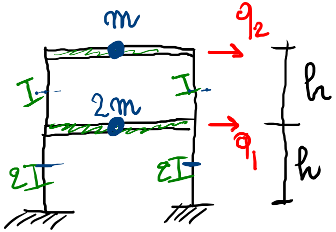
ES. ANALISI MODALE (SIST. 2 G.D.L. CON MOTO IMPRESSO ALLA BASE) 11/4/24



E : cost.

$$\underset{\sim}{M} = \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix}$$

$$\underset{\sim}{K} = \underbrace{\frac{24EI}{h^3}}_{K} \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \Rightarrow \underset{\sim}{K}^T$$

$$\underset{\sim}{\Phi} = \begin{bmatrix} 1/2 & 1 \\ 1 & -1 \end{bmatrix}$$

($\underset{\sim}{\Phi} = \underset{\sim}{\Phi}^T$ ORA PER UN CASO PARTICOLARE IN GENERALE NON VALE)

$$\underline{q} = \underset{\sim}{\Phi}\, \underline{z}$$

$$\hat{\underset{\sim}{M}} = \underset{\sim}{\Phi}^T \underset{\sim}{M} \underset{\sim}{\Phi} = \begin{bmatrix} 1/2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} 1/2 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 3/2\, m & 0 \\ 0 & 3m \end{bmatrix}$$

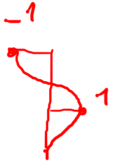
$$\hat{\underset{\sim}{K}} = \underset{\sim}{\Phi}^T \underset{\sim}{K} \underset{\sim}{\Phi} = K \;\; '' \;\; \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \;\; '' \;\; = K \begin{bmatrix} 3/4 & 0 \\ 0 & 6 \end{bmatrix}$$

PR. AUTOVALORI GENERALIZZ.:

$$\left(\underset{\sim}{K} - \omega^2 \underset{\sim}{M} \right) \underline{\phi} = \underline{0}$$

$$\omega_1^2 = \frac{1}{2} \frac{K}{m} \qquad \underline{\phi}^{(1)} = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

$$\omega_2^2 = 2 \frac{K}{m} \qquad \underline{\phi}^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

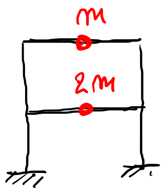


SIST. COORD. PRINCIPALI

$$\begin{cases} \frac{3}{2} m \ddot{z}_1 + \frac{3}{4} K z_1 = 0 \Rightarrow \ddot{z}_1 + \Omega_1^2 z_1 = 0 \\ 3m \ddot{z}_2 + 6K z_2 = 0 \Rightarrow \ddot{z}_2 + \Omega_2^2 z_2 = 0 \end{cases}$$

$$\Omega_1^2 = \frac{3}{4} K \cdot \frac{2}{3m} = \underbrace{\frac{1}{2} \frac{K}{m}}_{\text{OK} = \omega_1^2} \; ; \; \Omega_2^2 = \frac{6K}{3m} = \frac{K}{m} 2 \overset{\text{OK}}{=} \omega_2^2$$

MOTO IMPR. ALLA BASE



$\mapsto \ddot{y}(t)$

ASSEGNATO

$$\underline{F}(t) = -\begin{bmatrix}2\\1\end{bmatrix} m\,\ddot{y}(t) \;;\quad \hat{\underline{F}}(t) = \underset{\sim}{\Phi}^T \underline{F}(t) = -\begin{bmatrix}1/2 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} m\,\ddot{y}(t) = -\begin{bmatrix}2\\1\end{bmatrix} m\,\ddot{y}(t)$$

IN GEN. NON SONO UGUALI

SIST. COORD. PRINCIPALI

$$\begin{cases} \frac{3}{2} m \ddot{z}_1 + \frac{3}{4} K z_1 = -2m\,\ddot{y}(t) \\ 3m\ddot{z}_2 + 6K z_2 = -m\,\ddot{y}(t) \end{cases} \Rightarrow \begin{cases} \ddot{z}_1 + \omega_1^2 z_1 = -2\cdot\frac{2}{3}\,\ddot{y}(t) \\ \ddot{z}_2 + \omega_2^2 z_2 = -\frac{1}{3}\,\ddot{y}(t) \end{cases}$$

