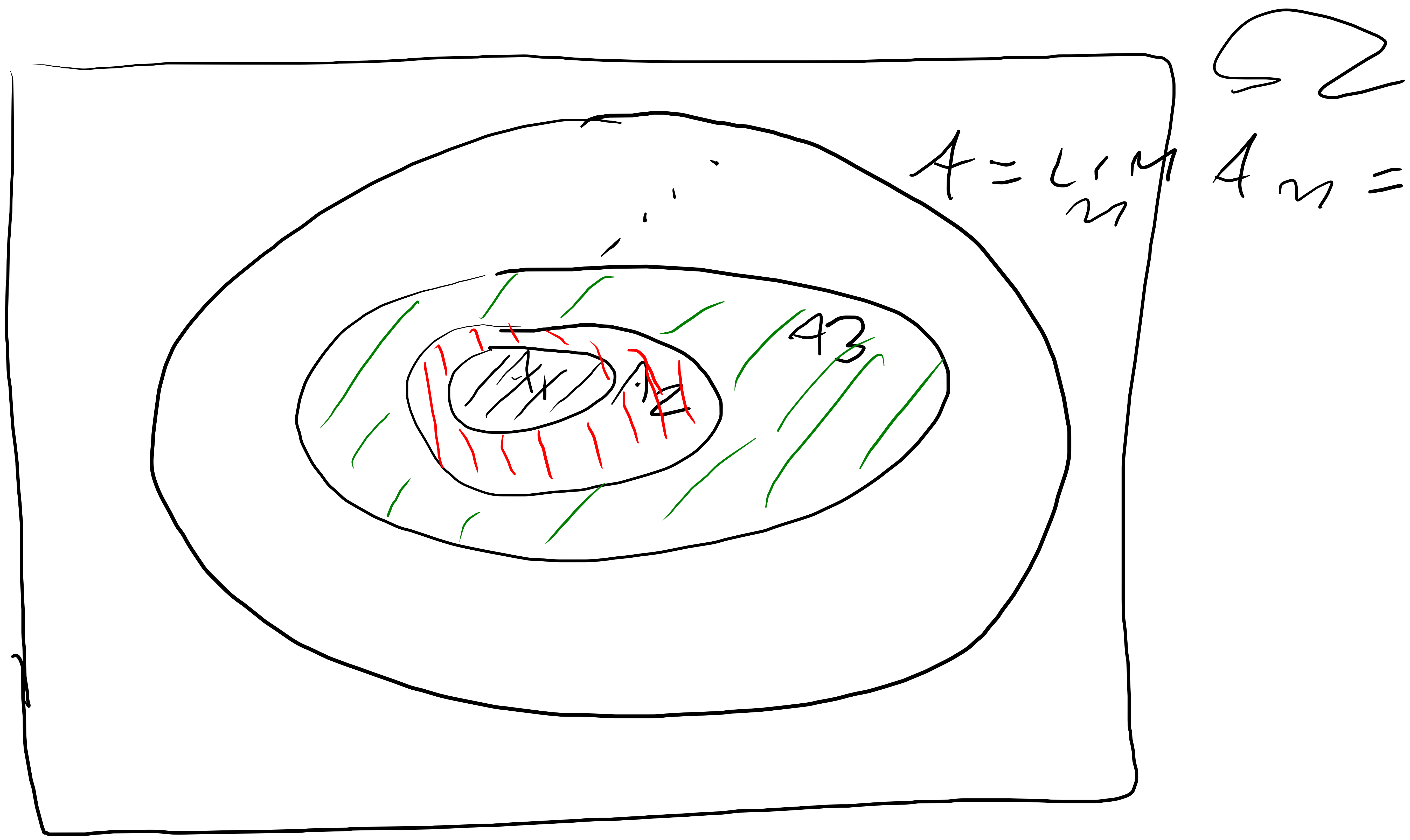
$A_1 \subset A_2 \subset A_3 \subset \ldots \subset A_n \subset A_{n+1} \subset \ldots$



