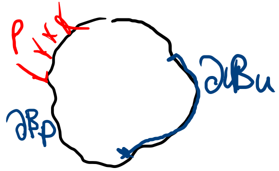
.... SUI PRINCIPI VARIAZIONALI



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EQUIL. $\quad \operatorname{div} \underset{\sim}{\sigma} + \underline{b} = \underline{0} \quad$ in B

$\qquad\qquad \underset{\sim}{\sigma}\, \underline{n} = \underline{p} \quad \partial B_p$

CONGR. $\quad \underset{\sim}{\varepsilon} = \frac{1}{2}\left(\nabla \underline{u} + \nabla \underline{u}^T\right) \quad$ in B

$\qquad\qquad \underline{u} = \underline{u}^0 \quad \partial B_u \quad (1)$

LEG. COST. $\quad \underset{\sim}{\sigma} = \mathbb{C}\, \underset{\sim}{\varepsilon} \quad$ in $B \quad (3)$

SCRIVO L'EPT $\Pi(\underline{u})$:

$$\Pi(\underline{u}) = \frac{1}{2}\int_B \underset{\sim}{\sigma} \cdot \underset{\sim}{\varepsilon}\, dV - \int_{\partial B_u} \underline{b} \cdot \underline{u}\, dV - \int_{\partial B_p} \underline{p} \cdot \underline{u}\, dS$$

$$\boxed{\Pi(\underline{u}) = \frac{1}{8}\int_B \mathbb{C}\left(\nabla \underline{u} + \nabla \underline{u}^T\right) \cdot \left(\nabla \underline{u} + \nabla \underline{u}^T\right) dV - \int \qquad - \int}$$

PRINCIPIO DI STAZ. EPT $\left(\delta \Pi(\underline{u}) = 0, \forall \delta \underline{u}\right)$:

NELL'INSIEME DELLE FUNZ. SPOST. $\underline{u}$ AMMISSIBILI, OVVERO REGOLARI E RISPETTOSE DELLE CONDIZ. AI LIMITI (1), QUELLA CHE RISOLVE IL PROBLEMA ELASTICO RENDE STAZIONARIA IL FUNZ. $\Pi(\underline{u})$

DIM: $\quad \delta \Pi = 0; \quad f(\delta \underline{u}) \neq 0$

ELABORANDO L'ESPR. SI OTTIENE UNA FORMA DEL P.L.V. CHE ASSICURA CHE I CAMPI DI TENSIONE $\underset{\sim}{\sigma}$ SIANO EQUILIBRATI, ATTRAVERSO L'UTILIZZO DI (3)

FUNZ. HU-WASHIZU (FUNZ. LIBERO)

SIMILE EPT

$$HW(\underline{u},\underset{\sim}{\sigma},\underset{\sim}{\varepsilon}) = \frac{1}{2}\int_B \mathbb{C}\underset{\sim}{\varepsilon}\cdot\underset{\sim}{\varepsilon}\, dV - \int_B \underline{b}\cdot\underline{u}\, dV - \int_{\partial B_p} \underline{p}\cdot\underline{u}\, dS + \int_B \underset{\sim}{\sigma}\cdot\left[\frac{1}{2}(\nabla\underline{u}+\nabla\underline{u}^T) - \underset{\sim}{\varepsilon}\right] dV - \int_{\partial B_u} \underset{\sim}{\sigma}\,\underline{n}\,(\underline{u}-\underline{u}^0)\, dS$$

TEOREMA DI STAZ. DEL FUNZ. DI HW:

LE FUNZ. $\underline{u}, \underset{\sim}{\sigma}, \underset{\sim}{\varepsilon}$ SOLUZIONE DEL PROBL. ELASTICO SONO QUELLE CHE, SENZA ALCUNA CONDIZ. AGGIUNTIVA, REND. STAZ. $HW(\underline{u},\underset{\sim}{\sigma},\underset{\sim}{\varepsilon})$.

