.... (CONDENSAZ. STATICA ....) 2/05/24

DALLE EQUAZ. "INFERIORI" DEL SIST. A BLOCCHI POSSIAMO DESUMERE

$$\underset{\sim}{K}_{BA}\,\underline{q}_A + \underset{\sim}{K}_{BB}\,\underline{q}_B = \underline{0} \quad ; \quad \boxed{\underline{q}_B = -\underset{\sim}{K}_{BB}^{-1}\,\underset{\sim}{K}_{BA}\,\underline{q}_A} \quad (*)$$

LA PARTE "SUPERIORE" DEL SIST. DIVENTA

$$\underset{\sim}{M}_A\,\ddot{\underline{q}}_A + \underset{\sim}{K}_{AA}\,\underline{q}_A + \underset{\sim}{K}_{AB}\,\underline{q}_B = \underline{F}_A(t) \implies \underset{\sim}{M}_A\,\ddot{\underline{q}}_A + \underbrace{\left[\underset{\sim}{K}_{AA} - \underset{\sim}{K}_{AB}\,\underset{\sim}{K}_{BB}^{-1}\,\underset{\sim}{K}_{BA}\right]}_{\underset{\sim}{K}_A}\underline{q}_A = \underline{F}_A(t)$$

$\underset{\sim}{K}_A$ MAT. RIGIDEZZA CONDENSATA DEI G.D.L. $\underline{q}_A$

$$\underset{\sim}{M}_A\,\ddot{\underline{q}}_A + \underset{\sim}{K}_A\,\underline{q}_A = \underline{F}_A(t) \implies \boxed{\underline{q}_A(t)}$$

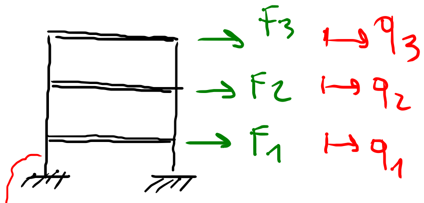
ATTRAVERSO (*) ottengo poi $\underline{q}_B(t)$.

DEFINITEZZA POSITIVA MATRICE DI RIGIDEZZA $\underset{\sim}{K}$

$$\underset{\sim}{K}\,\underline{q}\cdot\underline{q} > 0 \quad \text{per } \underline{q} \neq \underline{0}$$

$$\underset{\sim}{K}\,\underline{q}\cdot\underline{q} = 0 \quad \text{sse } \underline{q} = \underline{0}$$

DEFINIZIONE DI DEFINIT. POSITIVA



MAT. ELASTICO LINEARE

PER IL TH. DI CLAPEYRON, L'ENERGIA ELASTICA IMMAGAZZ. NELLA STRUTTURA VALE

$$\underset{\sim}{K}\,\underline{q} = \underline{F}$$

$$\Phi = \frac{1}{2}\left(F_1 q_1 + F_2 q_2 + F_3 q_3\right) = \frac{1}{2}\,\underline{F}\cdot\underline{q} > 0 \quad (\underline{q} \neq \underline{0})$$

$$\| $$

$$\frac{1}{2}\,\underset{\sim}{K}\,\underline{q}\cdot\underline{q} > 0 \quad (\underline{q} \neq \underline{0})$$

$$\text{se } \underline{q} = \underline{0} \Rightarrow \Phi = 0 \rightarrow \underset{\sim}{K}\,\underline{q}\cdot\underline{q} = 0 \quad (\underline{q} = \underline{0})$$

LA SIMMETRIA DI $\underset{\sim}{K}$ SI DIMOSTRA CON IL TH. DI BETTI $\Rightarrow \underset{\sim}{K} = \underset{\sim}{K}^T$

NOTA SUL METODO OPERATIVO PER CALCOLARE K_{ij} (OVVERO $q_1 = 1, q_2 = q_3 = \ldots 0$ ecc)

$$\left.\begin{aligned} K_{11} q_1 + K_{12} q_2 &= F_1 \\ K_{21} q_1 + K_{22} q_2 &= F_2 \end{aligned}\right\} \text{SIST. } 2\times 2$$

COSA SUCCEDE AL SIST. QUANDO $q_1 = 1, q_2 = 0$:

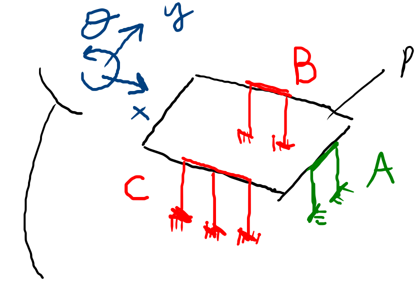
$K_{11} 1 + K_{12} \cdot \phi = F_1 \Rightarrow K_{11} = F_1$: FORZA APPLICATA AL GDL q_1 AL FINE DI AVERE $q_1 = 1$

$K_{21} \cdot 1 + K_{22} \cdot \phi = F_2 \Rightarrow K_{21} = F_2$: " " " " q_2 AL FINE DI AVERE $q_2 = 0$

(REAZ. VINCOLARE DEL VINCOLO AUSILIARIO CHE "BLOCCA" q_2)

STESSA COSA $q_1 = 0, q_2 = 1$

ESEMPIO DI CALCOLO DI UNA MATRICE DI RIGIDEZZA IN UN PROBL. 3D.

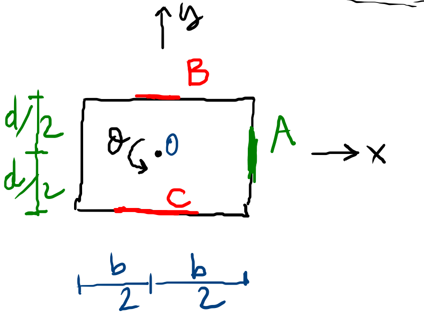


PIANO RIGIDO

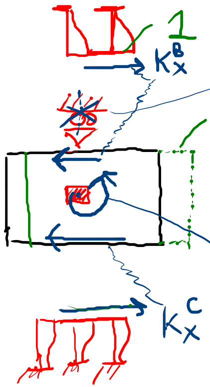
A, B, C: telai noti elastici

3 G.D.L. PIANO RIGIDO

$$\underset{\sim}{K} = \begin{bmatrix} K_{xx} & K_{xy} & K_{x\theta} \\ & K_{yy} & K_{y\theta} \\ SYM. & & K_{\theta\theta} \end{bmatrix}$$



$\boxed{x=1,\ y=\theta=0}$



$K_{yx}=0$

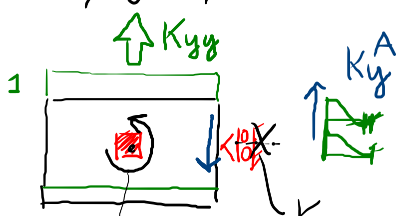
$K_{xx} = K_x^C + K_x^B$

$K_{\theta x}$: MOMENTO AL MORSETTO $\left(K_x^C \frac{d}{2} - K_x^B \frac{d}{2}\right) = \left(K_x^C - K_x^B\right)\frac{d}{2}$ (CON SEGNO POSITIVO)

TRASCURO IL CONTRIBUTO ALLA RIGIDEZZA DEL PIANO DEBOLE DEI TELAI

$K_x^C > K_x^B$ (ENTRAMBI NOTI)

$x=0,\ y=1,\ \theta=0$



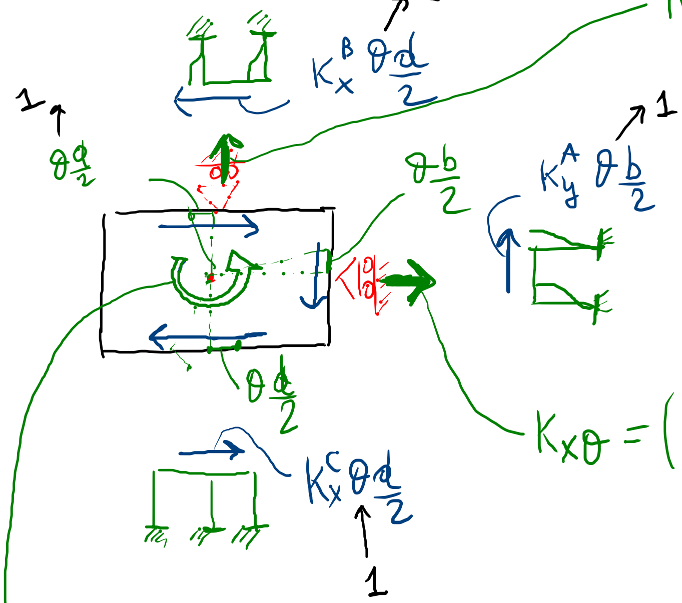
K_y^A (NOTO)

