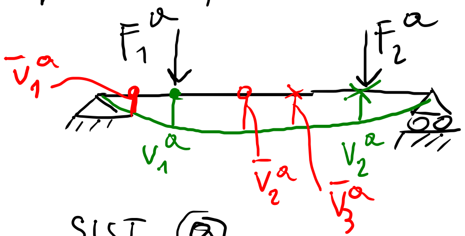
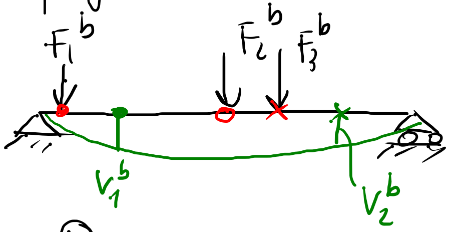
TEOREMA DI BETTI (O DI RECIPROCITÀ) 1872

MODS, 21/11/23

Ipotesi: quelle del th di Clapeyron.



SIST. (a)

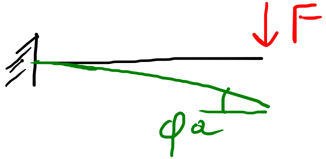


SIST (b)

$$\underbrace{F_1^a v_1^b + F_2^a v_2^b}_{\text{LAVORO MUTUO DELLE FORZE (a) × GLI SPOST. CORRENTI AL SIST (b)}} = \underbrace{F_1 \bar{v}_1^a + F_2 \bar{v}_2^a + F_3 \bar{v}_3^a}_{\text{.... (b) + GLI SPOST. --- SIST (a)}}$$

$$\boxed{L_{ab} = L_{ba}}$$

(a)



$$\varphi_a = \frac{F\ell^2}{2EI}$$

(b)

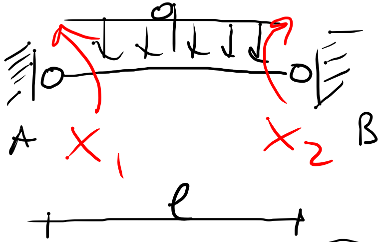
$$\eta^b = \frac{m\ell^2}{2EI}$$

$$F\eta^b \overset{?}{=} m\varphi^a$$

$$F\frac{m\ell^2}{2EI} = m\frac{F\ell^2}{2EI}$$

OK

LES: RIS. STR IPERST (METODO FORZE)



$\varphi_A = 0$ $\quad \curvearrowright^{+}$:

$\varphi_B = 0$

$$\begin{cases} +\dfrac{q\ell^3}{24EI} - \dfrac{X_1 \ell}{3EI} - \dfrac{X_2 \ell}{6EI} = 0 \\[2ex] -\dfrac{q\ell^3}{24EI} + \dfrac{X_2 \ell}{3EI} + \dfrac{X_1 \ell}{6EI} = 0 \end{cases}$$

$$\begin{cases} X_1 \dfrac{\ell}{3EI} + X_2 \underbrace{\dfrac{\ell}{6EI}}_{\eta_{12}} = \dfrac{q\ell^3}{24EI} \\[2ex] X_1 \underbrace{\dfrac{\ell}{6EI}}_{\eta_{21}} + X_2 \dfrac{\ell}{3EI} = \dfrac{q\ell^3}{24EI} \end{cases}$$

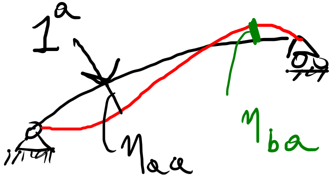
$\eta_{12} = \eta_{21}$ pe il theorema di Betti

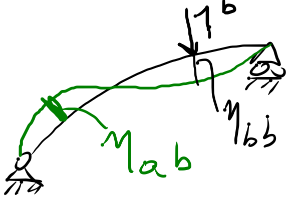
$$[H] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} [illegible] \end{bmatrix}$$

MATRICE DI CEDEVOLEZZE (η_{ij})

η_{ij} : COEFF DI INFLUENZA.