DIM. (TRACCIA)

$\delta_{\underline{u}} HW = 0$: ..... $\Rightarrow$ UN'ESPR. DEL PLV CHE, NOTI (*), (**), (***) ~~DI~~ ASSICURA L'EQUILIBRIO.

$\delta_{\underset{\sim}{\sigma}} HW = 0$: $\displaystyle\int_B \delta\underset{\sim}{\sigma}\cdot\underbrace{\left[\frac{1}{2}(\nabla\underline{u}+\nabla\underline{u}^T) - \underset{\sim}{\varepsilon}\right]}_{=0\ (*)} dV - \int \delta\underset{\sim}{\sigma}\,\underline{n}\cdot\underbrace{(\underline{u}-\underline{u}^0)}_{=0\ (**)} dS = 0, \ \forall\, \delta\underset{\sim}{\sigma}$

$\delta_{\underset{\sim}{\varepsilon}} HW = 0$:

$$\int_B \mathbb{C}\underset{\sim}{\varepsilon}\cdot\delta\underset{\sim}{\varepsilon}\, dV - \int \underset{\sim}{\sigma}\cdot\delta\underset{\sim}{\varepsilon}\, dV = 0, \ \delta\underset{\sim}{\varepsilon} \Rightarrow \boxed{\mathbb{C}\underset{\sim}{\varepsilon} - \underset{\sim}{\sigma} = \underset{\sim}{0}}\ (***)$$

LEG. EL. LINEARE

FUNZ. DI HELLINGER-REISSNER (HR)

$$\underset{\sim}{\sigma}\cdot\underset{\sim}{\varepsilon} = \frac{1}{2}\underset{\sim}{\sigma}\cdot\underset{\sim}{\varepsilon} + \frac{1}{2}\underset{\sim}{\sigma}\cdot\underset{\sim}{\varepsilon}$$
$$= \frac{1}{2}\mathbb{C}\underset{\sim}{\varepsilon}\cdot\underset{\sim}{\varepsilon} + \frac{1}{2}\underset{\sim}{\sigma}\cdot\mathbb{C}^{-1}\underset{\sim}{\sigma}$$
$$\Rightarrow -\underset{\sim}{\sigma}\cdot\underset{\sim}{\varepsilon} + \frac{1}{2}\mathbb{C}\underset{\sim}{\varepsilon}\cdot\underset{\sim}{\varepsilon} = -\frac{1}{2}\underset{\sim}{\sigma}\cdot\mathbb{C}^{-1}\underset{\sim}{\sigma}$$

FUNZ. LIBERO

SI È INS. IL LEG COSTITUTIVO

$$HR(\underline{u},\underset{\sim}{\sigma}) = \frac{1}{2}\int_B \underset{\sim}{\sigma}\cdot(\nabla\underline{u}+\nabla\underline{u}^T)\,dV - \frac{1}{2}\int_B \underset{\sim}{\sigma}\cdot\mathbb{C}^{-1}\underset{\sim}{\sigma}\,dV - \int \underline{b}\cdot\underline{u}\,dV - \int \underline{p}\cdot\underline{u}\,dS - \int \underset{\sim}{\sigma}\underline{m}(\underline{u}-\underline{u}^0)\,dS$$

TEOR DI STAZ. DI $HR(\underline{u},\underset{\sim}{\sigma})$

NELL'INS DELLE FUNZ. SPOST. $\underline{u}$ CONTINUE E REGOLARI E DELLE FUNZIONI DI TENSIONE $\underset{\sim}{\sigma}$, QUELLE CHE RISOLVONO IL PROBL. ELASTICO RENDONO STAZ. IL FUNZ. HR.