$g_1 = 4/3 \; (> g_2)$

$g_2 = 1/3$

(ESEMPI DI MODI DI VIBRARE DI STRUTTURE DA "RAMASCO" e "CHOPRA")

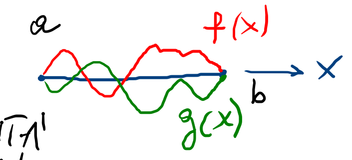
ORTOGONALITÀ DEI MODI DI VIBRARE

$$\boxed{i\neq j}\qquad \underbrace{\underline{\phi}^{(i)}\cdot \underset{\sim}{M}\,\underline{\phi}^{(j)}}_{\star} = \underbrace{\underline{\phi}^{(i)}\cdot \underset{\sim}{K}\,\underline{\phi}^{(j)}}_{*} = 0$$

$\underline{u} \perp \underline{v}$ $\qquad \underline{u}\cdot\underline{v}=0$

$\underline{u}\cdot \underset{\sim}{I}\,\underline{v}=0$

ORTOGONALITÀ GENERALIZZ. RISP. ALLA MATRICE $\underset{\sim}{A}$ $\qquad \underline{u}\cdot \underset{\sim}{A}\,\underline{v}=0$



ORTOGONALITÀ DI FUNZIONI $\quad \int_a^b f(x)\,g(x)\,dx = 0$

$\boxed{\text{DIM}}$

$$\left(\underset{\sim}{K}-\omega_i^2\,\underset{\sim}{M}\right)\underline{\phi}^{(i)}=\underline{0} \implies \underset{\sim}{K}\,\underline{\phi}^{(i)}=\omega_i^2\,\underset{\sim}{M}\,\underline{\phi}^{(i)}$$

$$\underline{\phi}^{(j)}\cdot \underset{\sim}{K}\,\underline{\phi}^{(i)}=\omega_i^2\,\underline{\phi}^{(j)}\cdot \underset{\sim}{M}\,\underline{\phi}^{(i)} \qquad \text{I}$$

$$\left(\underset{\sim}{K}-\omega_j^2\,\underset{\sim}{M}\right)\underline{\phi}^{(j)}=\underline{0} \implies \underset{\sim}{K}\,\underline{\phi}^{(j)}=\omega_j^2\,\underset{\sim}{M}\,\underline{\phi}^{(j)}$$

$$\underline{\phi}^{(i)}\cdot \underset{\sim}{K}\,\underline{\phi}^{(j)}=\omega_j^2\,\underline{\phi}^{(i)}\cdot \underset{\sim}{M}\,\underline{\phi}^{(j)} \qquad \text{II}$$

VISTO CHE $\underset{\sim}{K}$ e $\underset{\sim}{M}$ SONO SYM FACCIO LA DIFF. TRA I e II

$$0=\left(\omega_i^2-\omega_j^2\right)\underline{\phi}^{(i)}\cdot \underset{\sim}{M}\,\underline{\phi}^{(j)}\ ;\ \text{se } \omega_i^2\neq\omega_j^2 \implies \underline{\phi}^{(i)}\cdot \underset{\sim}{M}\,\underline{\phi}^{(j)}=0 \quad \star$$

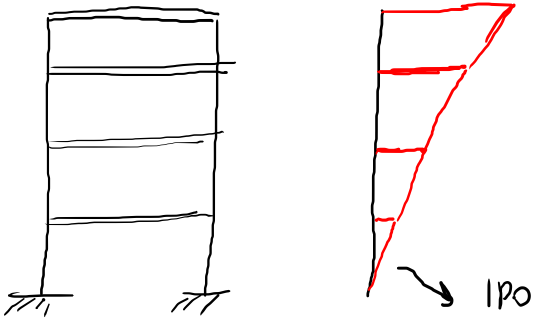
CONSIDERO LA II e SI RICAVA CHE $\underline{\phi}^{(i)}\cdot \underset{\sim}{K}\,\underline{\phi}^{(j)}=0$ $*$