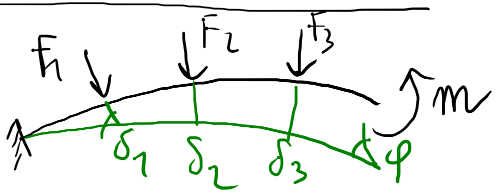
TH. DI MAXWELL (CASO PARTICOLARE DEL TH. DI BETTI)





$$-1^a \eta_{ab} = -1^b \eta_{ba} \Longrightarrow \eta_{ab} = \eta_{ba}$$

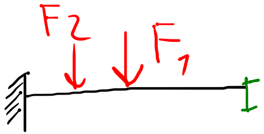
TH. DI CASTIGLIANO



$N = N(F_1, F_2, F_3, m)$

$M = M(\qquad\qquad)$

DIPENDENZE LINEARI



IPOTESI: QUELLE DI CLAPEYRON (NON SI POSSONO CONSIDERARE VARIAZ. TERMICHE)

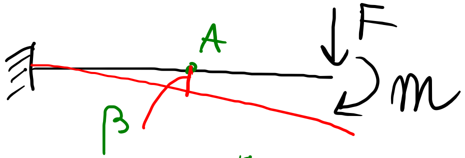
CARICHI CONCENTRATI

$$\Phi = \frac{1}{2} \int_{str} \frac{N^2}{EA} + \frac{M^2}{EI} = \Phi(F_1, F_2, \dots, m)$$

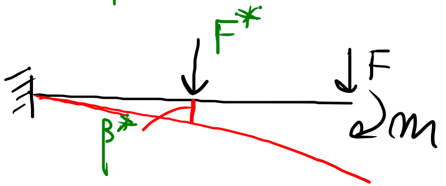
$$\frac{\partial \Phi}{\partial F_i} = \delta_i$$: SPOST. CORRISP. ALLA FORZA F_i

$$\frac{\partial \Phi}{\partial m} = \varphi$$: ROTAZ. CORRISP. A m

CALCOLO DI SPOST. DI PUNTI SENZA FORZA CORRISP.



$\Phi(F, m)$

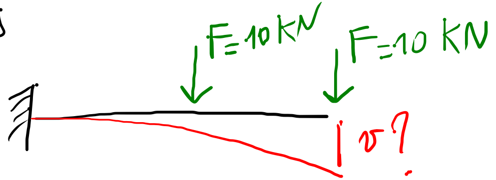


$\Phi(F, m, F^*)$

$$\beta^* = \frac{\partial \Phi(F, m, F^*)}{\partial F^*} \quad ; \quad \beta = \lim_{F^* \to 0} \beta^*$$

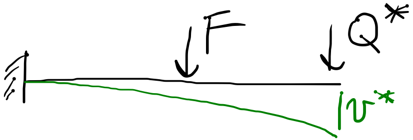
$$\beta^* = \beta^*(F, m, F^*)$$

ES

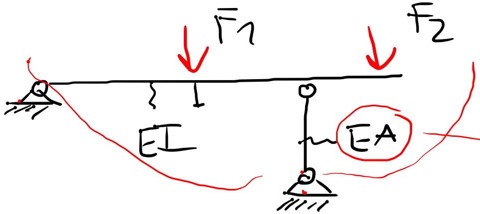


$$M = M(F) \quad ; \quad \Phi = \Phi(F) \qquad \frac{\partial \Phi}{\partial F}$$

$$\frac{1}{2} K x^2 = \frac{1}{2} K \frac{F^2}{K^2} = \boxed{\frac{1}{2} \frac{F^2}{K}} \qquad \boxed{x = \frac{F}{K}}$$

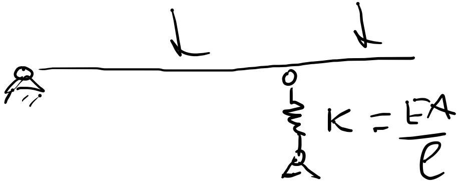


$$M(F, Q^*) \; ; \; \dots : \; v^* = \frac{\partial \Phi}{\partial Q^*} \qquad \lim_{Q^* \to F} v^* = v$$

