## B R I E F R E S E A R C H O V E R V I E W

LECTURE 2 - 03/10/2023



Technology in Mathematics Education

## • Theoretical framework



At the heart of the TPACK framework, is the complex interplay of three primary forms of knowledge: Content (CK), Pedagogy (PK), and Technology (TK). The TPACK approach goes beyond seeing these three knowledge bases in isolation. The TPACK framework goes further by emphasizing the kinds of knowledge that lie at the intersections between three primary forms: Pedagogical Content Knowledge (PCK), Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK), and Technological Pedagogical And Content Knowledge (TPACK).

http://tpack.org/

## • Technological Knowledge (TK)



Knowledge about certain ways of thinking about, and working with technology, tools and resources, and how working with technology can apply to all technology tools and resources. This includes understanding information technology broadly enough to apply it productively at work and in everyday life, being able to recognize when information technology can assist or impede the achievement of a goal, and being able to continually adapt changes in information technology (Koehler & Mishra, 2009)

## Pedagogical Knowledge (PK)



Teachers' deep knowledge about the processes and practices or teaching and learning methods. They encompass, among other things, overall educational purposes, values, and aims. This generic form of knowledge applies to understanding how students learn, general classroom management skills, lesson planning, and student assessment

## Content Knowledge (CK)



Teachers' knowledge about the subject matter to be learned or taught. The content to be covered in middle school science or history differs from that in an undergraduate course on art appreciation or a graduate seminar on astrophysics... As Shulman (1986) noted, this knowledge would include knowledge of concepts, theories, ideas, organisational frameworks, knowledge of evidence and proof, as well as established practices and approaches toward developing such knowledge



# Technological Content Knowledge (TCK)

An understanding of the manner in which technology and content influence and constrain one another. Teachers need to master more than the subject matter they teach; they must also have a deep understanding of how the subject matter (or the kinds of representations that can be constructed) can be changed by applying particular technologies. Teachers need to understand which specific technologies are best suited for addressing subject-matter learning in their domains and how the content dictates or perhaps even changes the technology-or vice versa



## Pedagogical Content Knowledge (PCK)



Consistent with and similar to Shulman's idea of knowledge of pedagogy that is applicable to teaching specific content. Central to Shulman's conceptualization of PCK is the notion of the transformation of the subject matter for teaching. Specifically, according to Shulman (1986), this transformation occurs as the teacher interprets the subject matter, finds multiple ways to represent it, and adapts and tailors the instructional materials to alternative conceptions and students' prior knowledge. PCK covers the core business of teaching, learning, curriculum, assessment and reporting, such as the conditions that promote learning and the links among curriculum, assessment, and pedagogy.

#### Technological Pedagogical Knowledge (TPK) **Technological Pedagogical Content** Knowledge (TPACK) An understanding of how teaching and learning can change when particular technologies are used in Technological Technological particular ways. This includes knowing the Technological Pedagogical Content Knowledae Knowledge Knowledge pedagogical affordances and constraints of a range (TK) (TPK) (TCK) of technological tools as they relate to disciplinarily and developmentally appropriate pedagogical designs and strategies Pedagogical Content Knowledge Knowledge (PK) (CK) (Koehler & Mishra, 2009) Pedagogical Content Knowledge (PCK) Contexts



## Technological Pedagogical and Content Knowledge (TPACK)



Underlying truly meaningful and deeply skilled teaching with technology, TPACK is different from knowledge of all three concepts individually. Instead, TPACK is the basis of effective teaching with technology, requiring an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge to develop new epistemologies or strengthen old ones

Integration efforts should be creatively designed or structured for particular subject matter ideas in specific classroom contexts.



## TPACK

Each situation presented to teachers is a unique combination of these three factors, and accordingly, there is no single technological solution that applies for every teacher, every course, or every view of teaching. Rather, solutions lie in the ability of a teacher to flexibly navigate the spaces defined by the three elements of content, pedagogy, and technology and the complex interactions among these elements in specific contexts.



### CONTENT

### TEACHERS NEED OF UNDERSTANDING OF

(Koehler & Mishra, 2009)

### TEACHING

### TECHNOLOGY





### CONTENT- DRIVEN Knoweldge

use technology to teach a particular mathematical concept or area, such as functions, or algebra

### OR BOTH?



### TOOL- DRIVEN knowledge

use a particular piece of software, such as the Computer Algebra System (CAS), Dynamic Geometry Environment (DGE) or spreadsheets (such as Excel)

(Tabach, M. & Trgalová, J., 2019)







### TPACK FRAMEWORK IS NOT CONTENT-SPECIFIC

• DOES TPACK FRAMEWORK SATISFY THE REQUIREMENT OF **BUILDING MATHS TEACHERS'** PREPARATION **KNOWLEDGE TOWARDS TECHNOLOGY INTEGRATION?** 



## • INSTRUMENTAL GENESIS (Haspekian, 2011)



## Personal

Teachers must first acquire basic skills to master the specific technology they intend to use and develop utilization schemes related to this technology



## FROM Mathematical Knowledge for Teaching TO MDigitalKT



(Ball et al., 2008)

(Tabach, M. & Trgalová, J., 2019)



(Tabach, M. & Trgalová, J., 2019)

## Teacher orientations and goals

These are related to affective aspects and teachers' beliefs regarding mathematics, teaching mathematics and technology.