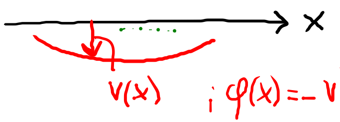
$\delta_u HR = 0 \Rightarrow$ PLV CHE ASSICURA L'EQUIL.

in B $\quad \underbrace{\mathbb{C}^{-1}\underset{\sim}{\sigma}}_{\underset{\sim}{\varepsilon}} = \frac{1}{2}(\nabla\underline{u}+\nabla\underline{u}^T)$; $\quad \underline{u}=\underline{u}^0$ su ∂B_u

$$\delta_{\sigma} HR = 0 : \frac{1}{2}\int_B \delta\underset{\sim}{\sigma}\cdot(\nabla\underline{u}+\nabla\underline{u}^T)\,dV - \int_B \delta\underset{\sim}{\sigma}\cdot\mathbb{C}^{-1}\underset{\sim}{\sigma}\,dV - \int \delta\underset{\sim}{\sigma}\underline{m}(\underline{u}-\underline{u}^0)\,dS = 0, \quad \forall\delta\underset{\sim}{\sigma}$$

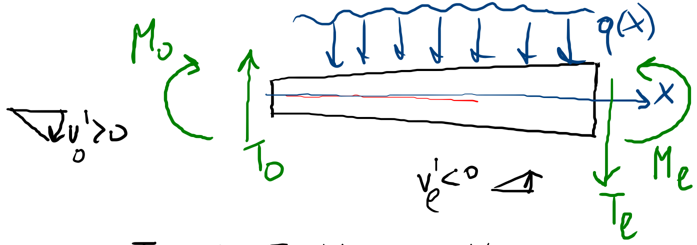
EQUILIBRIO DELLA TRAVE INFLESSA DI EULERO-BERNOULLI (MEDIANTE STAZ. E.P.T.)

$$\Pi(v) = \int_0^\ell \left(\frac{1}{2} EI v''^2 - q v\right) dx - \left[T v - M v'\right]_0^\ell$$



$; \varphi(x) = -v'(x)$

$$\boxed{\delta \Pi(v) = 0 \,;\ \delta v}$$



$$0 = \int_0^\ell \left(\cancel{\frac{1}{2}} EI \cancel{2} v'' \delta v'' - q \delta v\right) dx - \left[T \delta v - M \delta v'\right]_0^\ell$$

$T_\ell v(\ell) - T_0 v(0) - M_\ell v'(\ell) + M_0 v'(0)$

APPLICO INTEGR x PARTI $\int_0^\ell EI v'' \delta v'' \, dx = \left[EI v'' \delta v'\right]_0^\ell - \int (EI v'')' \delta v' \, dx$

$$\int f g' = [fg] - \int f' g$$

$$0 = \underbrace{-\int_0^\ell (EI v'')' \delta v' \, dx}_{\text{ANCORA PER PARTI}} - \int_0^\ell q \delta v \, dx - \left[T \delta v - (M + EI v'') \delta v'\right]_0^\ell$$

$$0 = \int_0^\ell \underbrace{\left[(EI v'')'' - q\right]}_{\neq 0} \delta v \, dx + \left[\overbrace{(M + EI v'')}^{0} \overbrace{\delta v'}^{0} - \overbrace{\left(T + (EI v'')'\right)}^{0} \overbrace{\delta v}^{0}\right]_0^\ell \quad \forall \delta v \ (\forall \delta v') \quad \Rightarrow$$

$(EI\,v'')'' = q \qquad x \in [0, \ell] \qquad$ EQ. DI CAMPO

$M = -EI\,v''$

$T = -EI\,v'''$

CONDIZ. AI LIMITI (VALIDE SIA IN $x=0$, SIA IN $x=\ell$)

