Recovering signals out of noise: the lock-in amplifier

Laboratorio di Fisica della Materia Condensata, a.a. 2023/24

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Noise is ubiquitous in experiments!

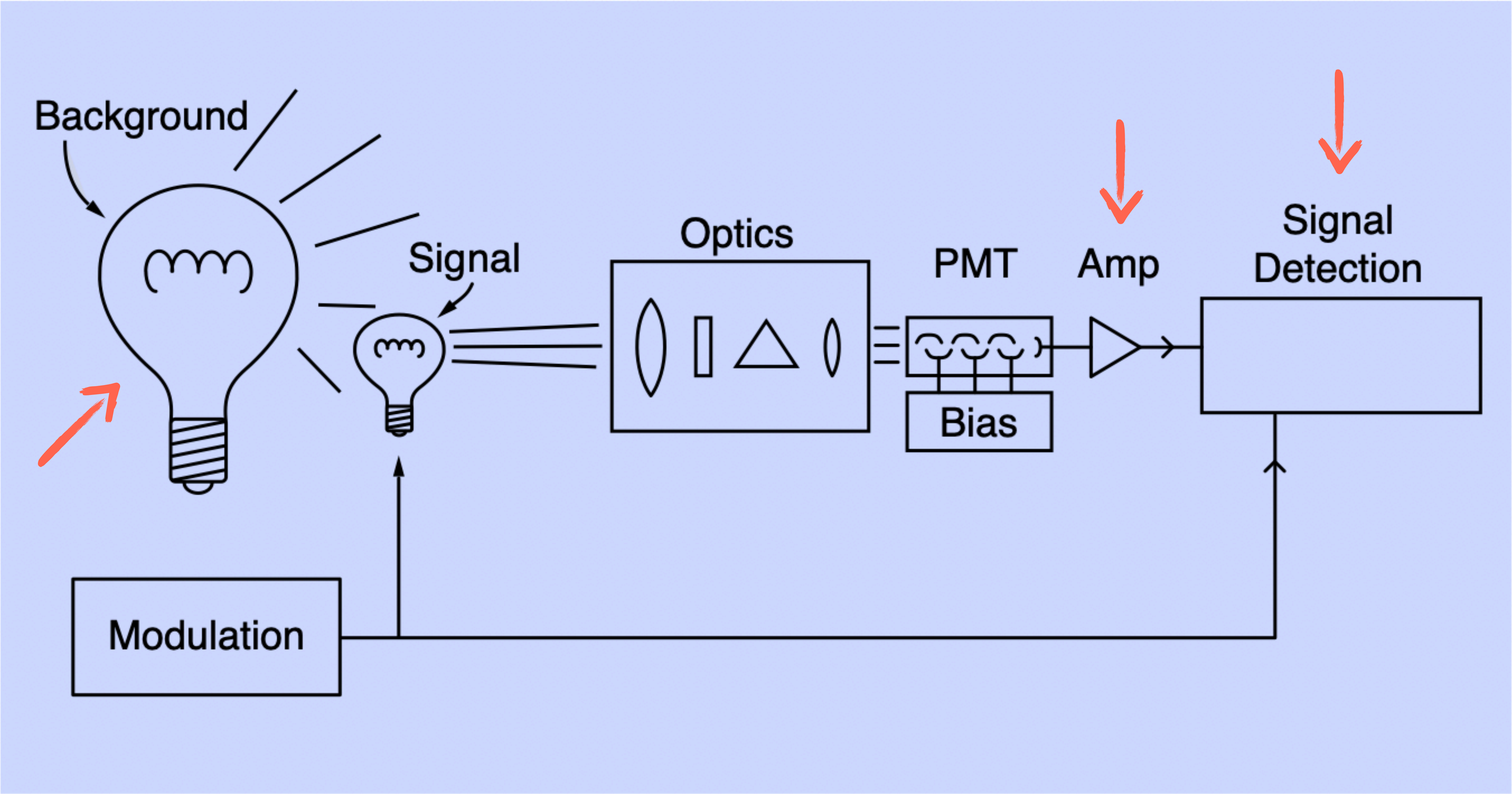
* Almost all measurements are done using electronic equipment. This means that experimental techniques rely on the **quantitative measurement of electrons** (voltages, currents, charge etc.)