What kind of discipline is mathematics and what are its norms and values?

What is the teacher's role in teaching mathematics, and how can this role be accomplished?

What added value will technology bring to learning mathematics?

What affordances and constraints does it involve?

How confident do teachers feel with respect to its use?



### WHAT KIND OF DISCIPLINE IS MATHEMATICS AND WHAT ARE ITS NORMS AND VALUES?

La matematica è una disciplina scientifica che si basa su formalizzazione e deduzione, getta le sue basi su problemi concreti di vita reale e punta all'estrazione più generale possibile degli stessi. Il rigore e la logida sono alla base degli studi matematici.

La matematica è una disciplina che richiede rigore, attenzione e che per essere imparata davvero va capita

I don't really understand what you want to ask here. But I think that mathematics is a discipline that is not taught well in schools and children are very discouraged by the discipline. It is necessary to change the way of seeing them and teaching

For me mathematics is a science with logic and rationality as building blocks. Mathematics is the science of structure, order and relations.

La matematica è una materia scientifica, i cui principi vengono usati durante tutta la vita anche se non si sa che fanno parte di essa. È una materia rigorosa ma necessita anche di creatività. Inoltre è universale, presente in ogni campo della vita e in ogni cultura.

#### **WHATIS THE** EACHER'S RO EAC Ν н G Ε Μ O W Ν D н HIS ROLE B Ε ACCOMPLISH Ε

Un docente dovrebbe guidare lo studente attraverso la comprensione degli argomenti, ma anche trasmettere interesse nei confronti della matematica. Questo può avvenire tramite lezioni coinvolgenti, lezioni frontali ma anche interattive, spesso variare i metodi di insegnamento può essere utile per mantenere viva l'attenzione e la curiosità.

Il ruolo dell'insegnante, secondo me, dovrebbe essere quello di trasmettere la propria passione per la matematica, cercando di renderla il più interessante possibile agli occhi di tutti, provando un po' a sfatare il mito che "la matematica è troppo difficile e solo alcuni ci possono riuscire", in quanto ritengo che chiunque si approcci alla materia nella materia giusta possa apprezzarla un minimo

I think It Is so important the role of the teacher because all the sensations and emotions are transmited to the students. It's interesting that the teacher have to be close to the students and feel comfortable to be able to talk and ask questions

The role of the teachers is the transmission of knowledge in a passionate manner with the goal of personal and professional growth of the student.

Teacher is a motivator for students and someone who students learn mathematics from. This role is accomplished by having passion for teaching and knowledge.

L'insegnante deve saper spiegare la teoria e le regole per poterla applicare a problemi reali e non, e quindi fornire dei mezzi affinché lo studente possa autonomamente trovare le risposte.

### WHAT ADDED VALUE WILL TECHNOLOGY BRING TO LEARNING MATHEMATICS?

Le nuove generazioni sono sempre più vicine alle tecnologie quindi penso possano essere un mezzo utile per far avvicinare i ragazzi alla matematica ma anche mezzi più efficienti per mostrare determinati risultati e argomenti.

Sicuramente, essendo la matematica un qualcosa di astratto, usare questi mezzi che ne permettono una visualizzazione più concreta può facilitarne la comprensione

As a computer science student, I also believe that it is very important to introduce technologies because it is the basis of today's world. Children grow up with technology and know how to use it, therefore it will be easier to learn mathematics

Nowadays technology is taking an increasingly role in our life and combining technology and learning math can help students to understand, apply and remember both aspects.

It is excellent for visualising, especially in geometry. Students will also see that many mathematical concepts can be more easily understood by using technology.

La tecnologia può aiutare a visualizzare visivamente e quindi capire meglio ciò di cui si sta parlando, per esempio figure geometriche o grafici di funzioni, oppure capire anche come funzionano gli algoritmi e i programmi.

### W H A T A F F O R D A N C E S A N D C O N S T R A I N T S D O E S I T I N V O L V E ?

Potrebbe rappresentare un costo per le scuole italiane, in quanto non tutte sono ancora dotate di dispositivi come lavagne interattive multimediali in ogni classe; alcuni docenti potrebbero anche mostrarsi restii, o contrari, ad un metodo alternativo di insegnamento.

Lati positivi sono il supporto "concreto" che riesce a dare ai concetti astratti e il fatto che indubbiamente rende più interattiva e curiosa la spiegazione

It is needed that there is enough technology in the teaching place so that each student can do it on their own and this is not something that is always possible. However, if it were possible, the possibilities would be endless because everything would become more visible to the student.

#### It requires for the teachers a continuous update

Schools and classrooms must be equipped with technological tools (projector, smart whiteboard etc.). Some teachers are not familiar with using technology in class. However, it makes lessons richer and more interesting for students.

