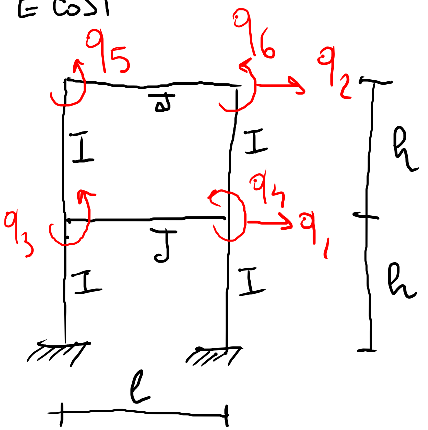
MATRICE DI RIGIDEZZA DI UNA STRUTTURA CON GDL ROTAZ. E TRASLAZIONALI

30/04/24

E cost



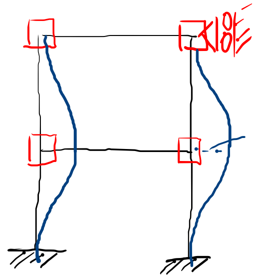
TRASC. DEFORM. ASSIALE
(PILASTRI + TRAVI INESTENS.)

$\underset{\sim}{K} \Rightarrow 6 \times 6$

STUDIAMO 2 COLONNE
DELLA MATR.

$\boxed{q_1 = 1;\ q_2 = \dots q_6 = 0} \Rightarrow K_{i1}\ (i = 1, \dots, 6)$

K_{ij}

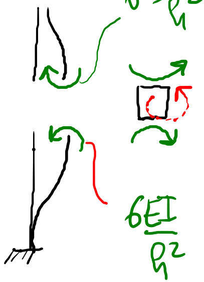


$K_{21} = -\ 2 \cdot 12 \frac{EI}{h^3} = -24\, EI/h^3$

$K_{11} = 4 \cdot 12 \frac{EI}{h^3} = 48 \frac{EI}{h^3}$

$\boxed{K_{31}:}$

(K_{41})

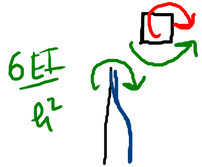


$K_{31} = 0$

CASO PARTICOLARE

(ANCHE $K_{41} = 0$)

$\boxed{K_{51}:}$



$K_{51} = -\frac{6EI}{h^2}$

K_{61} è ANALOGO A K_{51}

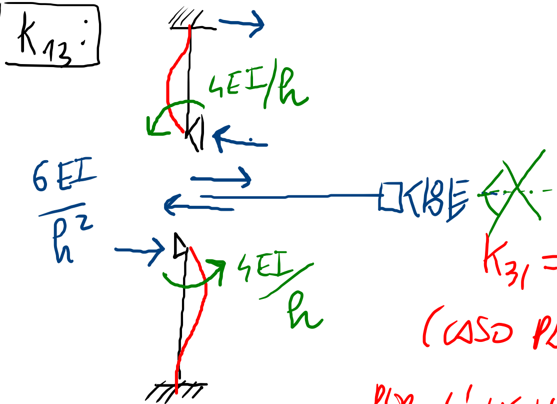
$K_{61} = -\frac{6EI}{h^2}$

$\boxed{1^0 \text{ COLONNA DI } \underset{\sim}{K}}$

$$
\underset{\sim}{K} = \begin{bmatrix}
48\frac{EI}{h^3} & \text{SYM} & 0 & & & \\
-24\frac{EI}{h^3} & & +\frac{6EI}{h^2} & & & \\
0 & & \frac{8EI}{h}+\frac{4EJ}{\ell} & & & \\
0 & & 2EJ/\ell & & & \\
-6EI/h^2 & & 2EI/h & & & \\
-6EI/h^2 & & 0 & & &
\end{bmatrix}
$$

K_{13} K_{31}

K_{12} :

