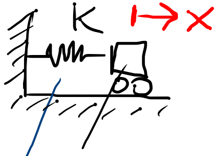
FONDAMENTI DI DINAMICA STRUTTURE (MECCANICA DELLE VIBRAZIONI) MADS, 24/11/23

- CHOPRA "DYNAMICS OF STRUCTURES"
- CLOUGH, PENZIEN "STRUCTURAL DYNAMICS"
- RAMASCO, GAVARINI, VIOLA, CASTIGLIONI (ITALIANO)

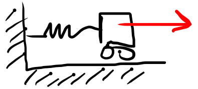
- OSCILLATORE SEMPLICE
  - OSC. LIBERE
  - " CON SMORZAMENTO
  - FORZANTI E RISONANZA
- SPETTRO DI RISPOSTA ELASTICO.

OSCILLATORE SEMPLICE



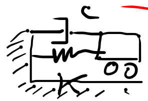
m (MASSA)

MOLLA ELASTICA



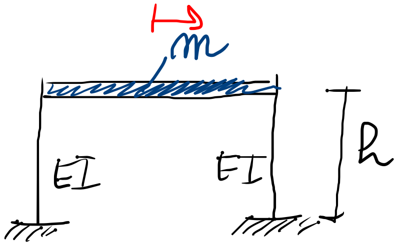
$F(t)$: OSCILLAZIONI FORZATE

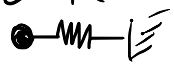
$F(t)=0$: OSCILLAZ. LIBERE



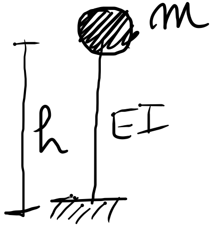
DISSIPATORE VISCOSO

(PERDITA DI ENERGIA NEL MOTO)

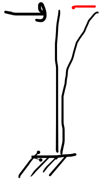




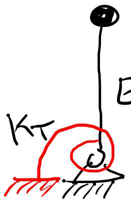
$K = f(EI, h \dots)$



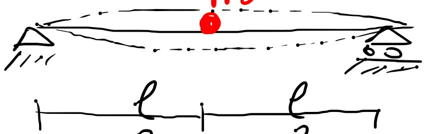
$$K = \frac{3EI}{h^3}$$

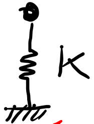


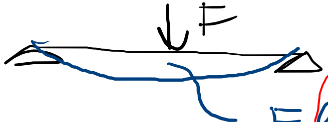
$$\delta = \frac{Fh^3}{3EI} \qquad \frac{1}{K}$$



$K = f(EI, h, K_T)$

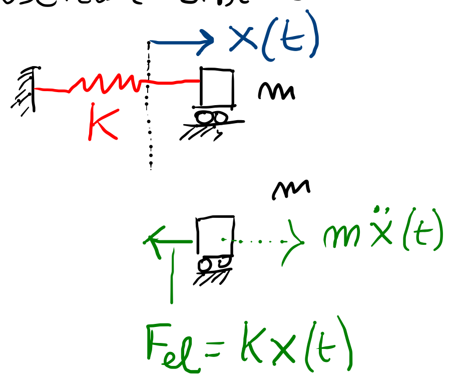






$$\frac{Fl^3}{48EI} \qquad \frac{1}{K}$$

OSCILLAZ LIBERE



EQ DELLA DINAMICA

$$m\ddot{x}(t) = -Kx(t)$$

$$m\ddot{x}(t) + Kx(t) = 0$$

[LEGGE MOTO ARMONICO]

$$\boxed{\ddot{x} + \omega^2 x = 0}; \quad \omega^2 = \frac{K}{m}; \quad \omega = \sqrt{\frac{K}{m}}$$

(*)

PULSAZIONE NATURALE o PROPRIA

(*) $x(t) = e^{\lambda t}$; $\ddot{x} = \lambda^2 e^{\lambda t}$

$$\lambda^2 \cancel{e^{\lambda t}} + \omega^2 \cancel{e^{\lambda t}} = 0 \; ; \; \lambda^2 = -\omega^2 \quad \lambda_1 = +i\omega, \quad \lambda_2 = -i\omega$$

INTEGRALE GENERALE:

$$x(t) = C_1 e^{i\omega t} + C_2 e^{-i\omega t}$$

I FORMA DELLA SOLUZIONE

VISTO CHE $e^{i\omega t} = \cos\omega t + i \sin\omega t$, ALLORA

$$x(t) = B_1 \sin\omega t + B_2 \cos\omega t$$

II FORMA " SOLUZIONE

$$x(t) = \underbrace{A}_{\text{AMPIEZZA}} \sin(\omega t + \underbrace{\varphi}_{\text{FASE}})$$

III FORMA ...