  → E.g. measurement of a DC anode current by an amperemeter, charge counting in the pixels of a charge-coupled device (CCD) camera chip

* Any electronic signal always shows random, uncorrelated fluctuations: **noise**

* The signal of interest may be **obscured by noise**! The noise may be fundamental to the process: e.g. discrete charges (as well as discrete light quanta, i.e. photons) are governed by Poisson statistics which gives rise to shot noise.

Prototype experiment



Two types of noise

* Sometimes noise is **extrinsic** and **“non-essential”**: it can be minimised by good laboratory practice

* Often noise sources are **intrinsic**, related to the physics (and the statistics) of the system and the probe used in the measurements: this cannot be acted upon

  Intrinsic sources of noise:

  - Shot noise
  - Johnson-Nyqist noise
  - Flicker noise (aka 1/f noise)

* Understanding the noise sources in a measurement is critical to a achieving a satisfactory signal-to-noise performance! ➙ The quality of a measurement may be substantially degraded by a trivial error…

Spectral noise density

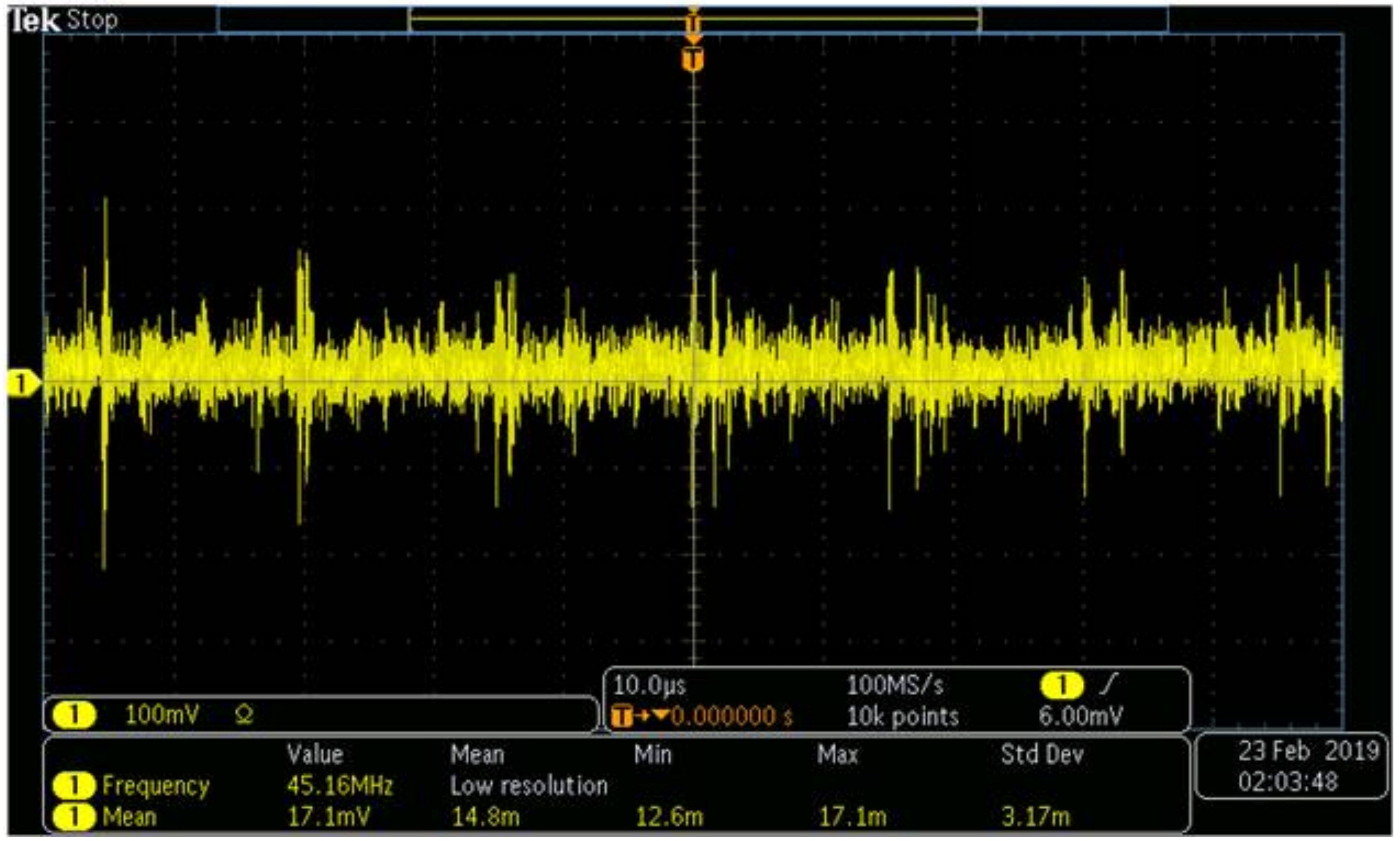
* Noise characteristics of a system are often represented by the noise spectral density (PSD)