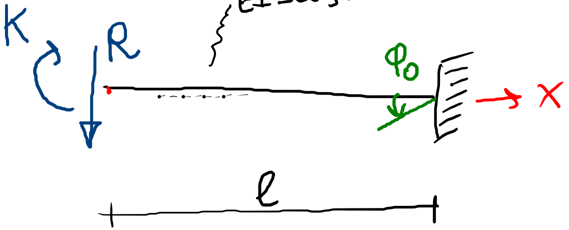
$M(0) + EI\,v''(0) = 0$ OPPURE $v'(0) = -\bar{\varphi}$ $(\delta v' = 0)$

$T(0) + (EI\,v'')'\big|_0 = 0$ " $v(0) = \bar{v}$ $(\delta v = 0)$

CONDIZ. NATURALI

COND. ESSENZIALI

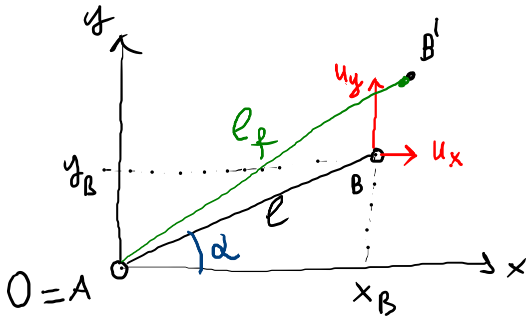
ES



$EI\,v^{IV} = 0 \qquad x \in [0, \ell]$

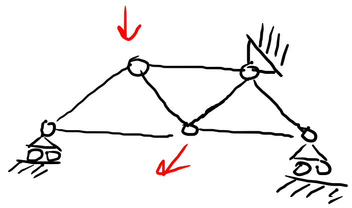
$$\begin{cases} EI\,v''(0) = -K \\ -R + EI\,v'''(0) = 0 \\ v(\ell) = 0 \\ v'(\ell) = -\varphi_0 \end{cases}$$

INTRODUZ. AL METODO DEGLI SPOST PER LE STR. RETICOLARI.



$$l_f = \sqrt{(x_B+u_x)^2 + (y_B+u_y)^2} = l_f(u_x, u_y)$$

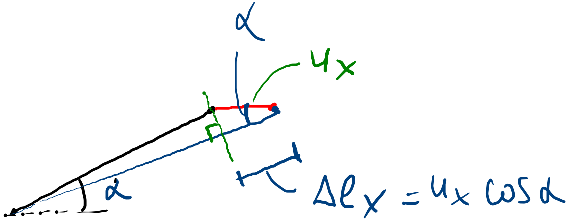
$$l = \sqrt{x_B^2 + y_B^2}$$

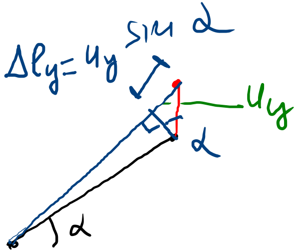


PICCOLI SPOST.: SVILUPPO IN SERIE $l_f(u_x, u_y)$:

$$l_f = l_f(0,0) + \frac{\partial l_f}{\partial u_x}\Big|_{(0,0)} u_x + \frac{\partial l_f}{\partial u_y}\Big|_{(0,0)} u_y + O\left(\sqrt{u_x^2+u_y^2}\right) = l + \overbrace{\cos\alpha\, u_x}^{\Delta l_x} + \overbrace{\sin\alpha\, u_y}^{\Delta l_y}$$

$$l_f = l + \left[\frac{1}{\cancel{2}\underbrace{\sqrt{\;}}_{l_{\cancel{f}}}}\cancel{2}(x_B+u_x)\right]_{(0,0)} u_x + \left[\frac{1}{\cancel{2}\underbrace{\sqrt{\;}}_{l_{\cancel{f}}}}\cancel{2}(y_B+u_y)\right]_{(0,0)} u_y + \dots = l + \overset{\cos\alpha}{\frac{x_B}{l}} u_x + \overset{\sin\alpha}{\frac{y_B}{l}} u_y$$

