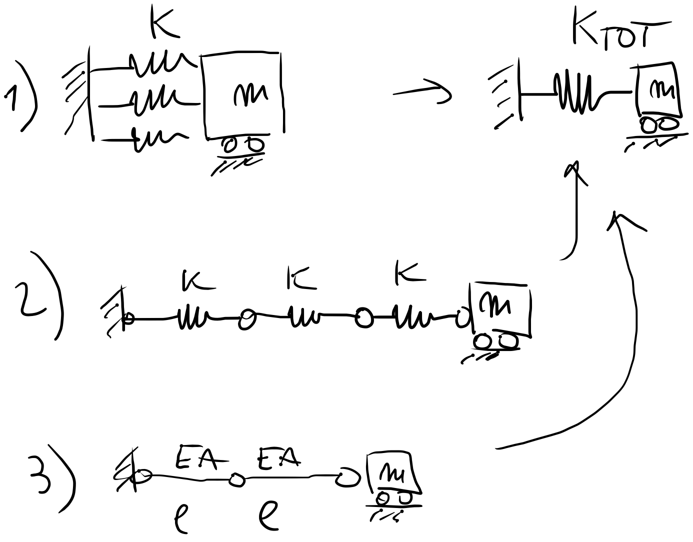
ES VOLTA SCORSA

MSDS, 27/11/23



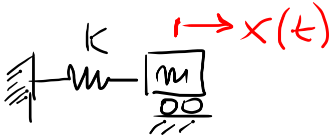

$$K = \frac{EA}{\ell}$$

1) $K_{TOT} = 3K$ (MOLLE IN PARALLELO)

2) $$\frac{1}{K_{TOT}} = \frac{1}{K} + \frac{1}{K} + \frac{1}{K}$$ (MOLLE IN SERIE)

3) $$\frac{1}{K_{TOT}} = \frac{\ell}{EA} + \frac{\ell}{EA}$$ (COME IL N. 2)

CONSIDERAZ. ENERGETICHE $\left(E_{mecc}(t) = E_{el}(t) + E_{cin}(t)\right)$ $\qquad \omega^2 = \frac{K}{m}$



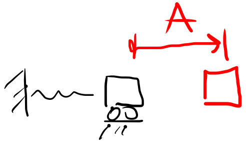

EN. ELASTICA : $E_{el}(t) = \frac{1}{2} K x(t)^2 = \frac{1}{2} K \left[A \sin(\omega t + \varphi)\right]^2$

EN. CINETICA : $E_{cin}(t) = \frac{1}{2} m \dot{x}(t)^2 = \frac{1}{2} m \Big[\underbrace{A\omega \cos(\omega t + \varphi)}_{\dot{x}(t)}\Big]^2$

$E_{mecc}(t) = \frac{1}{2} K A^2 \sin^2(\quad) + \frac{1}{2} m A^2 \omega^2 \cos^2(\quad) = \frac{1}{2} K A^2 \Big[\underbrace{\sin^2(\quad) + \cos^2(\quad)}_{1 \ (\forall t)}\Big] = \frac{1}{2} K A^2$ → $E_{mecc}$ = COSTANTE

($m\omega^2 = K$)

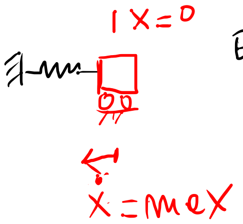
$\frac{1}{2} K A^2 = \frac{1}{2} m \omega^2 A^2$ (VELOCITA'$^2$)




$E_{el} = \frac{1}{2} K A^2 = E_{mecc}$

$E_{cin} = 0$ $(\dot{x} = 0)$ velocità nulla

MAX $\{\dot{x}(t)\} = A\omega \overbrace{\cos(\quad)}^{1}$




$E_{el} = 0$

$E_{cin} = \frac{1}{2} m (\omega A)^2 = E_{mecc}$

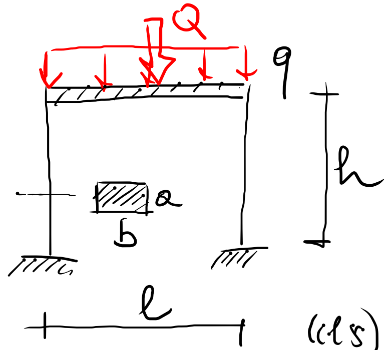
VELOCITA' MASSIMA

se $\max\{x(t)\} = A \Rightarrow \max\{\dot{x}(t)\} = A\omega$

$\max\{\ddot{x}(t)\} = \max\{-A\omega^2 \sin(\quad)\} = A\omega^2$

ES

$\boxed{\omega, T}$ ?