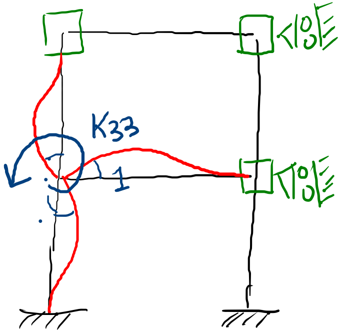


$K_{31}=0$

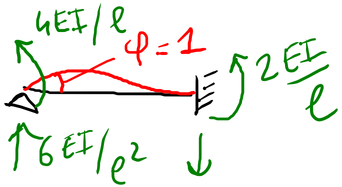
(CASO PARTIC. PER L'UGUAGLIANZA DEI TAGLI NEI 2 PILASTRI)

K_{i3} : 3° COLONNA

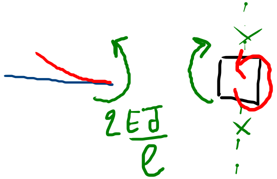
$q_3=1$; $q_1=q_2=q_4=q_5=q_6=0$



$$K_{33} = 2\cdot\frac{4EI}{h}+4\frac{EJ}{\ell}$$

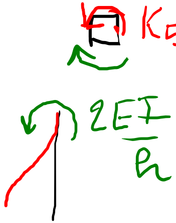


K_{43} :



$K_{43} = +\frac{2EJ}{\ell}$

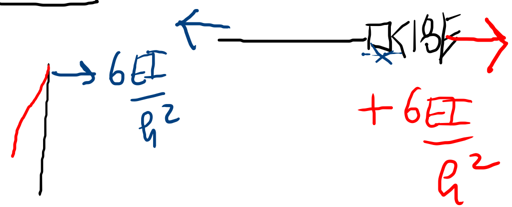
K_{53} :



$K_{53} = +\frac{2EI}{h}$

$K_{63}=0$

K_{23} :



K_{23}

$+\frac{6EI}{h^2}$

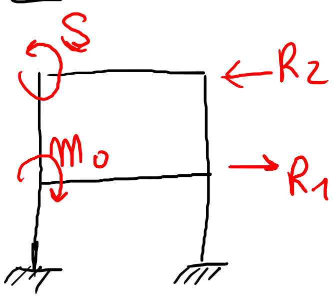
Le dimens. di K_{ij} sono funzionali al sist. lineare $\underset{\sim}{K}\,\underline{q}=\underline{F}$ (ESEMPIO PAG. PREC.)

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{Bmatrix} \left.\begin{matrix} q_1 \\ q_2 \end{matrix}\right\} \text{SPOST} \\ \left.\begin{matrix} q_3 \\ \vdots \end{matrix}\right\} \text{ROTAZ.} \end{Bmatrix} = \begin{Bmatrix} \left.\begin{matrix} F_1 \\ F_2 \end{matrix}\right\} \text{DIM. FORZE} \\ \left.\begin{matrix} F_3 \\ \vdots \end{matrix}\right\} \text{MOMENTI} \end{Bmatrix}$$

$$\Rightarrow K_{11}\overset{\text{SPOST.}}{q_1} + K_{12}\overset{\downarrow}{q_2} + K_{13}\overset{\text{ROTAZ.}}{q_3} + \dots = \text{FORZA}$$

$$K_{31}q_1 + K_{32}q_2 + K_{33}q_3 + \dots = \text{MOMENTO}$$

ES.



$$\underset{\sim}{K}\,\underline{q} = \underline{F} = \begin{bmatrix} +R_1 \\ -R_2 \\ -M_0 \\ 0 \\ +S \\ 0 \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right] \text{FORZE DI PIANO} \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right] \text{MOM. NODALI} \end{matrix}$$