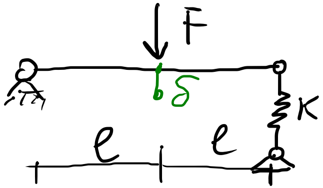


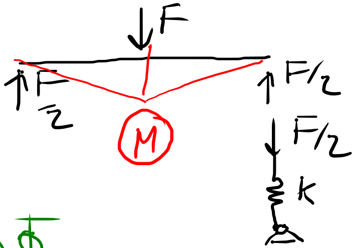
$$\Phi = \Phi(EI) + \Phi(EA)$$

TUTTI GLI ELEMENTI ELASTICI DELLA STRUTTURA DEVONO ENTRARE NELLA FORMULAZ DI Φ.



$$\Phi = \Phi(EI) + \Phi(K)$$





$M(F)$

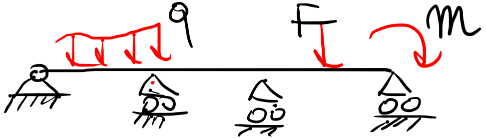
$$\Phi = \frac{1}{2} \int \frac{M^2}{EI} dz + \frac{1}{2} \left(\frac{F}{2}\right)^2 \frac{1}{K}$$

$$\delta = \frac{\partial \Phi}{\partial F}$$

TH DI MENABREA

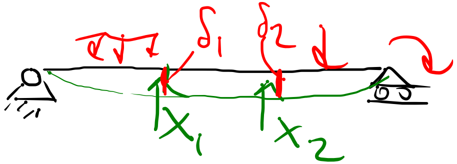
"LEAST ACTION"
(TH. DEL MINIMO LAVORO)

ENERGIA ELASTICA
=
LAVORO DI DEFORMAZIONE



VINCOLI FISSI

STR IPERST. (2 V. IPERST.)



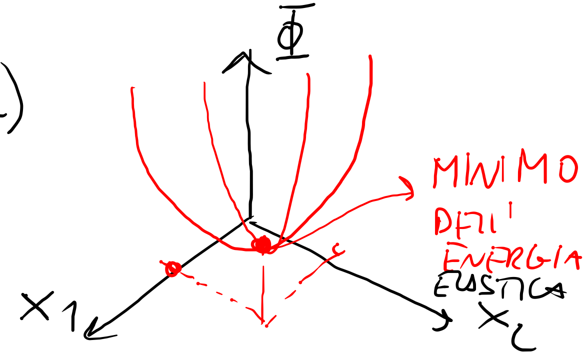
STR PRINCIPALE

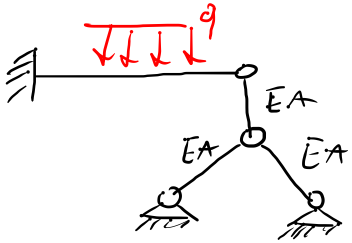
$\Phi(q, F, m; X_1, X_2)$

$\delta_i = \dfrac{\partial \Phi}{\partial X_i}$ PER CASTIGLIANO

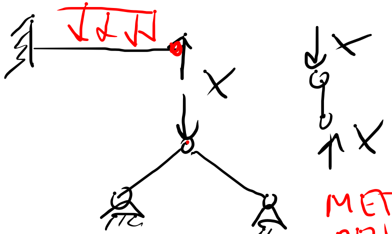
$\begin{cases} \delta_1 = 0 \\ \delta_2 = 0 \end{cases}$ EQ DI CONGRUENZA $\Rightarrow$ $\left. \begin{aligned} \dfrac{\partial \Phi}{\partial X_1} = 0 \\ \dfrac{\partial \Phi}{\partial X_2} = 0 \end{aligned} \right\}$ STAZIONARIETÀ DI Φ

$\Phi(\ldots\ldots, X_1, X_2)$



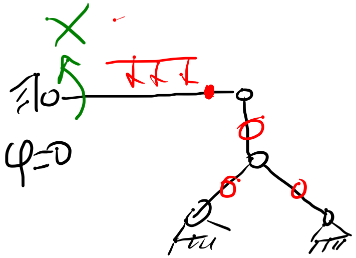


1)



METODO DELLE FORZE

2)



$\Phi(EI, EA)$

$\boxed{\varphi = \dfrac{\partial \Phi}{\partial X} = 0}$

T. DI MENABREA