* Let's assume a quantity of interest $X(t)$, the noise PSD is defined as the squared modulus of the Fourier transform of $X(t)$

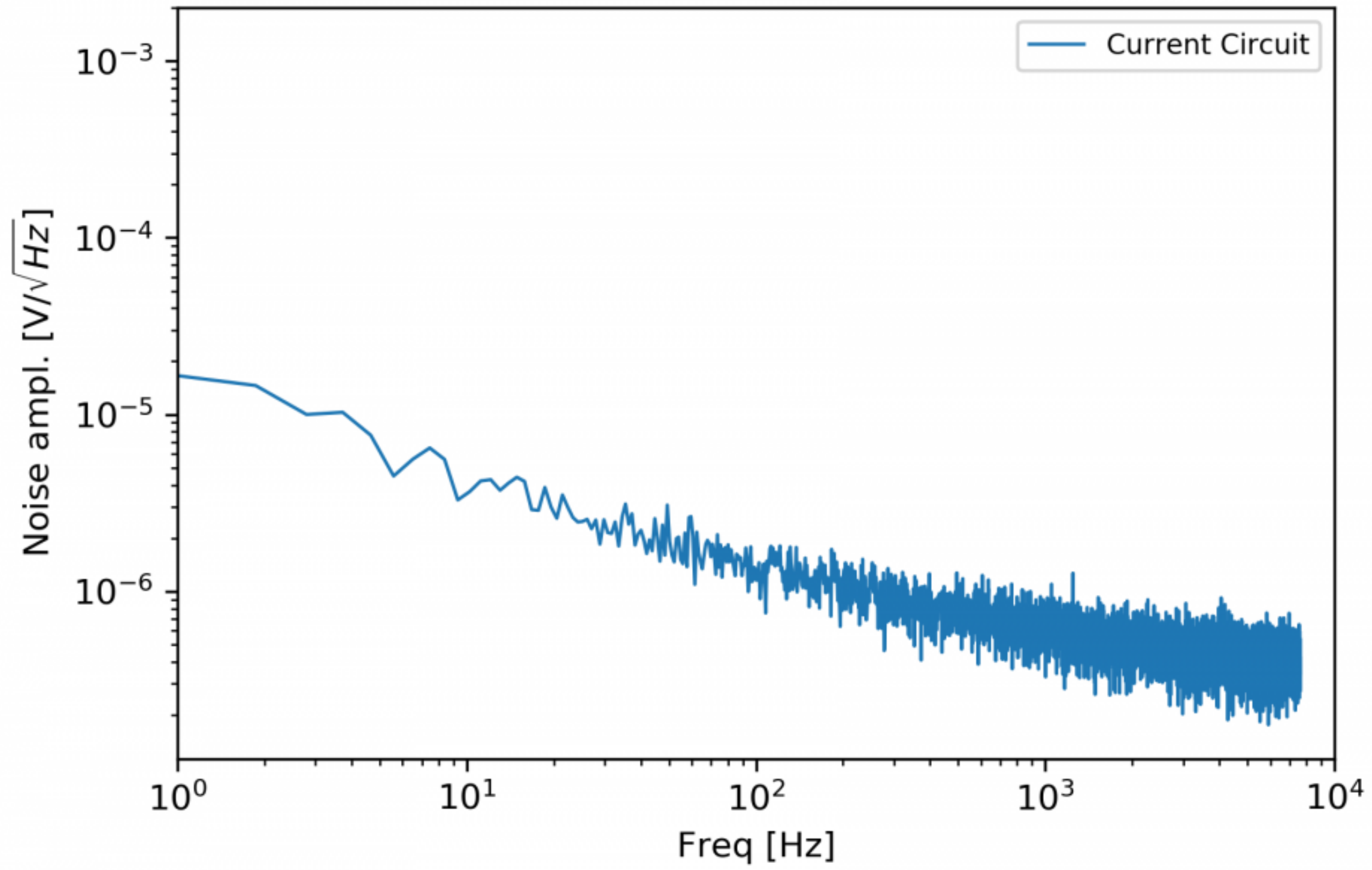
$$S_X(f) = \lim_{T\to\infty} \frac{1}{T} \left\langle \left| \int_{-T/2}^{+T/2} X(t)\, e^{+i2\pi f t}\, \mathrm{d}t \right|^2 \right\rangle$$

