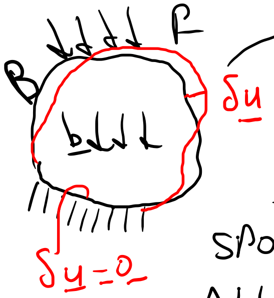
PRINCIPIO DI STAZIONARIETÀ DELL'EN. POT. TOTALE

MADS, 6/10/23

NELLA MECCANICA DEI SIST. DISCRETI QUESTO PRINCIPIO PERMETTE DI STUDIARE LE <u>CONFIGURAZIONI DI EQUILIBRIO</u>.



NELLA CONFIG. EQUILIBRATA

$\underline{u}(\underline{x})$: CAMPO DI SPOSTAM IN B ALL'EQUILIBRIO

$$\underline{u} \rightsquigarrow \underset{\sim}{\varepsilon} = \frac{1}{2}\left(\nabla \underline{u} + \nabla \underline{u}^T\right)$$

$$\underset{\sim}{\sigma} = \mathbb{C}\, \underset{\sim}{\varepsilon}$$

$$\Pi(\underline{u}) = \underbrace{\frac{1}{2}\int_B \underset{\sim}{\sigma} \cdot \underset{\sim}{\varepsilon}\, dV}_{\Phi \text{ EN. ELASTICA}} - \underbrace{\left\{ \int_B \underline{b} \cdot \underline{u}\, dV + \int_{\partial B} \underline{p} \cdot \underline{u}\, dS \right\}}_{\text{POTENZIALE DEI CARICHI}}$$

E.P.T

APPLICO UNA 'VARIAZIONE' DELLO SPOST. $\delta \underline{u}$

($\delta \underline{u}$: ANALOGO ALLO SPOSTAM. VIRTUALE)

$$\Pi(\underline{u}+\delta\underline{u}) = \frac{1}{2}\int_B \underbrace{\mathbb{C}(\underset{\sim}{\varepsilon}+\delta\underset{\sim}{\varepsilon})}_{\underset{\sim}{\sigma}}\cdot(\underset{\sim}{\varepsilon}+\delta\underset{\sim}{\varepsilon})\,dV - \int_B \underline{b}\cdot(\underline{u}+\delta\underline{u})\,dV - \int_{\partial B}\underline{p}\cdot(\underline{u}+\delta\underline{u})\,dS$$

$\forall\,\delta\underline{u}$

$$\underbrace{\Pi(\underline{u})}_{\substack{\text{EPT}\\ \text{PG PREC}}}+\underbrace{\delta\Pi}_{\text{VARIAZ}} = \frac{1}{2}\int_B \mathbb{C}\underset{\sim}{\varepsilon}\cdot\underset{\sim}{\varepsilon} + 2\,\mathbb{C}\underset{\sim}{\varepsilon}\cdot\delta\underset{\sim}{\varepsilon} + \mathbb{C}\,\delta\underset{\sim}{\varepsilon}\cdot\delta\underset{\sim}{\varepsilon}\;dV - \int_B \underline{b}\cdot\underline{u} - \int_B \underline{b}\cdot\delta\underline{u} - \int_{\partial B}\underline{p}\cdot\underline{u} - \int_{\partial B}\underline{p}\cdot\delta\underline{u}$$

$\Pi = \Pi(\delta\underline{u})$

$$\delta\Pi = \underbrace{\int_B \overset{\underset{\sim}{\sigma}}{\mathbb{C}\underset{\sim}{\varepsilon}}\cdot\delta\underset{\sim}{\varepsilon} - \int_B \underline{b}\cdot\delta\underline{u} - \int_{\partial B}\underline{p}\cdot\delta\underline{u}}_{\delta\Pi^{(1)}\ (\text{lineare in } \delta\underline{u})} + \underbrace{\frac{1}{2}\int_B \mathbb{C}\,\delta\underset{\sim}{\varepsilon}\cdot\delta\underset{\sim}{\varepsilon}}_{\delta\Pi^{(2)}\ (\text{quadratico in } \delta\underline{u})}$$

VARIAZ. SECONDA

ALL' EQUILIBRIO $\delta\Pi^{(1)}$ (VARIAZ. PRIMA) è $= 0$

Per il T.L.V. $\delta\Pi^{(1)} = 0 \quad (\forall\,\delta\underline{u})$

STAZIONARIETÀ DELL' ENERGIA ALL' EQUILIBRIO

SISTEMA A $\quad \doteq \quad$ SISTEMA B

$\mathrm{div}\,\underset{\sim}{\sigma}^A + \underline{b}^A = \underline{0}$

$\underset{\sim}{\sigma}^A\underline{m} = \underline{p}$

$\delta\underset{\sim}{\varepsilon}^B = \frac{1}{2}\left(\nabla\delta\underline{u}^B + \nabla\delta\underline{u}^{B\,T}\right)$