$K_{yy} = K_y^A$

$K_{xy}=0$

$K_{\theta y} = + K_y^A \frac{b}{2}$

$\boxed{x=y=0,\quad \theta=1}$



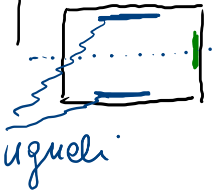
$$K_{y\theta} = K_y^A \frac{b}{2}$$

$$K_{x\theta} = \left(K_x^C - K_x^B\right)\frac{d}{2}$$

$$K_{\theta\theta} = K_x^C \frac{d}{2}\cdot\frac{d}{2} + K_x^B \frac{d}{2}\cdot\frac{d}{2} + K_y^A \frac{b}{2}\,\frac{b}{2}$$

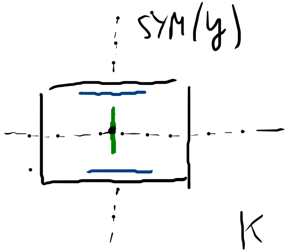
$$[\underset{\sim}{K}] = \begin{bmatrix} K_x^B + K_x^C & 0 & (K_x^C - K_x^B)\frac{d}{2} \\ 0 & K_y^A & K_y^A \frac{b}{2} \\ (K_x^C - K_x^B)\frac{d}{2} & K_y^A \frac{b}{2} & \boxed{} \end{bmatrix}$$

COSA SUCC. QUANDO $K_x^C = K_x^B$



SYMMETRIA (x)

$$\underset{\sim}{K} = \begin{bmatrix} x & 0 & 0 \\ 0 & x & x \\ 0 & x & x \end{bmatrix}$$



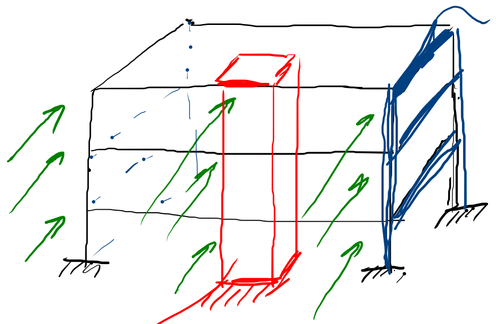
SYM(y)

SYM (x)

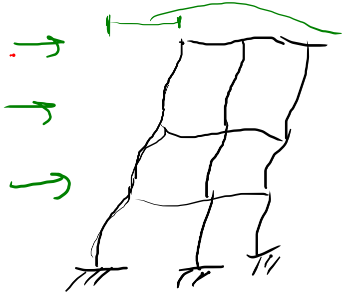
$$\underset{\sim}{K} = \begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}$$

$\underset{\sim}{K}$ DIAGONALE

PROBLEMA DI RIPARTIZ. DELLE AZIONI ORIZZONTALI TRA UN TELAIO E UNA MENSOLA IRRIGIDENTE (PROBLEMA DEL CONTROVENTO) continuaz. lez. 2/05



TELAIO RESISTENTE PER L'AZIONE ORIZZ. INDICATA



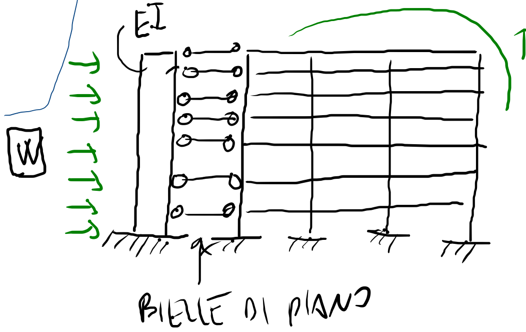
SPOST. MODERATAM. GRANDI PER AZIONI ORIZZ.