* PSDs are statistical measures: they can be estimated from real data by averaging over many measurements ➞Taking a single measurement trace gives only a very rough estimate of the PSD

Noise PSD in practice…



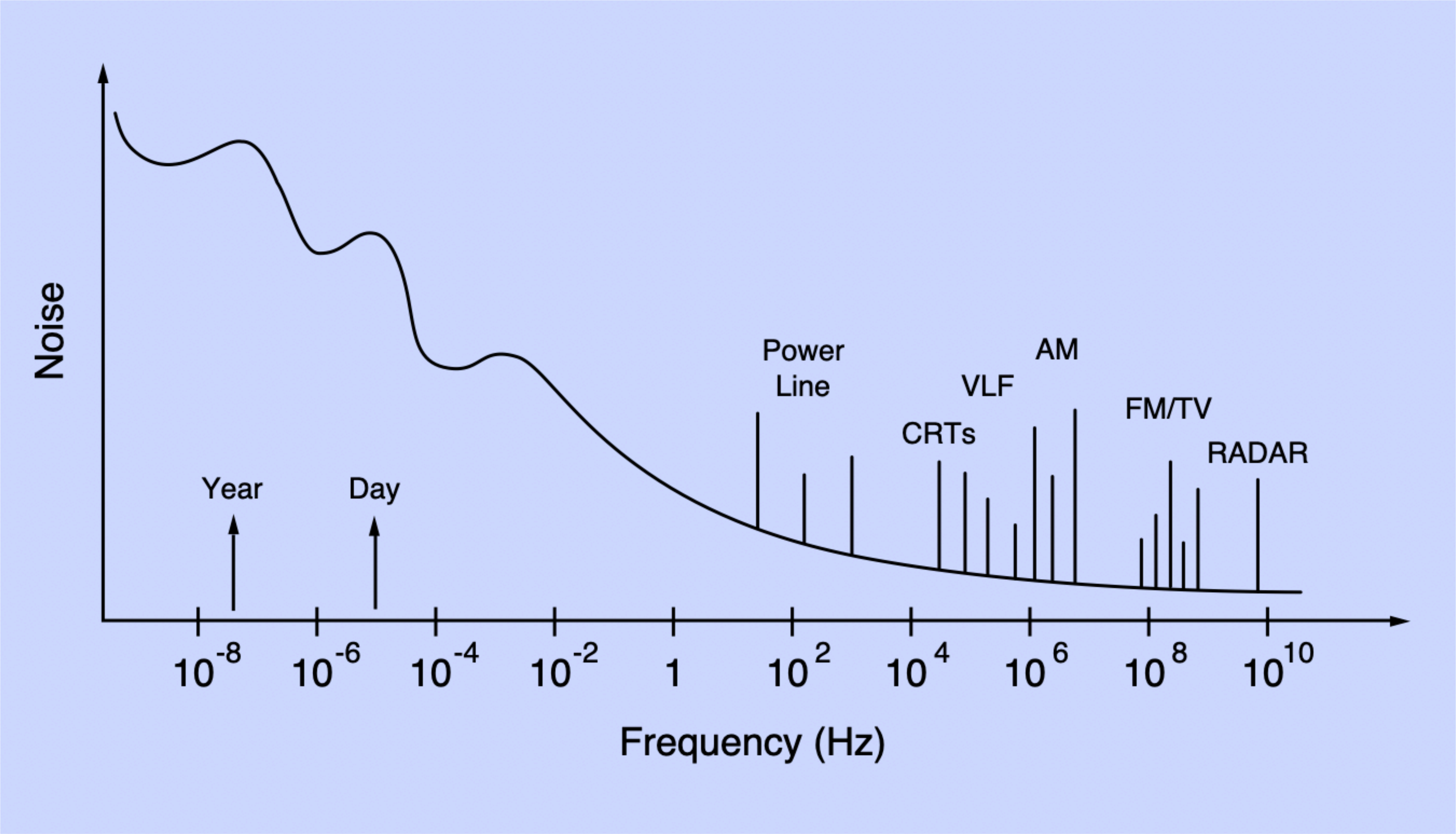
FT



Remove signal mean,
