SPETTRO DI RISPOSTA ELASTICO.

MADS, 15/12/23

ASSEGNATO UN OSCILLATORE (m, K, C EVENTUALE), LO SPETTRO DI RISPOSTA ELASTICO È IL GRAFICO DELLA "MASSIMA RISPOSTA" DEL SISTEMA ECCITATO DA UNA FORZANTE $F(t)$

NOTA.

→ PER "MASS. RISPOSTA" SI INTENDE IL MAX
- SPOSTAM.
- VELOCITÀ
- ACCELERAZIONE

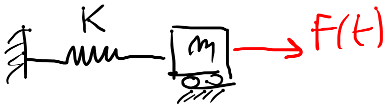
$R_d(T)$

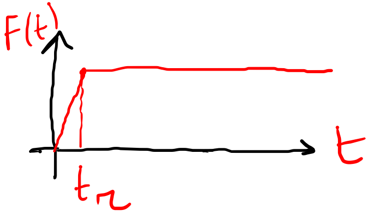
$R_d(\omega)$: SPETTRO DI RISPOSTA IN TERMINI DI SPOST. (DISPLACEMENT)

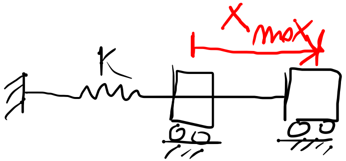
$$\omega = \sqrt{\frac{K}{m}}$$

$$T = \frac{2\pi}{\omega}$$

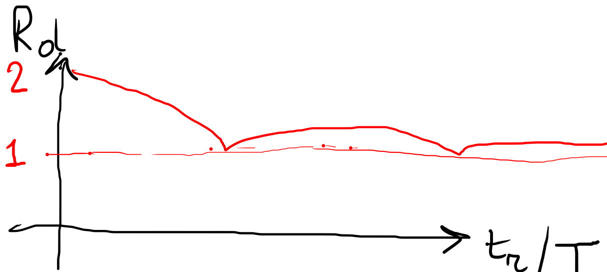
PROBLEMA







$$R_d = \frac{x_{max}}{x_{STATICO}}$$



$$F_{max} = K\, x_{max} = K\, R_d\, x_{ST}$$

$$= R_d\, \underbrace{K\, x_{ST}}_{F_{STATICO}}$$

R_d: AMPLIFICAZ. DINAMICA DELLA FORZA APPLIC. STATICAM.

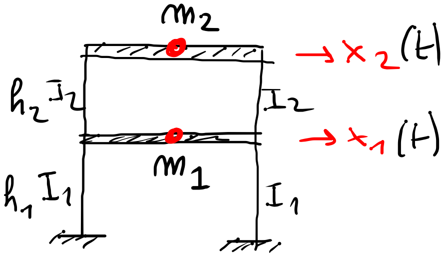
Per il moto armonico: $x(t) = A \sin(\omega t + \varphi)$ $\quad$ $\max x(t) = A \longrightarrow R_d$

$\dot{x}(t) = A\omega \cos(\omega t + \varphi)$ $\quad$ $\max \dot{x}(t) = A\omega \longrightarrow R_v = \omega R_d$

$\ddot{x}(t) = -\underline{A\omega^2} \sin(\omega t + \varphi)$ $\quad$ $\max \ddot{x}(t) = A\omega^2 \longrightarrow R_a = \omega^2 R_d$

· PRESENT. DEL PROF. GHERSI (SPETTRO ELASTICO PER L'ACCELEROGRAMMA DI TOLMEZZO 1976)

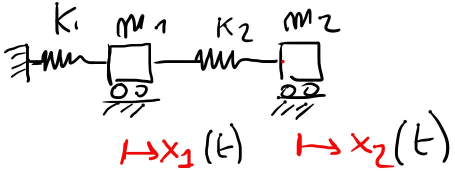
OSCILLAZ. LIBERE DI UN SIST. A 2GDL.

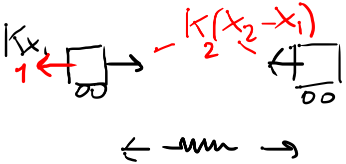


MASSE CONC. AL PIANO

$$K_2 = 24\,\frac{EI_2}{h_2^3}$$

$$K_1 = 24\,\frac{EI_1}{h_1^3}$$





$x_2 > x_1$

$$\begin{cases} m_1\ddot{x}_1 = -K_1 x_1 + K_2(x_2-x_1) \\ m_2\ddot{x}_2 = -K_2(x_2-x_1) \end{cases}$$

$$\begin{cases} m_1\ddot{x}_1 + K_1 x_1 - K_2(x_2-x_1) = 0 \\ m_2\ddot{x}_2 + K_2(x_2-x_1) = 0 \end{cases}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} K_1+K_2 & -K_2 \\ -K_2 & +K_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{M}\,\underline{\ddot{x}} + \underset{\sim}{K}\,\underline{x} = \underline{0}$$

OSCILLAZ. LIBERE IN UN SIST. A N GDL

$\underset{\sim}{M}$: matrice delle masse

$\underset{\sim}{K}$: " di rigidezza

$\Big\}\ (2\times 2)$

Cerco soluz $\begin{cases} x_1(t) = U_1 \sin \omega t \\ x_2(t) = U_2 \sin \omega t \end{cases} \Longleftrightarrow \begin{cases} x_1(t) = C_1 e^{\lambda t} \\ x_2(t) = C_2 e^{\lambda t} \end{cases}$

$$\begin{cases} \left[m_1 U_1 (-\omega^2) + k_1 U_1 + k_2 (U_1 - U_2)\right] \sin \omega t = 0 \\ \left[m_2 U_2 (-\omega^2) + k_2 (U_2 - U_1)\right] \sin \omega t = 0 \end{cases}$$