2 CONDIZ. INIZIALI

$x(0)$, $\underbrace{\dot{x}(0)}_{\text{VELOCITA'}}$ ← $t=0$

RAPPORTO TRA PULSAZ, FREQUENZA, PERIODO PROPRIO

$$[\omega] = \left[\frac{1}{T}\right] \rightsquigarrow \frac{RAD}{S} \quad \text{(U.D. MISURA)}$$

$$f = \frac{\omega}{2\pi} \rightsquigarrow \frac{1}{S} = Hz \quad \text{(HERTZ)}$$

T: periodo proprio o naturale $= \dfrac{1}{f} = \dfrac{2\pi}{\omega} \rightsquigarrow s$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \; ; \quad T = 2\pi\sqrt{\frac{m}{K}}$$

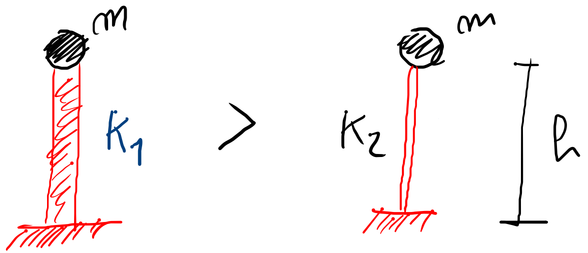
$$F = Kx \qquad K = \frac{F}{x}$$

$$\left[\frac{K}{m}\right] = \left[\frac{F}{x}\frac{a}{F}\right] = \left[\frac{a}{x}\right]$$

$$\underbrace{= \left[\frac{L}{T^2}\frac{1}{L}\right] = \left[\frac{1}{T^2}\right]}_{\text{DIMENSIONI}}$$

$$F = ma \qquad m = \frac{F}{a}$$

$$[\omega] = \left[\sqrt{\frac{K}{m}}\right] = \left[\frac{1}{T}\right]$$



$$\omega_1 > \omega_2$$

$$T_1 < T_2$$

UTILIZZATO PIÙ FREQUENT. IN ING. SISMICA.

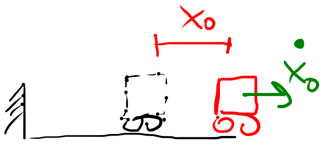
Confronto tra ⊙ $x(t) = B_1 \sin\omega t + B_2 \cos\omega t$ e $x(t) = A\sin(\omega t + \varphi)$ ⊛

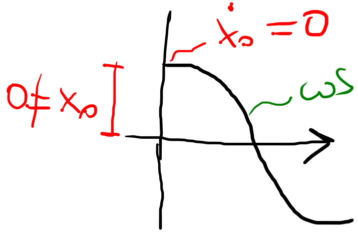
⊛ $x(t) = A\cos\varphi \sin\omega t + A\sin\varphi \cos\omega t$, nolo anche $\frac{B_2}{B_1} = \mathrm{tg}\,\varphi$; $A = \sqrt{B_1^2 + B_2^2}$

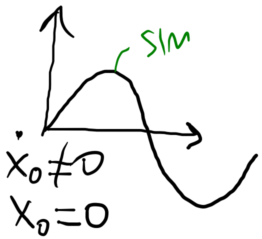
CONDIZ. INIZIALI PIÙ GENERALI

$$\begin{cases} x(0) = x_0 \ (\neq 0) \Rightarrow ⊙ \quad B_2 = x_0 \\ \dot{x}(0) = \dot{x}_0 \ (\neq 0) \Rightarrow ⊙ \quad B_1\omega = \dot{x}_0 \end{cases}$$

$$\dot{x}(t) = B_1\omega\cos\omega t + B_2\omega(-\sin\omega t) \quad ⊙$$

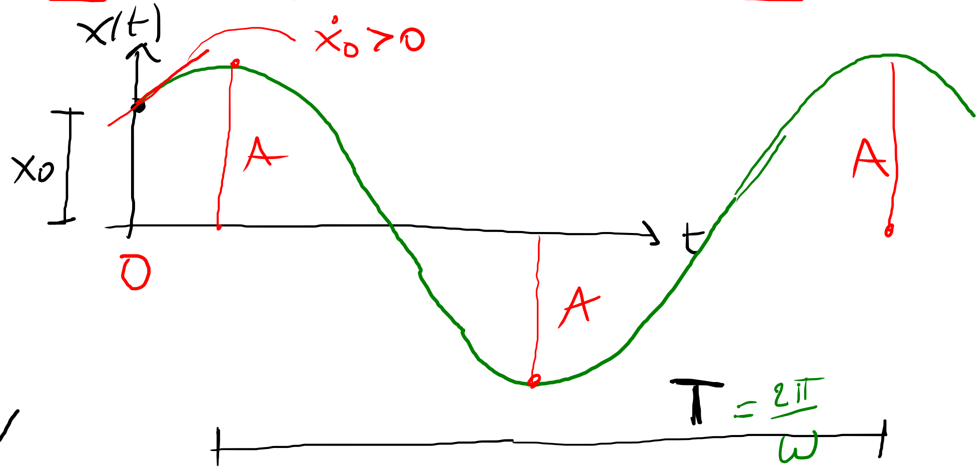






LA SOLUZ. PIÙ GENERALE

$$\boxed{x(t) = \frac{\dot{x}_0}{\omega}\sin\omega t + x_0\cos\omega t}$$



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