$$\underline{q} = \underset{\sim}{K}^{-1}\,\underline{F}$$

LA SOLUZ. DEL SISTEMA $\underset{\sim}{K}\,\underline{q}=\underline{F}$

CORRISPONDE A QUELLA DELLO SCHEMA

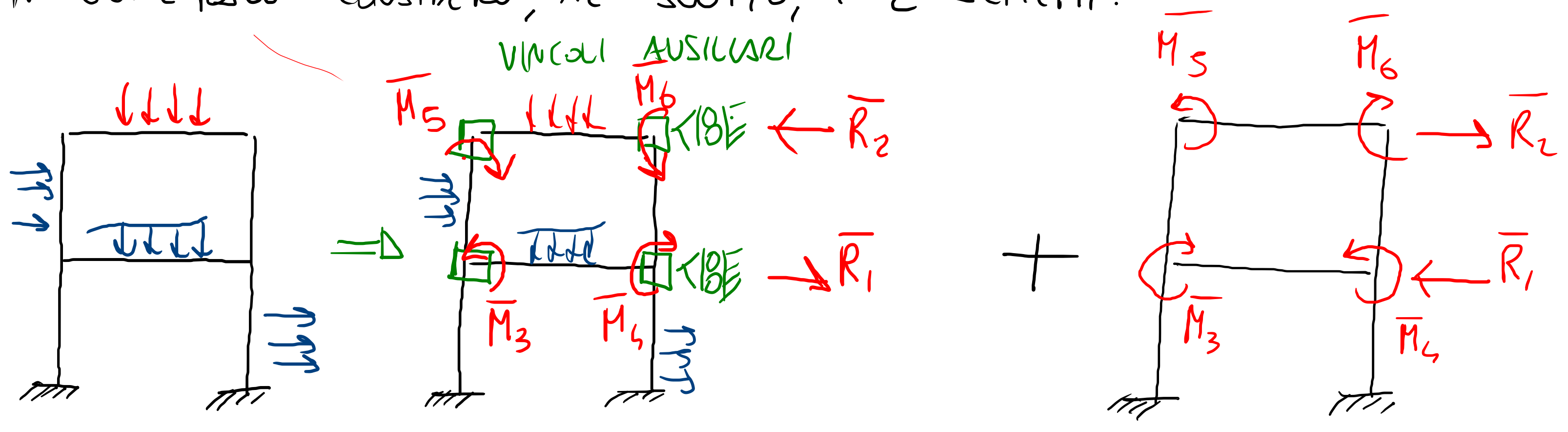
II NEL METODO DEGLI SPOSTAMENTI

DOVE CONSIDERO SOLO CONDIZIONI

DI CARICO NEI "NODI" A CUI

ASSOCIO UN G.D.L.

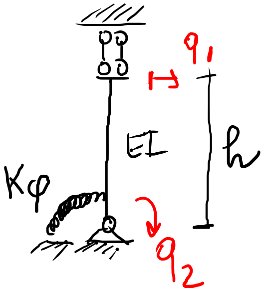
IN GENERALE CONSIDERO, AL SOLITO, I 2 SCHEMI:



SCH. (I) $(\underline{q} = \underline{0})$

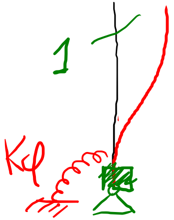
SCH (II) : $K\underline{q} = \underline{F}$

OSSERVAZ. SULLA PRESENZA DI VINCOLI CEDEVOLI.

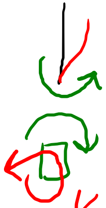


2 GDL $\Rightarrow$ $\underset{\sim}{K}$: 2x2

$\boxed{q_1 = 1,\ q_2 = 0}$



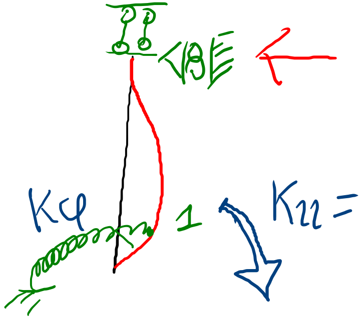
$\Rightarrow K_{11} = 12\dfrac{EI}{h^3}$