only fluctuations around the mean

An RMS is obtained by integrating the PSD over a chosen frequency window

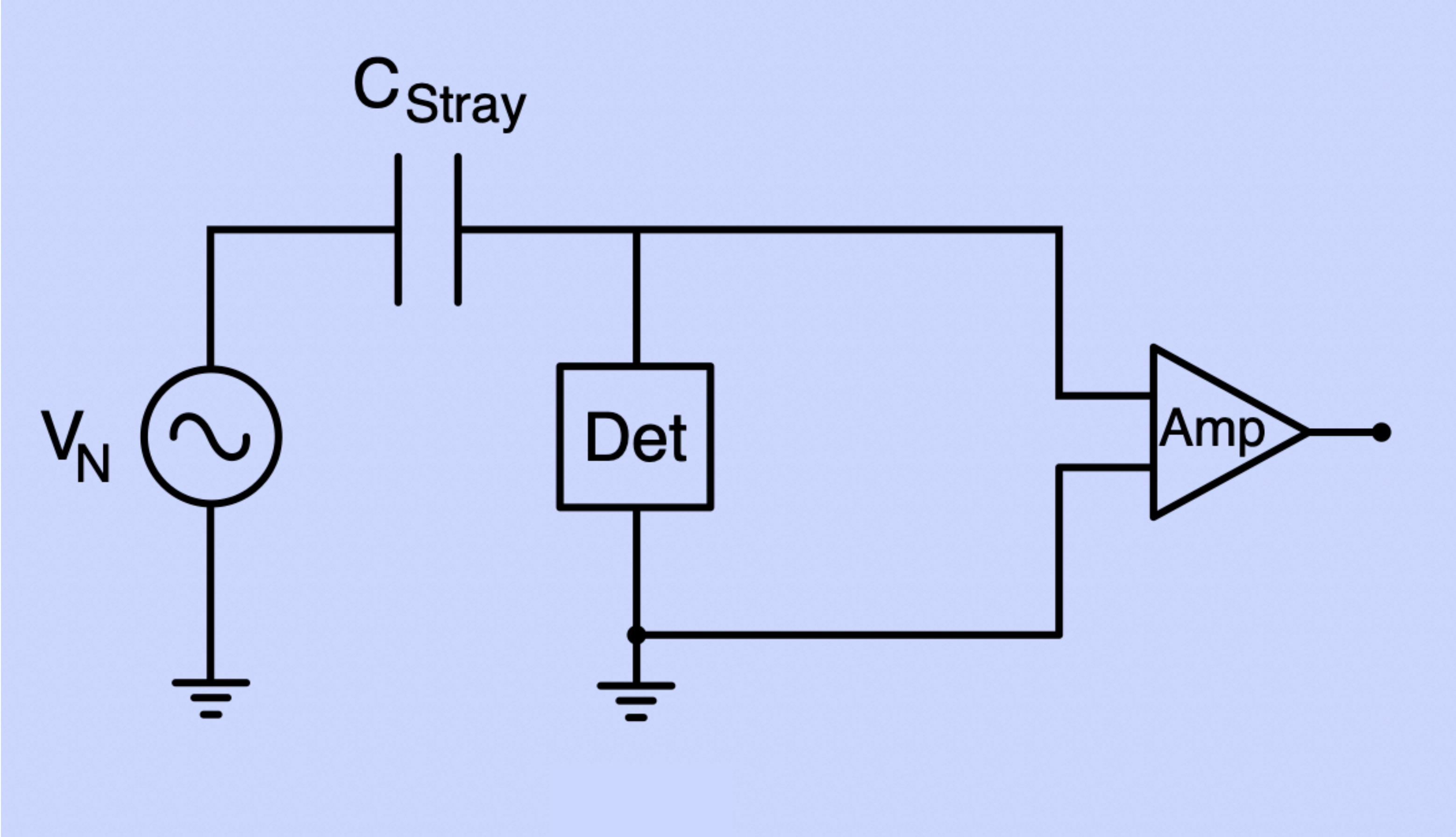
Examples of noise sources



Examples of extrinsic noise sources

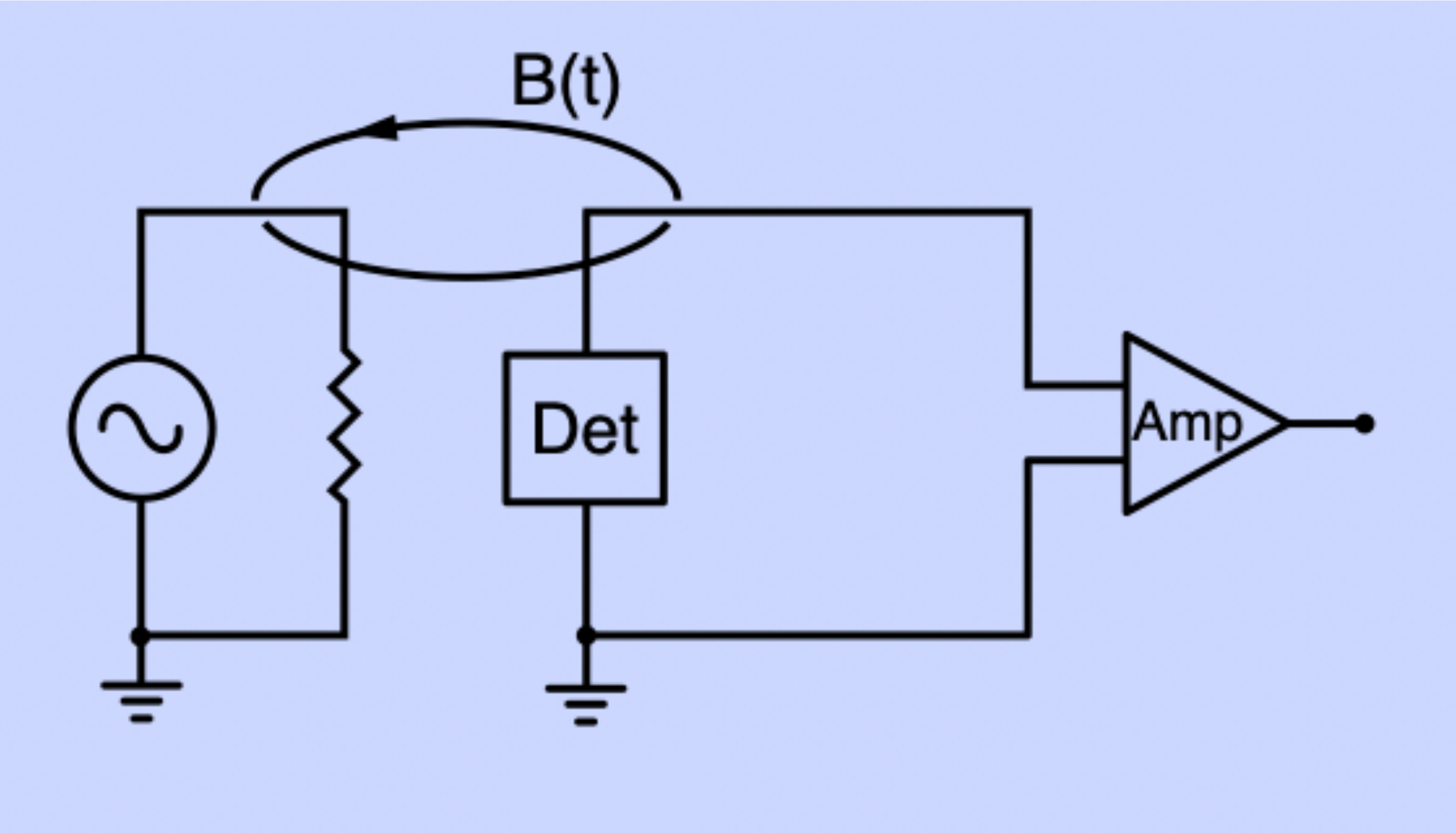
Capacitive and inductive couplings

- Noise can be picked up through the **capacitive coupling** with a nearby apparatus with varying voltage



Cure: shielding the detector

- Noise can be picked up through the **inductive coupling** to a time-varying magnetic field, which induces a e.m.f. in the detection circuit