$\sin \omega t \neq 0$

$$\begin{cases} -m_1 U_1 \omega^2 + k_1 U_1 + k_2 (U_1 - U_2) = 0 \\ -m_2 U_2 \omega^2 + k_2 (U_2 - U_1) = 0 \end{cases}$$

SIST. OMOGENEO, PER AVERE SOLUZ. DIVERSE DALLA OVVIA $(U_1 = U_2 = 0)$

$$\begin{vmatrix} -m_1 \omega^2 + k_1 + k_2 & -k_2 \\ -k_2 & -m_2 \omega^2 + k_2 \end{vmatrix} = 0$$

AUTOVAL DI $\underset{\sim}{A}$

$$|\underset{\sim}{A} - \lambda \underset{\sim}{I}| = 0$$

$$|\underset{\sim}{K} - \omega^2 \underset{\sim}{M}| = 0$$ IN TERMINI MATRICIALI

$\Uparrow$

LE INCOGNITE SONO NEI TERMINI ω^2 (AUTOVALORE GENERALIZZATO) (DUE SOLUZ.)

$$\left| \quad \right| = 0 \rightsquigarrow \omega^4 - \left(\frac{k_1+k_2}{m_1} + \frac{k_2}{m_2} \right) \omega^2 + \frac{k_1 k_2}{m_1 m_2} = 0$$

$$\left.\begin{array}{l} \omega_1^2 \\ \omega_2^2 \end{array}\right\} = -b \mp \sqrt{b^2 - 4ac} \quad \begin{array}{l} >0 \\ >0 \end{array}$$

$$\omega_1^2 < \omega_2^2 \quad \text{MODO UTILE PER L'ORDINAMENTO}$$

$$\Downarrow$$

$$\omega_1 < \omega_2 \quad \in \mathbb{R}$$

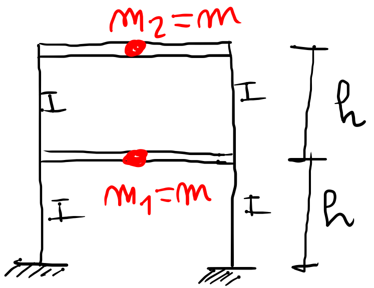
ω_1: PULSAZIONE FONDAMENTALE DEL SISTEMA

$T_1 = \frac{2\pi}{\omega_1}$ PERIODO " " " $\quad T_1 > T_2$

GLI AUTOVETTORI $\underline{U}^{(1)} = \begin{bmatrix} U_1^{(1)} \\ U_2^{(1)} \end{bmatrix}$ e $\underline{U}^{(2)} = \begin{bmatrix} U_1^{(2)} \\ U_2^{(2)} \end{bmatrix}$: MODI DI VIBRARE DEL SISTEMA

2 per un sist. A 2 GDL

ES.



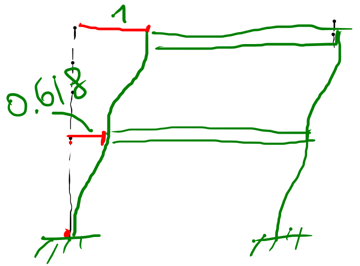
$$K = K_1 = K_2 = 24 \frac{EI}{h^3}$$

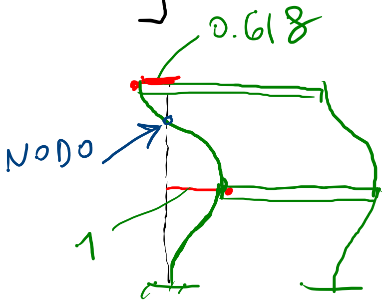
$$\omega_1 = 0.618 \sqrt{\frac{K}{m}} \qquad T_1 = 10.17 \sqrt{\frac{m}{K}}$$

$$\omega_2 = 1.618 \sqrt{\frac{K}{m}} \qquad T_2 = 3.88 \sqrt{\frac{m}{K}}$$

$$U^{(1)} = \begin{bmatrix} 0.618 \\ 1 \end{bmatrix} \qquad U^{(2)} = \begin{bmatrix} 1 \\ -0.618 \end{bmatrix}$$

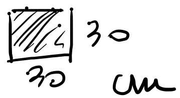
NORMALIZZ. $\max \left| U_j^{(i)} \right| = 1$



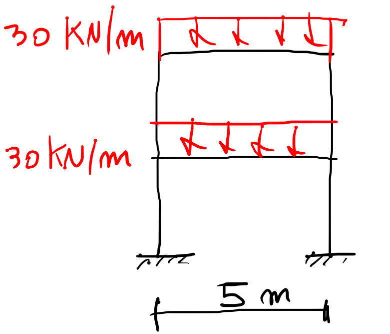


ES. NUM. $E = 20\ GPa$

$$I = \frac{30^4}{12} = 67500\ cm^4$$



$h = 3\ m$



$T_1 = 0.36\ s$ PERIODO FONDAM.

$T_2 = 0.14\ s$

$$g\,m = 30000 \cdot 5\ N$$

$$m = \frac{30000 \cdot 5}{g}$$

$$\simeq 15000\ Kg$$