$h = 3\,m$

$a = b = 30\,cm$

$l = 5\,m$

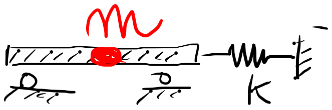
$q = 40\,KN/m$

(cls) $E = 25000\,\frac{N}{mm^2}$

$g = 9.81\,m/s^2$

$I = \frac{ab^3}{12}$

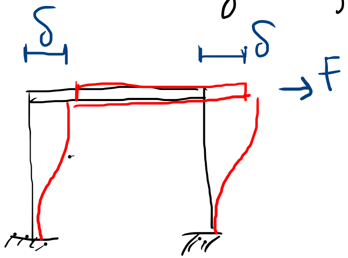
TRASCURO LA MASSA DELLA STRUTTURA




$Q = ql = 40 \cdot 5 = 200\,KN$

$Q = mg \implies m = \frac{Q}{g} = \frac{200\,000}{9.81} \simeq 20\,000\,Kg$ (m OK)

$\boxed{K?}$




$\frac{12EI}{h^3}$

$\delta = \frac{Fh^3}{24EI}$ $\frac{1}{K}$

(K ok)

$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{1}{m} \frac{24\,E\,ab^3}{12} \frac{1}{h^3}} = 27.4\,\frac{rad}{s}$

$T = \frac{2\pi}{\omega} = \frac{6.28}{27.4} \simeq 0.23\,s$

$f = \frac{1}{T} \simeq 4.35\,Hz$

con $h = 4\,m$ $\quad T = 0.35\,s$

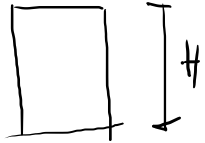
$h = 5\,m$ $\quad T = 0.49\,s$

$\omega = \hat{f}\left(\sqrt{\left(\frac{b}{h}\right)^3}\right)$ ; $T = \hat{g}\left(\sqrt{\left(\frac{h}{b}\right)^3}\right)$

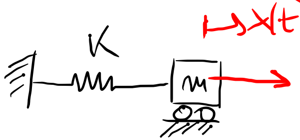
FORMULA EMPIRICA PER CALCOLARE T NEGLI EDIFICI IN FUNZ DI h:

$T = c_1 H^{3/4}$

dip. dal materiale

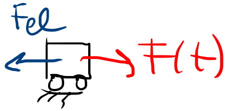


# OSCILLATORE SEMPLICE CON FORZANTE ARMONICA (FENOMENO DELLA RISONANZA)




$$F(t) = \overbrace{F}^{\text{AMPIEZZA}} \sin\bar\omega t$$

$(\Omega)$: PULSAZ. DELLA FORZANTE




EQ - DIFF NON OMOGENEA

EQ DEL MOTO: $m\ddot{x} + Kx = F\sin\bar\omega t$

$$x(t) = \underbrace{x^{hom}(t)}_{\text{OMOGENEO}} + \overbrace{x^{p}(t)}^{\text{particolare}}$$

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$x^p$?

$$x^p(t) = X\sin\bar\omega t = \frac{F}{K}\frac{1}{1-\left(\frac{\bar\omega}{\omega}\right)^2}\sin\bar\omega t$$

$$m(-X\bar\omega^2)\cancel{\sin\bar\omega t} + KX\cancel{\sin\bar\omega t} = F\cancel{\sin\bar\omega t}$$

$$X(-m\bar\omega^2 + K) = F$$

$$X = \frac{F}{K - m\bar\omega^2} = \frac{F}{K}\frac{1}{1-\overbrace{\frac{m}{K}}^{\frac{1}{\omega^2}}\bar\omega^2} = \frac{F}{K}\frac{1}{1-\left(\frac{\bar\omega}{\omega}\right)^2}$$

$$x(t) = B_1\sin\omega t + B_2\cos\omega t + \frac{F}{K}\frac{1}{1-\left(\frac{\bar\omega}{\omega}\right)^2}\sin\bar\omega t$$

DALLE CONDIZ. INIZIALI $x(0)$, $\dot x(0)$

1) $x(0) = x_0 \Rightarrow \boxed{B_2 = x_0}$

2) $\dot x(t) = B_1\omega\cos\omega t - B_2\omega\sin\omega t + (\quad)\bar\omega\cos\bar\omega t$

$$\dot x(0) = \dot x_0 = B_1\omega + \frac{F}{K}\frac{1}{1-\left(\frac{\bar\omega}{\omega}\right)^2}\bar\omega$$

$$\boxed{B_1 = \frac{\dot x_0}{\omega} - \frac{F}{K}\frac{1}{1-\left(\frac{\bar\omega}{\omega}\right)^2}\frac{\bar\omega}{\omega}}$$

IL MOTO SI INNESCA ANCHE QUANDO

$$x_0 = \dot x_0 = 0 \quad (B_2 = 0;\ B_1 \neq 0)$$

Con queste condiz iniziali:

$$x(t) = \underbrace{\frac{F}{K}}^{X_{st}} \frac{1}{1-\left(\frac{\bar{\omega}}{\omega}\right)^2} \left[ \sin \bar{\omega} t - \frac{\bar{\omega}}{\omega} \sin \omega t \right]$$

OSSERVAZIONI: 1) $x(t) \to \infty$ quando $\bar{\omega} \to \omega$ ; SOLUZ. NON VALIDA PER $\frac{\bar{\omega}}{\omega} = 1$

2) $\frac{F}{K} = X_{st}$ : SPOST. X SOTTO L'AZIONE STATICA DI F.