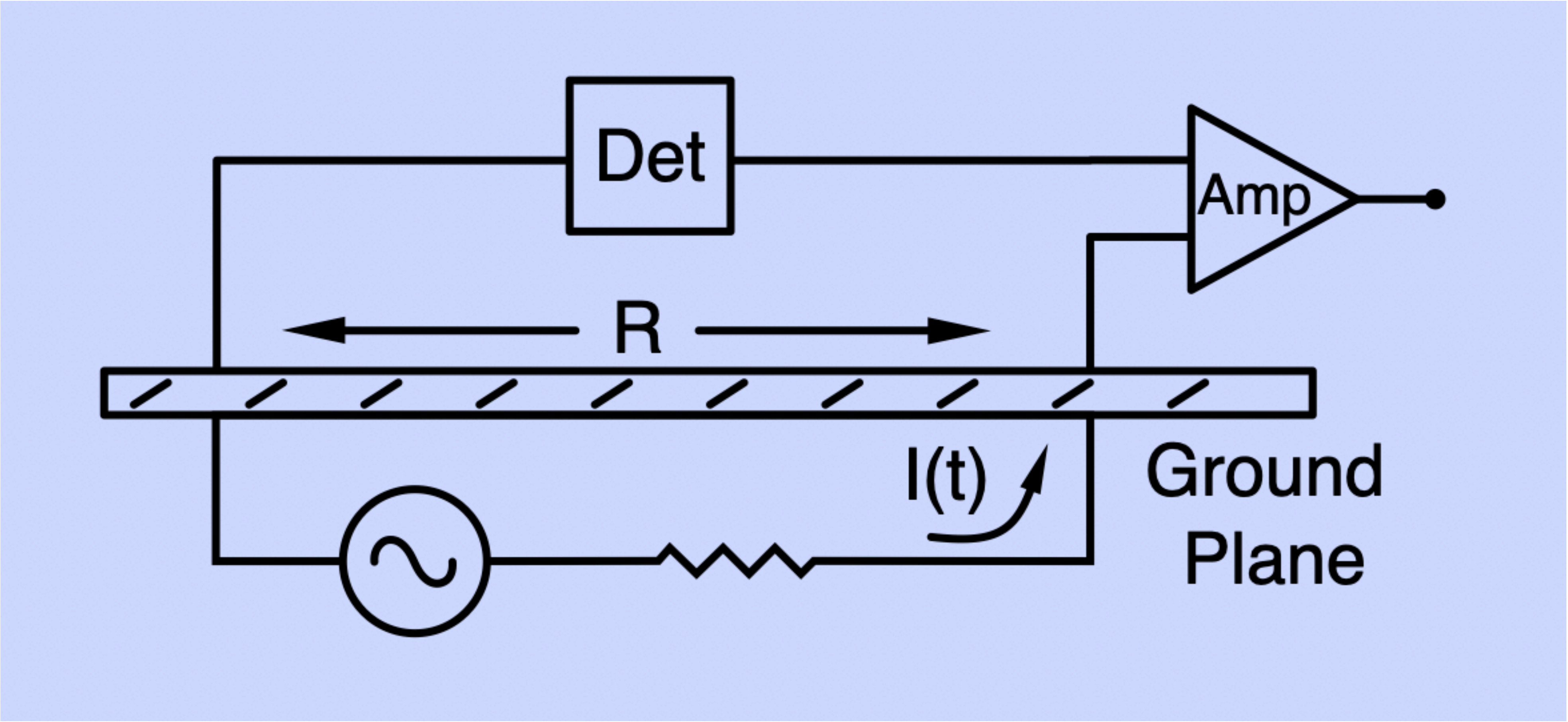


Cure: use twisted pairs or coaxial cables

Examples of extrinsic noise sources

Resistive couplings: ground loops

* Currents through common connections can give rise to noisy voltages. E.g. the detector can be contaminated by the noise on the ground bus.