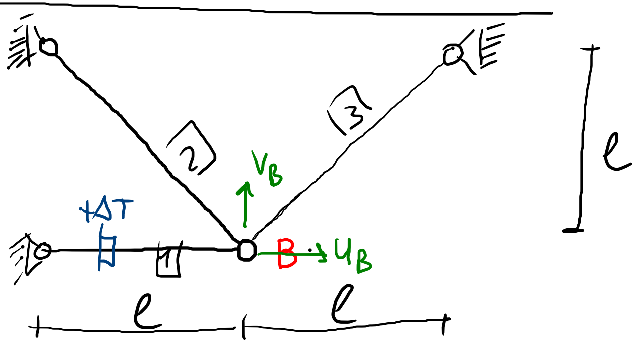




$$l_f - l = \Delta l = \Delta l_x + \Delta l_y$$

(SOMMA DEI 2 CONTRIBUTI INDIPENDENTI DOVUTI A u_x e A u_y)

LES



$A_1 = A, \quad A_2 = A_3 = \sqrt{2}\,A$

$l_1 = l, \quad l_2 = l_3 = \sqrt{2}\,l$

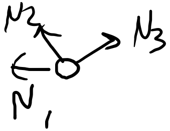
STR IPERST SOGGETTA A DISTORSIONE

SPOST. INCOGNITI: u_B, v_B (GLI ALTRI NODI SONO FISSI)

(2 EQUAZ. IN u_B, v_B)

EQUAZIONI A DISPOSIZ.

EQUIL DEL NODO B



LEG. COSTITUTIVO: $\left(\Delta l_1 = \Delta l_1^{el} + \Delta l^{term}\right)$

$$N_1 = \frac{EA_1}{l_1}\Delta l_1^{el} = \frac{EA_1}{l_1}\left(\Delta l_1 - \alpha \Delta T l\right)$$

$$N_2 = \frac{EA_2}{l_2}\Delta l_2, \quad N_3 = \frac{EA_3}{l_3}\Delta l_3$$

PER RISOLV IL PROBLEMA SCRIVO LE 2 EQ DI EQUIL. DEL NODO IN FUNZ. DI N_1, N_2, N_3 ; POI SOSTITUISCO IL LEG. COSTITUTIVO COSÌ CHE COMPAIANO $\Delta \ell_1, \Delta \ell_2, \Delta \ell_3$ e DAT ; ESPRIMO $\Delta \ell_1, \Delta \ell_2, \Delta \ell_3$ IN FUNZIONE DI u_B e v_B PER AVERE COSÌ UN SIST. DI 2 EQ IN 2 INCOGNITE.

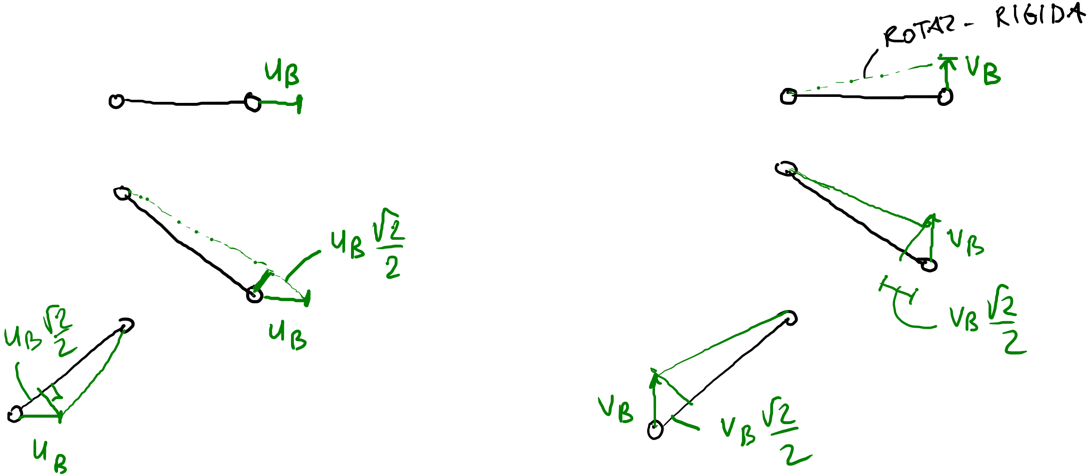
ESPRIMO $\Delta \ell_1, \Delta \ell_2, \Delta \ell_3$ IN FUNZ. DI

u_B $\quad\quad$ v_B

1) $\Delta \ell_1$

2)