$A = \lim_n A_n = \bigcup_n A_n$

$B_1 = A_1$, $B_2 = A_2 - A_1$, $B_3 = A_3 - A_2$, $\ldots$, $B_n = A_n - A_{n-1}$

(B_n) A DUE A DUE DISGIUNTI

$$\bigcup_{n\geq 1} B_n = \bigcup_{n\geq 1} A_n \left(= A\right)$$

$B_n \in \mathcal{A}$ PER OGNI $n \geq 1$

PER LA σ- ADDITIVITÀ

$$\underbrace{P\left(\bigcup_{n\geq 1} B_n\right)} = \sum_{n\geq 1} P(B_n) = \lim_{n\to+\infty} \underbrace{\overbrace{\sum_{k=1}^{n} P(B_k)}^{P(A_n)}}_{(*)}$$

$$= P(A)$$

$$= P\left(\lim_{n\geq 1} A_n\right)$$

$$\otimes = \sum_{k=1}^{n} P(B_k) = P(B_1) + P(B_2) + \ldots + P(B_n)$$

$$= P(\cancel{A_1}) + \underbrace{P(A_2 - A_1)}_{P(\cancel{A_2}) - P(\cancel{A_1})} + \ldots + \underbrace{P(A_n - A_{n-1})}_{P(A_n) - P(\cancel{A_{n-1}})}$$

MONOTONIA $\longrightarrow$

$$= P(A_n)$$

$$A_1 \supset A_2 \supset A_3 \ldots \supset A_n \supset \ldots$$

ALLORA

$$\overline{A_1} \subset \overline{A_2} \subset \overline{A_3} \subset \ldots \subset \overline{A_n} \subset \ldots$$

$$P(\underbrace{\lim_n \overline{A_n}}_{\substack{\underbrace{\cup \overline{A_n}}\\ \overline{\cap A_n}}}) = \lim_n \underbrace{P(\overline{A_n})}_{1 - P(A_n)}$$

QUINDI

$$1 - P(\bigcap_n A_n) = \lim_n (1 - P(A_n))$$

$$= 1 - \lim_n P(A_n)$$

SUBADDITIVITÀ $\quad (A_n) \quad A_n \in \mathcal{F}$

$$\underbrace{P\left(\bigcup_n A_n\right) \leq \sum_n P(A_n)}_{\text{POTREBBE ESSERE } +\infty}$$

$B_1 = A_1, \quad B_2 = A_1 \cup A_2, \quad B_3 = A_1 \cup A_2 \cup A_3$

$\ldots \quad B_n = A_1 \cup A_2 \cup \ldots \cup A_n \qquad B_n \subset B_{n+1}$

$$P\left(\lim_n B_n\right) = \lim P(B_n)$$

$$\lim_n B_n = \bigcup_{n\geq 1} B_n = \bigcup_{n\geq 1} A_n$$

$$P\left(\bigcup_n A_n\right) = P\left(\lim_n B_n\right) = \lim_n P(B_n)$$

$$\leq \lim_n \sum_{k=1}^{n} P(A_k) = \sum_{n\geq 1} P(A_n)$$

$$P(B_n) = P(A_1 \cup \dots \cup A_n) \leq P(A_1) + \dots + P(A_n)$$

(FINITA SUBADDITIVITÀ)

THM PAG. 42

$P \quad (\Omega, \mathcal{F})$

- $P(\emptyset) = 0 \quad P(\Omega) = 1$
- P FINITAMENTE ADDITIVA
- P CONTINUA DAL BASSO

$\Rightarrow P$ È σ- ADDITIVA (P È UNA PROBABILITÀ)