$K_{21} = -\dfrac{6EI}{h^2}$

$$\underset{\sim}{K} = \begin{bmatrix} 12\dfrac{EI}{h^3} & -\dfrac{6EI}{h^2} \\ -\dfrac{6EI}{h^2} & \dfrac{4EI}{h} + K_\varphi \end{bmatrix}$$

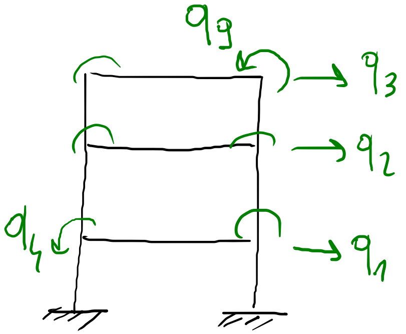
$\boxed{q_1 = 0,\ q_2 = 1}$

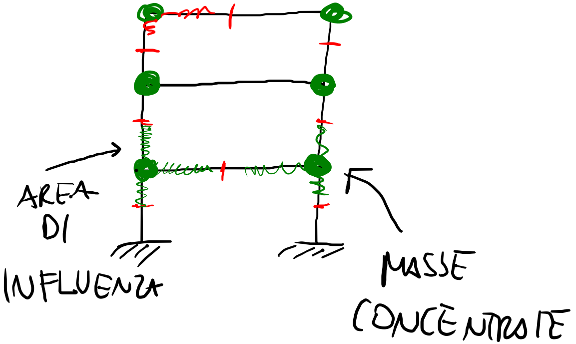


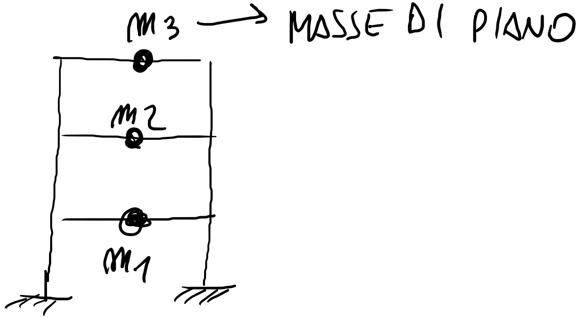
$K_{12} = -\dfrac{6EI}{h^2}$

$K_{22} = 4\dfrac{EI}{h} + K_\varphi$

CONDENSAZIONE DELLA MATRICE DI RIGIDEZZA (IN DINAMICA)







$\underset{\sim}{K} \Rightarrow 9 \times 9 \qquad \underline{q} = \begin{bmatrix} \underline{q}_A \\ \underline{q}_B \end{bmatrix}$

$\underline{q}_A = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} ; \quad \underline{q}_B = \begin{bmatrix} q_4 \\ \vdots \\ q_9 \end{bmatrix}$

$$E_{cin} = \frac{1}{2} \left(m_1 \dot{q}_1^2 + m_2 \dot{q}_2^2 + m_3 \dot{q}_3^2 \right)$$

$$\underset{\sim}{M} \ddot{q} + \underset{\sim}{K} q = \underline{F}(t)$$

$$\left[\begin{array}{c|c} \underset{\sim}{M}_A & \underset{\sim}{0} \\ \hline \underset{\sim}{0} & \underset{\sim}{0} \end{array}\right] \begin{bmatrix} \ddot{\underline{q}}_A \\ \ddot{\underline{q}}_B \end{bmatrix} + \left[\begin{array}{c|c} \underset{\sim}{K}_{AA} & \underset{\sim}{K}_{AB} \\ \hline \underset{\sim}{K}_{BA} & \underset{\sim}{K}_{BB} \end{array}\right] \begin{bmatrix} \underline{q}_A \\ \underline{q}_B \end{bmatrix} = \begin{bmatrix} \underline{F}_A(t) \\ \underline{0} \end{bmatrix}$$

$$\underset{\sim}{M}_A = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

LA CONDENSAZ. PERMETTE DI ESPRIMERE IL SIST. LINEARE NELLE SOLE INCOGNITE $\underline{q}_A$.