3)



$$\Delta \ell_1 = +u_B$$

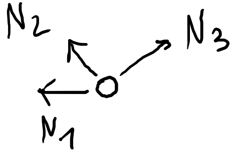
$$\Delta \ell_2 = +u_B \frac{\sqrt{2}}{2} - v_B \frac{\sqrt{2}}{2}$$

$$\Delta \ell_3 = -u_B \frac{\sqrt{2}}{2} - v_B \frac{\sqrt{2}}{2}$$

2 EQ. DI EQUIL. (SF DI TRAZ. POSITIVA)

NOTO CHE $\frac{EA_1}{\ell_1} = \frac{EA}{\ell} = K$

$\frac{EA_2}{\ell_2} = \frac{EA}{\ell} = K$; $\frac{EA_3}{\ell_3} = \frac{EA}{\ell} = K$



$$\left\{\begin{array}{l} \xrightarrow{+} : \; -N_1 - N_2\frac{\sqrt{2}}{2} + N_3\frac{\sqrt{2}}{2} = 0 \\ +\uparrow : \; +N_2\cancel{\frac{\sqrt{2}}{2}} + N_3\cancel{\frac{\sqrt{2}}{2}} = 0 \end{array}\right.$$

$$\left\{\begin{array}{l} -K(\Delta\ell_1 - \alpha\Delta T\ell) - K\frac{\sqrt{2}}{2}\Delta\ell_2 + K\frac{\sqrt{2}}{2}\Delta\ell_3 = 0 \\ K\Delta\ell_2 + K\Delta\ell_3 = 0 \end{array}\right.$$

$$\left\{\begin{array}{l} -K(u_B - \alpha\Delta T\ell) - K\frac{1}{2}(u_B - v_B) - K\frac{1}{2}(u_B + v_B) = 0 \\ K\cancel{\frac{\sqrt{2}}{2}}(u_B - v_B) - K\cancel{\frac{\sqrt{2}}{2}}(u_B + v_B) = 0 \end{array}\right.$$

SOL: $\left\{\begin{array}{l} u_B = \frac{1}{2}\alpha\Delta T\ell \\ v_B = 0 \end{array}\right.$

$$+K\frac{1}{2}\alpha\Delta T\ell - K\frac{1}{4}\alpha\Delta T\ell - K\frac{1}{4}\alpha\Delta T\ell = 0$$

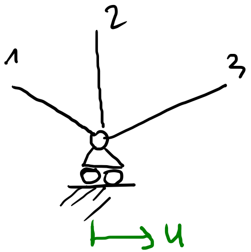
OK!

N_1? $N_1 = K(u_B - \alpha\Delta T\ell) = -K\frac{1}{2}\alpha\Delta T\ell$

N_3? $= K\Delta\ell_3 = K\frac{\sqrt{2}}{2}(-u_B - v_B) = -K\frac{\sqrt{2}}{4}\alpha\Delta T\ell$

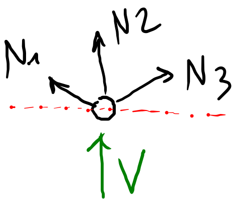
N_2 ANALOGO (>0)

CASO PARTICOLARE



u: UNICO GDL NODALE

1 INCOGNITA



EQ DI EQUIL. "UTILE"

$\xrightarrow{+}$ (NON COMPARE V)

1 EQ. A DISPOSIZ.