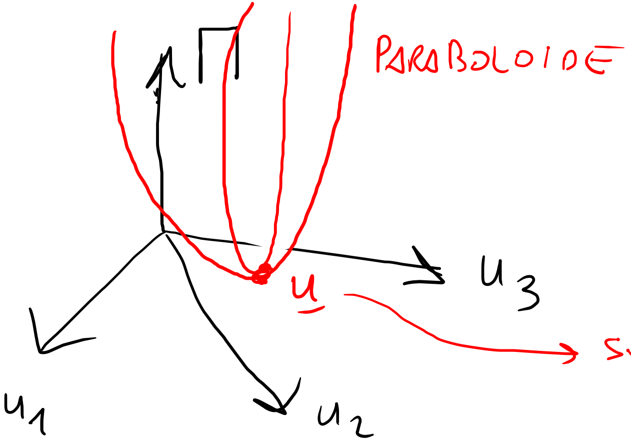
$$\int \underset{\sim}{\sigma}^A\cdot\delta\underset{\sim}{\varepsilon}^B = \int \underline{b}^A\cdot\delta\underline{u}^B + \int \underline{p}^A\cdot\delta\underline{u}^B$$

ALL' EQUIL.

ALL'EQUIL. $\delta\Pi^{(1)} = 0$; $\delta\Pi = \delta\Pi^{(2)}$

$\delta\Pi^{(2)} > 0$ $\quad \left(\int_B \mathbb{C}\, \delta\underset{\sim}{\varepsilon} \cdot \delta\underset{\sim}{\varepsilon}\, dV > 0 \right)$ perché $\mathbb{C}$ è un tensore definito positivo.

$(\forall\, \delta\underline{u})$



$\Downarrow$

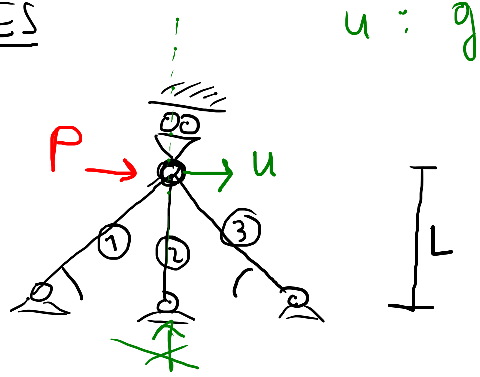
$\underline{u}$ è UN MINIMO DELL'ENERGIA

SOLUZ. EQUILIBRIO (STAZIONARIETÀ ENERGIA)

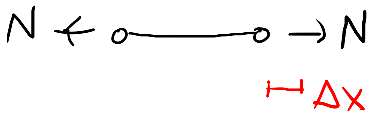
PRINCIPIO DI STAZ. DELL'E.P.T: TRA TUTTE LE CONFIG. CONGRUENTI, LA SOLUZ. EQUILIBRATA RENDE STAZIONARIA L'E.P.T.

ES

u : gdl del sist. ; determino mediante staz. E.P.T. la soluz. u^*



EA cost



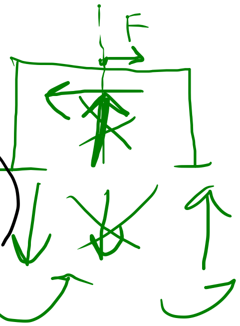
$$N = \underbrace{\frac{EA}{\ell}}_{k} \Delta x$$

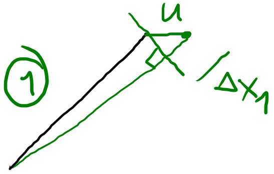
$$E_{el}(\text{biella}) = \frac{1}{2} K \Delta x^2 = \frac{1}{2} \frac{EA}{\ell} \Delta x^2$$

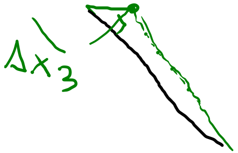
$$\Pi(u) = \text{EN. ELASTICA} - Pu$$

$$= \frac{1}{2} \sum_{i=1}^{3} \left(\frac{EA}{\ell} \Delta x^2 \right)_i \quad , \text{ con } \Delta x_i = f(u)$$

$$\Pi(u) = \frac{1}{2} EA \left(\frac{u^2}{2} \frac{1}{L\sqrt{2}} + 0 + \left(-\frac{u}{\sqrt{2}}\right)^2 \frac{1}{L\sqrt{2}} \right) - Pu$$







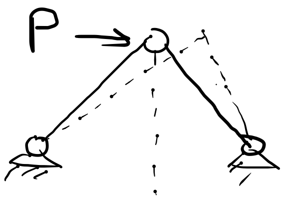
$$\Pi(u) = \frac{1}{2} EA \frac{u^2}{L\sqrt{2}} - Pu$$

