MADS, 3/10/23

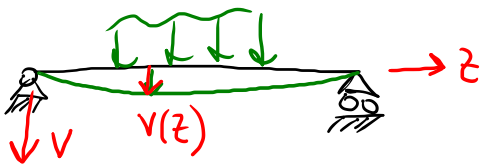
- LINEA ELASTICA DEL IV ORDINE
- TEOREMA DELLA STAZIONARIETA' DELL'E.P.T.
- TEOREMI ENERGETICI

} APPROFONDIMENTI DELLA TEORIA DELLE STRUTTURE

LINEA ELASTICA DEL IV ORDINE

LINEA EL. DEL II ORDINE (TRAVI SNELLE)

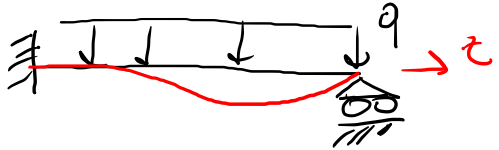
$$\begin{cases} v''(z) = -\dfrac{M(z)}{EI} \\ 2\ \text{C. LIMITI} \end{cases}$$



$\varphi(z) = -v'(z)$ $\quad \varphi\circlearrowleft$

OTTIMA PER STRUTTURE ISOSTATICHE.

SIAMO IN GRADO DI RISOLVERE MEDIANTE "LINEA ELASTICA":

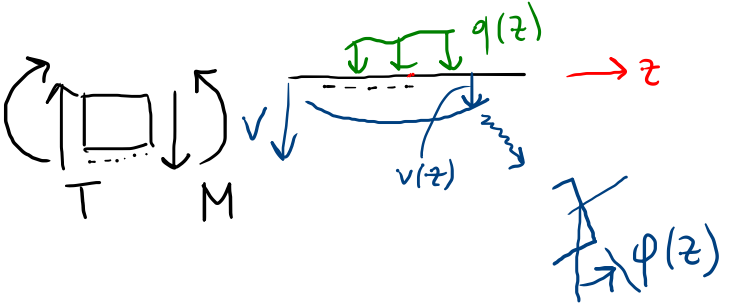


Per ora io so studiare la struttura solo con il METODO DELLE FORZE.

IPERST: $M(z)$ non è noto solo con EQUAZ. DI EQUIL.

CONCETTI NOTI DELLE TRAVI SNELLE INFLESSE

- EQUILIBRIO : $\frac{dM}{dz} = T$; $\frac{dT}{dz} = -q(z)$



- CONGRUENZA (INTERNA) : $\kappa = \frac{d\varphi}{dz}$ $\qquad \frac{dv}{dz} = -\varphi$

κ ↑ CURVATURA

- LEGAME COSTITUTIVO : $\kappa = \frac{M}{EI}$ $\qquad EI(z)$

$$\frac{M}{EI} = \kappa = -\frac{d^2v}{dz^2} \rightsquigarrow M = -EIv'' \rightsquigarrow M' = -(EIv'')' \rightsquigarrow T = -(EIv'')' \rightsquigarrow$$

$$\rightsquigarrow T' = -(EIv'')'' \longrightarrow \boxed{q(z) = (EI(z)\, v''(z))''}$$

EQ. LINEA EL DEL IV ORDINE -GENERALE-

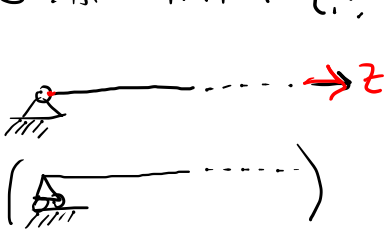
L'INCOGNITA NEI PROBLEMI STRUTT. È $v(z)$ [CARICHI NOTI]

- PER L'INTEGR SERVONO 4 COND. AI LIMITI.