Quando tutto funziona può risultare molto utile per capire meglio le cose, per fare esempi veloci. Però potrebbe anche risultare una perdita di tempo, in quanto per usare alcuni programmi ci vuole una certa familiarità e ci vuole tempo per impostare tutti i dati.

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HOW
CONFIDENT DO
TEACHERS
FEEL WITH
RESPECT TO
ITS USE?
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Credo possa dipendere in parte dall'età del docente, o quantomeno dalla dimestichezza che hanno con la tecnologia, alcuni non sono ancora molto esperti e potrebbero avere difficoltà. In generale ritengo più plausibile che preferiscano i classici metodi di insegnamento

Al momento, almeno nella mia esperienza personale, sono davvero pochi gli insegnanti che ne usufruiscono

They should know how to use it very well so as not to confuse students and know how to answer all the questions they may have. If you feel confident, that feeling will be transmitted to the student I think the teachers are not so much confident. I think some teachers already use it and they also feel confident respect to its use. However, some of them (especially older ones) are not familiar with it and therefore do not use it during their lessons. Se un insegnante è esperto in questo campo allora I'utilizzo della tecnologia diventa naturale e utile. Ma non tutti sono in grado di utilizzare perfettamente tutti i programmi disponibili.

## Teacher ICT (Information and Communication Technology) Competencies







### **Technology Literacy**

Enabling students to use ICT in order to learn more efficiently



### **Knowledge Deepening**

Enabling students to acquire indepth knowledge of their school subjects and apply it to complex, real-world problems



### **Knowledge Creation**

Enabling students, citizens and the workforce they become, to create the new knowledge required for more harmonious, fulfilling and prosperous societies



(UNESCO, 2011-2017)

### "Basic digital literacy" can be thought of as a result of the teacher's personal instrumental genesis and ability to select appropriate resources to "complement standard didactic teaching methods" as part of the mathematical knowledge for teaching requiring professional instrumental genesis

basic digital literacy skills and digital citizenship

ability to select and use appropriate off-the-shelf educational tutorials, games, drill-and-practice software, and web content in computer laboratories or with limited classroom facilities to complement standard curriculum objectives, assessment approaches, unit plans, and didactic teaching methods.

use ICT to manage classroom data and support their own professional learning

The continuous instrumental genesis of teachers, both personal and professional

(UNESCO, 2011-2017; Tabach, M. & Trgalová, J., 2019)

## Knowledge Deepening

Teachers' mathematical knowledge for teaching with technology

teacher competences... include the ability to manage information, structure problem tasks, and integrate open-ended software tools and subject-specific applications [SDCK] with student-centred teaching methods and collaborative projects in support of students' indepth understanding of key concepts and their application to complex, real-world problems [KDCS]. To support collaborative projects, teachers should use networked and web-based resources to help students collaborate, access information, and communicate with external experts to analyze and solve their selected problems [KDCS]. Teachers should also be able to use ICT to create and monitor individual and group student project plans [KDCT], as well as to access information and experts and collaborate with other teachers to support their own professional learning (ibid., p. 11).

UNESCO, 2011-2017

(Tabach, M. & Trgalová, J., 2019)

## Knowledge Creation Stage



Teachers...

will be able to design ICT-based learning resources and environments [KDCT]; use ICT to support the development of knowledge creation and the critical thinking skills of students [KDCS]; support students' continuous, reflective learning [KDCS]; and create knowledge communities for students and colleagues (ibid., p. 14).

UNESCO, 2011-2017

## From TPACK to MDKT

There is a need for "content-specific ideas that address what students or teachers should know about using technology for learning mathematics"

(Niess et al. 2009)



Designing and developing digital-age learning environments and experiences.



Teaching, learning and the mathematics curriculum



Assessment and evaluation



**Productivity and professional practice** 



Teachers design and develop authentic learning environments and experiences while incorporating appropriate digital-age tools and resources to maximize mathematical learning in context. Teachers implement curriculum plans that include methods and strategies for applying appropriate technologies to maximize student learning and creativity in mathematics.

Teachers apply technology to facilitate a variety of effective assessment and evaluation strategies.









Evaluate and reflect on the effective use of existing and emerging technologies to enhance all students' mathematical learning

## Revelopment of Habits

teachers are able to use the technology and recognize the alignment of the technology with mathematics content yet do not integrate the technology in teaching and learning of mathematics.

of mind and practices (Tabach, M. & Trgalová, J., 2019)

teachers engage in activities that lead to a choice to adopt or reject teaching and learning mathematics with appropriate technology.

teachers evaluate the results of the decision to integrate teaching and learning mathematics with an appropriate technology teachers form a favourable or unfavourable attitude toward teaching and learning mathematics with appropriate technology.

teachers actively integrate teaching and learning of mathematics with an appropriate technology

Teachers must first express an interest in using technology for teaching and learning mathematics in order to develop a positive attitude (personal orientation) toward it and use it for personal purposes (personal instrumental genesis leading to the development of SDCK). Only after that can they integrate technology in their professional practice (professional instrumental genesis).





Mathematical knowledge for teaching with technology (MDKT) framework

(Tabach, M. & Trgalová, J., 2019)