PRENDO LA I e LA VALUTO PER $i = j$

$$\underline{\phi}^{(i)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(i)} = \omega_i^2\, \underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(i)} \Longrightarrow \omega_i^2 = \frac{\underline{\phi}^{(i)} \cdot \underset{\sim}{K}\, \underline{\phi}^{(i)}}{\underline{\phi}^{(i)} \cdot \underset{\sim}{M}\, \underline{\phi}^{(i)}}$$

RAPPORTO DI RAYLEIGH $\odot$

$\odot$ PUÒ ESSERE UTILE PER STIMARE SPERIMENTALMENTE ω_i^2 DI UNA STRUTTURA:



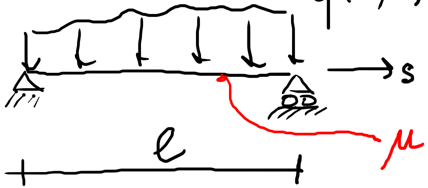
$$\omega^2_{1\,APPR} \approx \frac{\underline{\phi}^{(1)}_{APPR} \cdot \underset{\sim}{K}\, \underline{\phi}^{(1)}_{APPR}}{\underline{\phi}^{(1)}_{APPR} \cdot \underset{\sim}{M}\, \underline{\phi}^{(1)}_{APPR}}$$

IPOTESI I MODO DI VIBRARE

$\underline{\phi}^{(1)}_{APPR}$

- ESPERIENZA
- INDAGINI SPERIMENTALI

DINAMICA DEI SISTEMI CONTINUI A 1 G.D.L. GENERALIZZATO



$q(s,t)$: FORZANTE ESTERNA

μ : MASSA PER UNITÀ DI LUNGHEZZA

EI: RIGIDEZZA ELASTICA

$y(s,t)$: LINEA ELASTICA "DINAMICA" (INCOGNITA)

$$y(s,t) = \underbrace{\psi(s)}_{\text{FUNZ. DI FORMA}} \, x(t)$$

$$\dot{y}(s,t) = \psi(s)\,\dot{x}(t)$$

$$\chi(s) = -y''(s,t) = -\psi''(s)\,x(t)$$

$$M(s,t) = -EI y'' = -\psi'' x\, EI$$

AL FINE DI DETERMINARE L'EQ. DEL MOTO SCRIVO IL BILANCIO DELLE POTENZE

$$\frac{d}{dt}\left[E_{cin} + E_{pot}\right] = \text{Potenza del carico } q$$

$$E_{cin} = \frac{1}{2}\int_0^\ell \mu\, ds\, \dot{y}^2 = \frac{1}{2}\underbrace{\int_0^\ell \mu\, \psi^2 ds}_{m_{eq}\ \text{MASSA EQUIV.}}\, \dot{x}^2$$

$$E_{el} = \frac{1}{2}\int_0^\ell M\chi\, ds = \frac{1}{2}\int_0^\ell (-\psi'' x\, EI)(-\psi'' x)\, ds$$

$$= \frac{1}{2}\underbrace{\int_0^\ell EI\, \psi''^2\, ds}_{K_{eq}\ :\ \text{RIGIDEZZA EQ.}}\ x^2 = \frac{1}{2} K_{eq}\, x^2$$

POT. CARICO q : $\int_0^\ell q\, ds\, \dot{y} = \underbrace{\int_0^\ell q\,\psi\, ds}_{F_{eq}\ :\ \text{FORZA EQUIV.}}\ \dot{x}$

$$\frac{d}{dt}\left[\frac{1}{2} m_{eq}\,\dot{x}(t)^2 + \frac{1}{2} K_{eq}\,x(t)^2\right] = F_{eq}\,\dot{x}(t)$$

$$m_{eq}\,\cancel{\dot{x}(t)}\,\ddot{x}(t) + \frac{1}{2}\,2\,K_{eq}\,x(t)\,\cancel{\dot{x}(t)} = F_{eq}\,\cancel{\dot{x}(t)} \quad : \quad \boxed{m_{eq}\,\ddot{x}(t) + K_{eq}\,x(t) = F_{eq}}$$

EQ. DELLA DINAMICA DI UN SIST. CON 1 G.D.L. GENERALIZZATO.

$\psi(s)$: FUNZ. DI FORMA CHE DEVE ESSERE SCELTA OPPORTUNAMENTE