PROVA : $(A_n) \in \mathcal{F}$ A DUE A DUE DISGIUNTI

DEVO MOSTRARE CHE $P\left(\bigcup_n A_n\right) = \sum_n P(A_n)$

DEFINISCO

$B_1 = A_1$, $B_2 = A_1 \cup A_2$, ..., $B_n = A_1 \cup \dots \cup A_n$ - - - -

$B_n \subset B_{n+1}$

QUINDI $\quad P(\underbrace{\lim_n B_n}_{\bigcup_n B_n = \bigcup_n A_n}) = \lim_n P(B_n) = \lim_n \sum_{k=1}^{n} P(A_k) = \sum_{n \geq 1} P(A_n)$

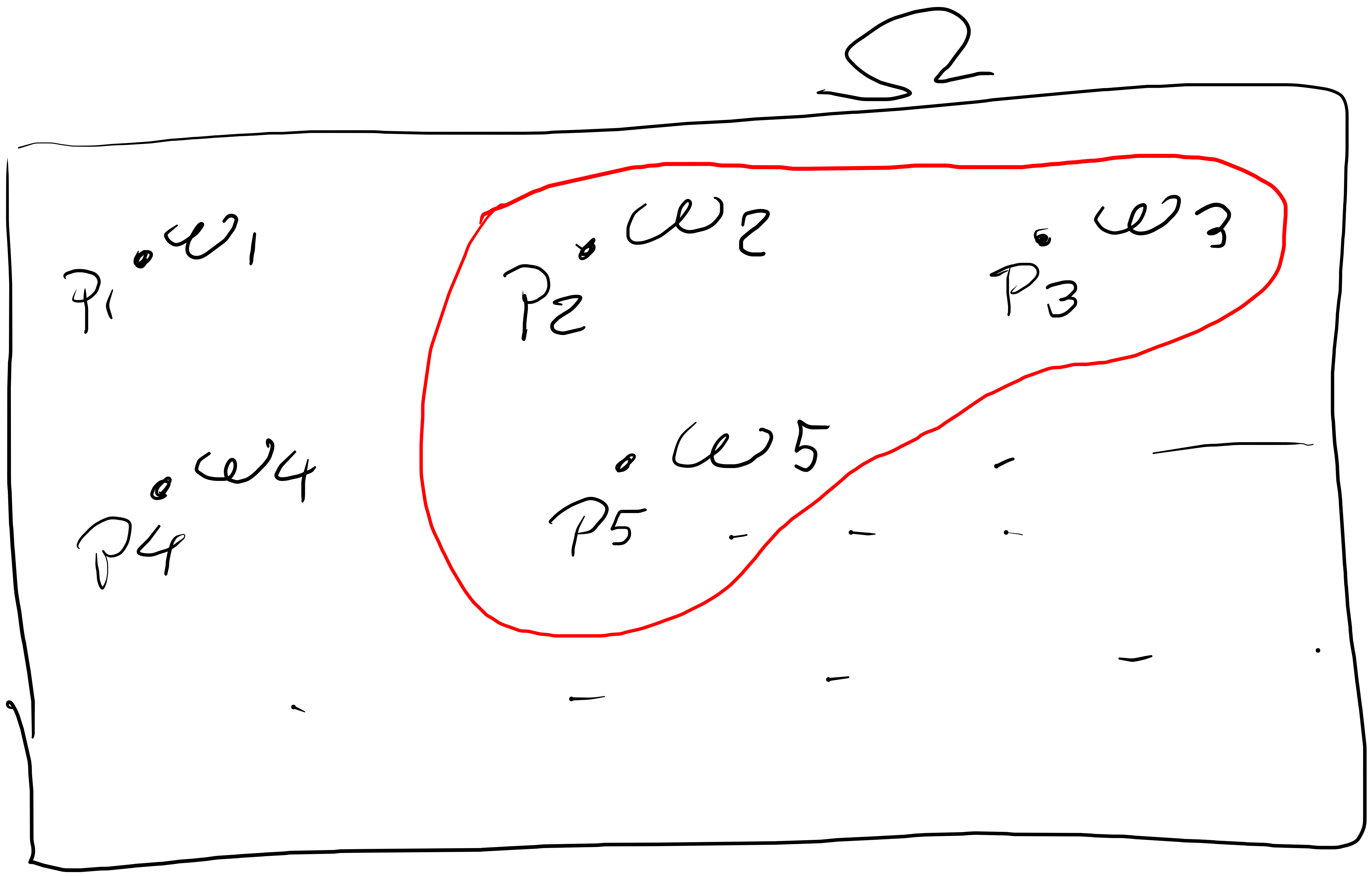
$$P(B_n) = P(A_1 \cup A_2 \cup \dots \cup A_n)$$