NOTA: Se $EI(z) = \text{cost} \Rightarrow EI\, v''''(z) = q(z)$

$\Rightarrow T(z) = -EI\, v'''(z)$

C. AI LIMITI ?!



$v(0) = 0$

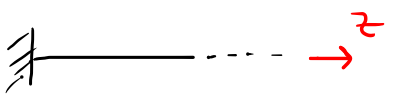
$M(0) = -EI\, v''(0) = 0$

CERNIERA o CARRELLO

$\varphi(0) = -v'(0) = 0$

$T(0) = -EI\, v'''(0) = 0$

DOPPIO PENDOLO



$v(0) = 0$

$-v'(0) = 0$

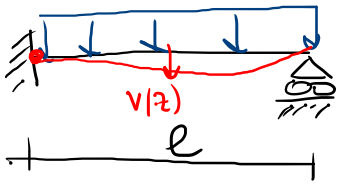
INCASTRO

$M(0) = -EI\, v''(0) = 0$

$T(0) = -EI\, v'''(0) = 0$

ESTREMO LIBERO

LES



q → z EI cost

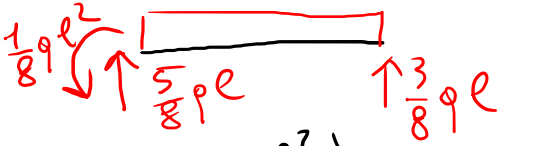
$$\begin{cases} EI\, v''''(z) = q \\ v(0)=0 \\ v'(0)=0 \\ v(\ell)=0 \\ v''(\ell)=0 \end{cases} \quad \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{matrix} \quad \begin{cases} C_3 = 0 \\ C_4 = 0 \\ C_1 = -\frac{5}{8}\frac{q\ell}{EI} \\ C_2 = \frac{1}{8}\frac{q\ell^2}{EI} \end{cases}$$

$$v'''(z) = \frac{q}{EI} z + C_1$$

$$v''(z) = \frac{q}{EI}\frac{z^2}{2} + C_1 z + C_2$$

$$v'(z) = \frac{q}{EI}\frac{z^3}{6} + C_1\frac{z^2}{2} + C_2 z + C_3 \qquad v(z) = \frac{q}{EI}\frac{z^4}{24} + C_1\frac{z^3}{6} + C_2\frac{z^2}{2} + C_3 z + C_4$$

$M(z) < 0$



$$M(z) = -\left(q\frac{z^2}{2} - \frac{5}{8}q\ell z + \frac{1}{8}q\ell^2\right)$$

↑□↓

$$M(0) = -\frac{1}{8}q\ell^2$$

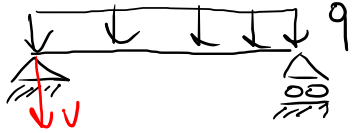
$$M(\ell) = -\left(\frac{q\ell^2}{2} - \frac{5}{8}q\ell^2 + \frac{1}{8}q\ell^2\right) = 0 \quad \text{OK}$$

$$T(z) = -\left(qz - \frac{5}{8}q\ell\right)$$

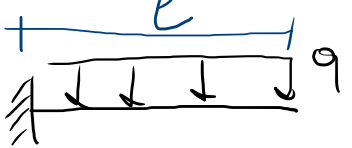
$$T(0) = \frac{5}{8}q\ell \quad \text{OK}$$

$$T(\ell) = -\left(q\ell - \frac{5}{8}q\ell\right) = -\frac{3}{8}q\ell$$

ES



ISOST -



$$\begin{cases} EI\,v^{IV}(z) = q \\ v(0) = 0 \\ v''(0) = 0 \\ v(\ell) = 0 \\ v''(\ell) = 0 \end{cases}$$

SI OTTIENE LA STESSA SOLUZ. DI $v''(z) = -\frac{M(z)}{EI}$

$$\begin{cases} EI\,v^{IV}(z) = q \\ v(0) = 0 \\ v'(0) = 0 \\ v''(\ell) = 0 \\ v'''(\ell) = 0 \end{cases}$$