$\Delta x_1 = +\frac{u}{\sqrt{2}}$ $\qquad \Delta x_3 = -\frac{u}{\sqrt{2}}$ $\qquad \Delta x_2 = 0$

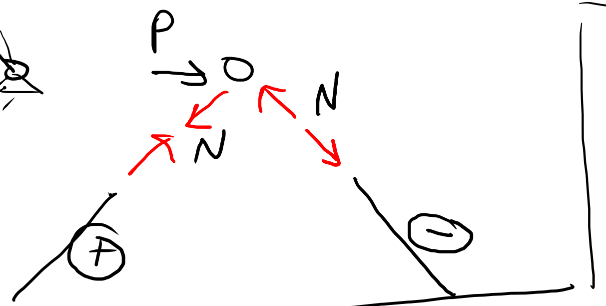
$$\delta \Pi^{(1)}(u) = \frac{\partial \Pi}{\partial u} \delta u \; , \; \delta \Pi^{(1)} = 0 \; ; \; \frac{\partial \Pi}{\partial u} = 0$$

$$\frac{d\Pi}{du} = 0 \;: \quad EA\,\frac{u^*}{L\sqrt{2}} - P = 0 \;; \quad \boxed{u^* = \frac{P}{EA}\,L\sqrt{2}}$$

$(u \Rightarrow u^*)$



STR SYM CARIC ANTISYM



LA STAZ. DELL' EPT $\left(\delta\Pi^{(1)}(\underline{u}) = 0\right)$ CONDUCE ALLA FORMULAZ. DEL METODO DEGLI SPOSTAMENTI.

RICORDIAMOCI CHE ESISTE L'EN. ELASTICA COMPLEMENTARE... $\Phi^c = \frac{1}{2}\int \underset{\sim}{\sigma}\cdot \mathbb{C}^{-1}\underset{\sim}{\sigma}\, dV$

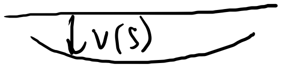
$\Phi^c(\underset{\sim}{\sigma})$

$$\Pi^c(\underset{\sim}{\sigma}) = \frac{1}{2}\int_B \underset{\sim}{\sigma}\cdot \mathbb{C}^{-1}\underset{\sim}{\sigma}\, dV - \int_B \underline{b}\cdot\underline{u} - \int_{\partial B} \underline{p}\cdot\underline{u}$$

↳ EPT COMPLEMENT.

DA VINCOLI

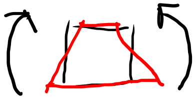
EPT PER STRUTTURE INFLESSE (EULERO-BERNOULLI)



$\rightarrow s$

$\Phi(\kappa)$: EN. ELASTICA DI UNA TRAVE INFESSA IN FUNZ. DELLA CURVATURA κ.

$\Phi = \frac{1}{2}\int \underset{\sim}{\sigma}\cdot\underset{\sim}{\varepsilon}$

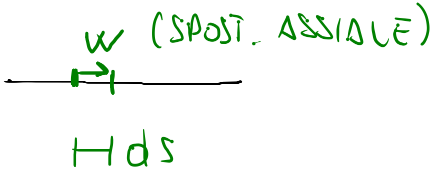
M  M

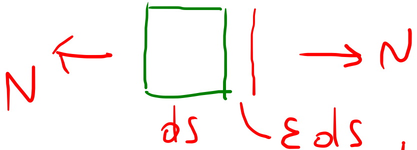
$\kappa = \frac{M}{EI}$; $\kappa = -v''(s)$

$$\Phi(v) = \frac{1}{2}\int_s EI\, v''^2$$

$$\Phi(\kappa) = \frac{1}{2}\int_s M \underbrace{\kappa\, ds}_{d\varphi} = \frac{1}{2}\int_s EI\kappa^2\, ds = \frac{1}{2}\int_s EI\,(v'')^2\, ds \Rightarrow$$

IL CONTRIBUTO a Φ della FORZA NORMALE è il seguente





$$\varepsilon = \frac{dw}{ds} = w'$$

$$d\Phi(w) = \frac{1}{2} N \varepsilon \, ds = \frac{1}{2} N w' \, ds$$

$$\Phi(w) = \int_S \frac{1}{2} N w' \, ds$$

$$\varepsilon = \frac{N}{EA} = w' \quad , \quad N = EA\, w'$$

$$\Phi(w) = \frac{1}{2} \int_S EA (w')^2 \, ds$$

EN ELASTICA CORRELATA AD EFFETTI DI FORZA NORMALE.

IN CONCLUSIONE NELLE TRAVI SNELLE

$$\Phi(w,v) = \frac{1}{2} \int_S EA\, w'^2 + EI\, v''^2 \, ds$$

E.P.T. per una trave snella:

$$\Pi(w,v) = \frac{1}{2}\int_s EA\, w'^2 + EI\, v''^2 \, ds$$

$$- \int_s q v + p w \, ds$$

POTENZ. CARICHI ESTERNI