$A_1 \ldots A_n$ A DUE A DUE DISGIUNTI

$$= P(A_1) + \ldots + P(A_n)$$

FINITA ADDITIVITÀ

SPAZIO DI PROB. DISCRETO



$p_1, p_2, \dots$

$$p_i = P(\{\omega_i\})$$

$$A = \{\omega_2, \omega_3, \omega_5\}$$

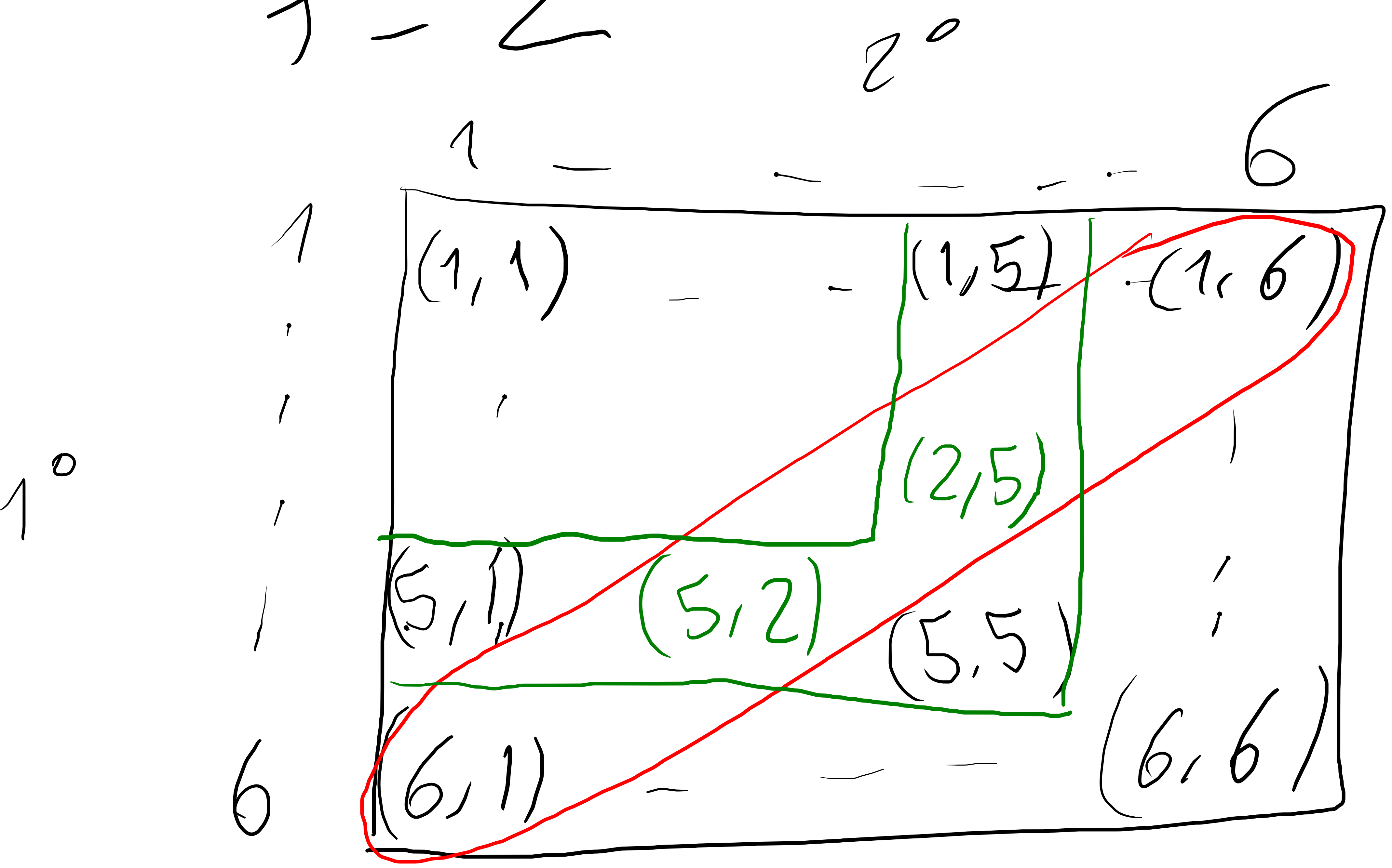
$$P(A) = p_2 + p_3 + p_5$$

$$P(\underbrace{A_1 \cup A_2 \cup \dots \cup A_n \dots}_{\text{A DUE A DUE DISGIUNTI}}) = \sum P(A_n)$$

LANCIO DI DUE DADI

$$\Omega = \{(i,j) \mid 1 \le i,j \le 6\} \qquad 6^2 = 36$$

$$\mathcal{F} = 2^{\Omega} \qquad P(\{(i,j)\}) = \frac{1}{36}$$



A = "SOMMA È 7"

B = "MAX È 5"

$$P(A) = P(\{(1,6), \dots, (6,1)\}) = \frac{6}{36} = \frac{1}{6}$$

$$P(B) = P(\underbrace{\{(1,5),(2,5),\ldots,(5,5),(5,4),(5,3),\ldots(5,1)\}}_{9 \text{ CASI}})$$

$$= \frac{9}{36} = \frac{1}{4}$$

$$\underline{P(B)} \quad P(\overbrace{\text{"PRIMO DADO È 5"}}^{B_1} \cup \overbrace{\text{"IL 2° DADO È 5"}}^{B_2})$$

$$= P(B_1) + P(B_2) - P(B_1 \cap B_2)$$

P("ALMENO UN DADO È 5")