$M(z) = -EI\,v''(z)$

$M(\ell) = -m$

$m = EI\,v''(\ell)$

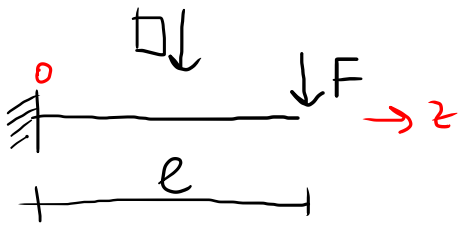
$EI\,v^{IV} = 0$

$v(0) = 0$

$v'(0) = 0$

$v'''(\ell) = 0$

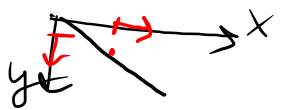
ES



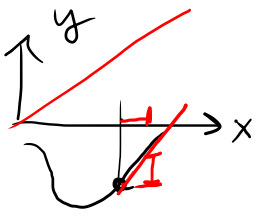
$T(z) = -EIv'''(z)$

$T(\ell) = +F$

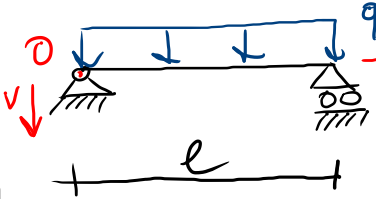
$F = -EIv'''(\ell)$



$$\begin{cases} EIv^{IV}(z) = 0 \\ v(0) = 0 \\ v'(0) = 0 \\ v''(\ell) = 0 \\ v'''(\ell) = -\dfrac{F}{EI} \end{cases}$$



LES (CEDIMENTO ANELASTICO)



CARICO AGGIUNTIVO

(VALORE ASSEGNATO DEL CEDIMENTO)

$$\begin{cases} EIv^{IV}(z) = q \\ v(0) = 0 \\ v''(0) = 0 \\ v(\ell) = \Delta \\ v''(\ell) = 0 \end{cases}$$

$$\begin{cases} EIv^{IV}(z) = 0 \\ v(0) = 0 \\ v''(\ell) = 0 \\ v'''(\ell) = 0 \\ v'(0) = \varphi_0 \end{cases}$$

$0 = C_4$

$C_2 = 0 \;\&\; C_1\ell + C_2 = 0$

$C_1 = 0$

$\varphi_0 = C_3$

$\varphi(z) = -v'(z)$

$-\varphi_0 = -v'(0)$

$v''' = C_1$

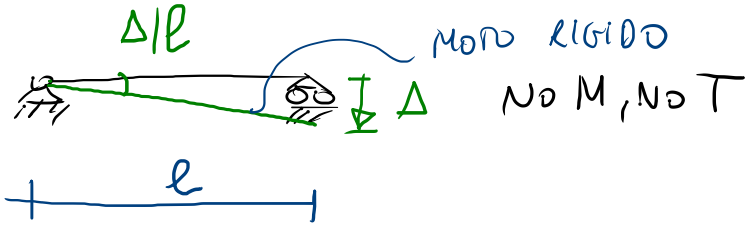
$v'' = C_1 z + C_2$

$v' = C_1\frac{z^2}{2} + C_2 z + C_3$

$v = C_1\frac{z^3}{6} + C_2\frac{z^2}{2} + C_3 z + C_4$

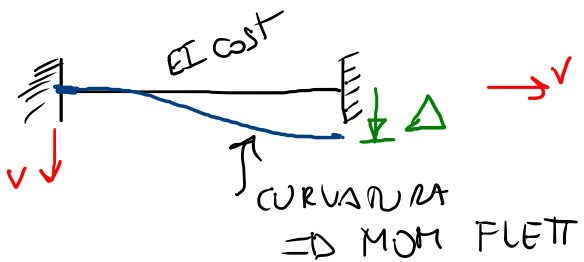
$v(z) = \varphi_0 z \;;\; v'(z) = \varphi_0 \;;\; v''(z) = v'''(z) = 0$

STRUTTURE ISOSTATICHE : I CEDIMENTI ANELASTICI NON PROVOCANO SOLLECITAZIONI MA SOLO ATTI DI MOTO RIGIDO.



NO M, NO T

LES



STR IPERST.
SOLLECITATA DA
UN CEDIMENTO ANELASTICO

$$\begin{cases} EI\, v^{IV}(z) = 0 \\ v(0) = 0 \\ v'(0) = 0 \\ v'(\ell) = 0 \\ v(\ell) = \Delta \end{cases}$$