BLOCCO ASCENSORI (MENSOLA CHE SI DEFORMA A FLESSIONE): ASSORBE LE AZIONI ORIZZONTALI

NUCLEO DI CONTROVENTO

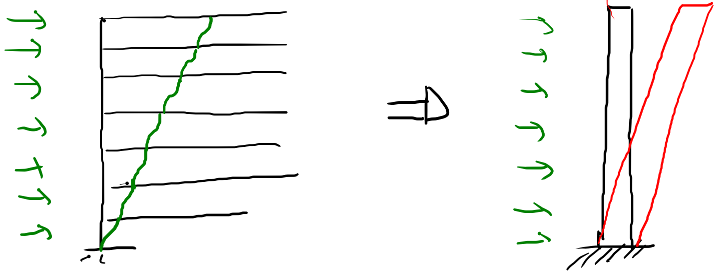
COME RIPARTISCO LE AZIONI ORIZZONTALI TRA TELAIO E MENSOLA IRRIGIDENTE?

PROBLEMA MODELLO PIANO

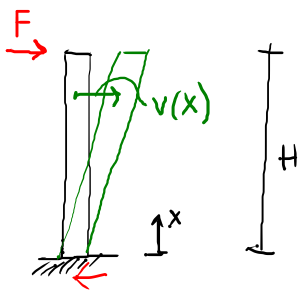


COME SI RIPARTISCONO LE FORZE $\boxed{W}$ TRA MENSOLA E TELAIO?

I PROBLEMA FONDAMENTALE: COMPORTAMENTO DEL TELAIO CON "MOLTI" PIANI SOTTO L'AZIONE DI FORZE ORIZZONTALI

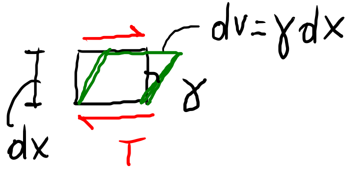


MENSOLA A DEFORMABILITÀ TAGLIANTE (GA_{eq})



T: cost $= +F$

CONCIO DI TRAVE



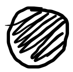
DALLA TEORIA DI S.V. SO CHE

$$\gamma = \frac{T}{GA_{eq}} = \frac{\kappa\, T}{GA}$$

A_{eq}

κ: FATTORE DI TAGLIO (>1)

$\kappa = \frac{6}{5}$ 

$= \frac{32}{27}$ 

$$dv = \frac{T}{GA_{eq}}\,dx \Rightarrow \frac{dv}{dx} = \frac{T(x)}{\underbrace{GA_{eq}(x)}_{cost}}$$

PER IL NOSTRO PROBLEMA

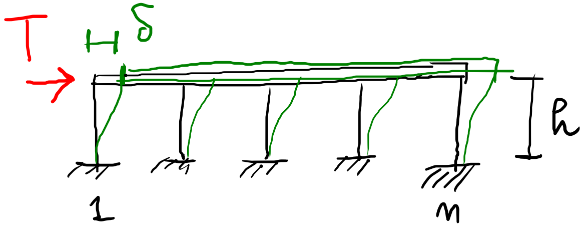
$T(x) = +F$

$$\begin{cases} \dfrac{dv}{dx} = \dfrac{F}{GA_{eq}} \\ v(0) = 0 \end{cases}$$

$\rightarrow v(x) = \frac{F}{GA_{eq}}x + C$

$\rightarrow C = 0$; $\boxed{v(x) = \frac{F}{GA_{eq}}x}$

IMPORTANTE: assegnato un telaio, quanto vale GA_{eq}?
(n pilastri)



$$\gamma = \frac{\delta}{h}$$

(SHEAR-TYPE)

$$\delta = \frac{T h^3}{12 \sum_{i=1}^{n} EI_i}$$

T: TAGLIO DEL PIANO CHE STO CONSIDERANDO

$$\gamma = \frac{\delta}{h} = T \frac{h^2}{12 \sum_{i=1}^{n} EI_i} \qquad \frac{1}{GA_{eq}}$$

RIGIDEZZA TRAVI

$$\frac{\delta}{h} = T\, f(h, EI_i, EJ, \ldots) \qquad \frac{1}{GA_{eq}}$$