$$= \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$$

LANCIO DI n MONETE

$$\Omega = \{ \underbrace{L_1 L_2 \dots L_n}_{\omega} \mid L_i = T \text{ o } L_i = C \} \quad 2^n$$

$$\mathcal{F} = 2^{\Omega} \qquad 2^{2^n} \text{ INSIEMI IN } \mathcal{F}$$

$$P(E_i) = \frac{2^{n-1}}{2^n} = \frac{1}{2}$$

$$E_i = \{ L_1 \dots L_n \mid L_i = T,\ L_j = T \text{ o } L_j = C \ \ j \neq i \}$$

$A_i =$ "ESCE T PER LA PRIMA VOLTA AL LANCIO i"

$$= \left\{ \underbrace{C\,C \ldots C}_{i-1} \; \underbrace{T}_{i} \; \underbrace{L_{i+1} \ldots L_n}_{n-i} \;\middle|\; L_j = T \text{ o } C,\ j > i \right\}$$

$$P(A_i) = \frac{2^{n-i}}{2^n} = \frac{1}{2^i}$$

$$\underbrace{L_1}_{2}\ \underbrace{L_2}_{2} \cdots \underbrace{L_{i-1}}_{2}\ T\ \underbrace{L_{i+1}}_{2} \cdots \underbrace{L_n}_{2}$$

$n-1$
PRODOTTI

$$P(\underbrace{A \cap B}) = \frac{2}{36} = \frac{1}{18}$$

$$\{(2,5),(5,2)\}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{36} + \frac{9}{36} - \frac{2}{36} = \frac{13}{36}$$