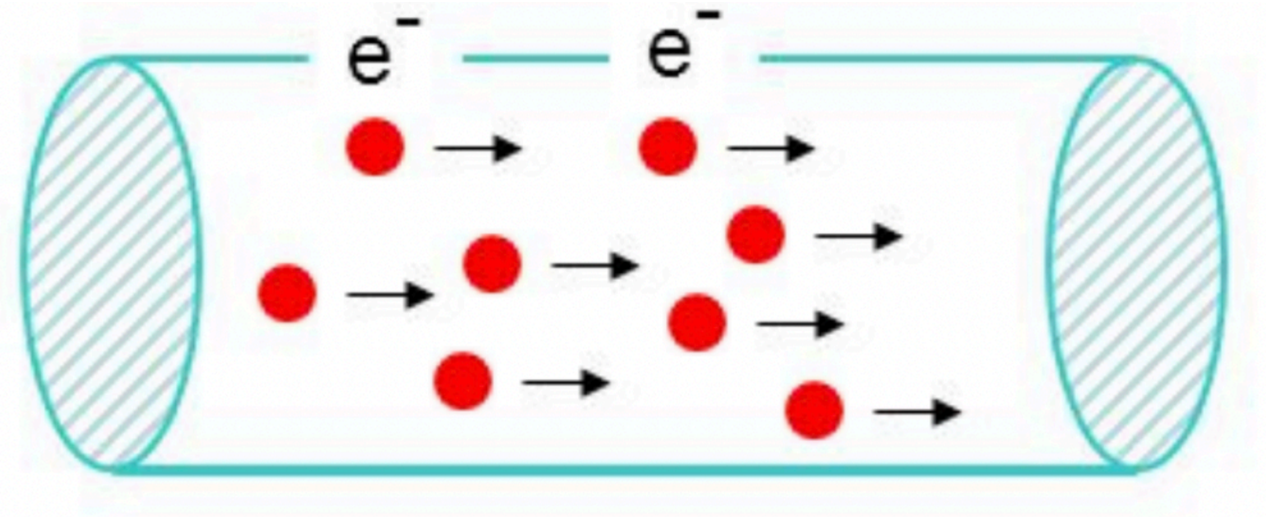


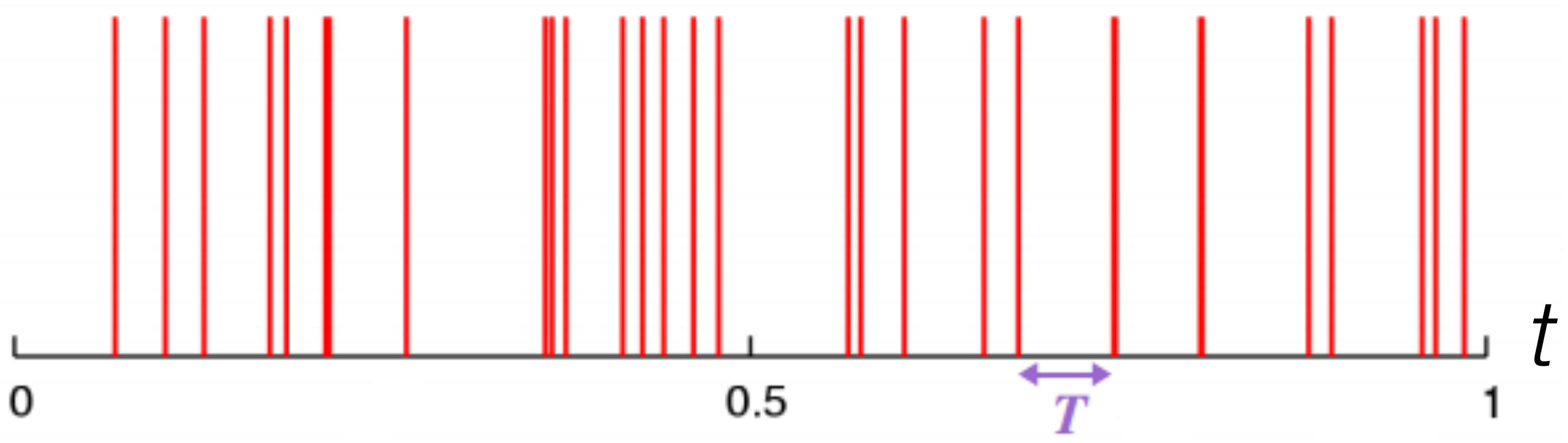
Cure: ground all instruments to the same point, remove sources of large currents from ground wires used for small signals!

Examples of intrinsic noise sources

Shot noise

- Light and electrical charge are quantized: so the number of photons or electrons which pass a point during a period of time are subject to Possonian statistical fluctuations.





→ if the signal mean is M photons, the standard deviation (noise) will be $\sqrt{M}$, hence the S/N $= M/\sqrt{M} = \sqrt{M}$.

- M may be increased by increasing the photon rate (laser power) or increasing the integration time.

Examples of intrinsic noise sources

Johnson-Nyqist noise (or thermal noise)

* In a conductor there are a large number of moving electrons. Point-by-point their density shows statistical fluctuations at finite temperature as a function of time, like the local density of air in a given point of a room. These density fluctuations give rise to voltage fluctuations:

$$V_{\mathrm{JN,rms}} = 4k_B RT\,\Delta f$$

where R is resistance of the conductor, k_B is Boltzmann's constant, T is the temperature, and Δf is the bandwidth over which the noise is measured.

* This is a *white* noise, since its PSD does not depend on the frequency, i.e. this noise contains Fourier components at any frequency. Example: for 1MΩ resistor the J-N noise is $V_{\mathrm{JN,rms}} \simeq 100\mu\mathrm{V}$ between 0 and 100 kHz

Examples of intrinsic noise sources

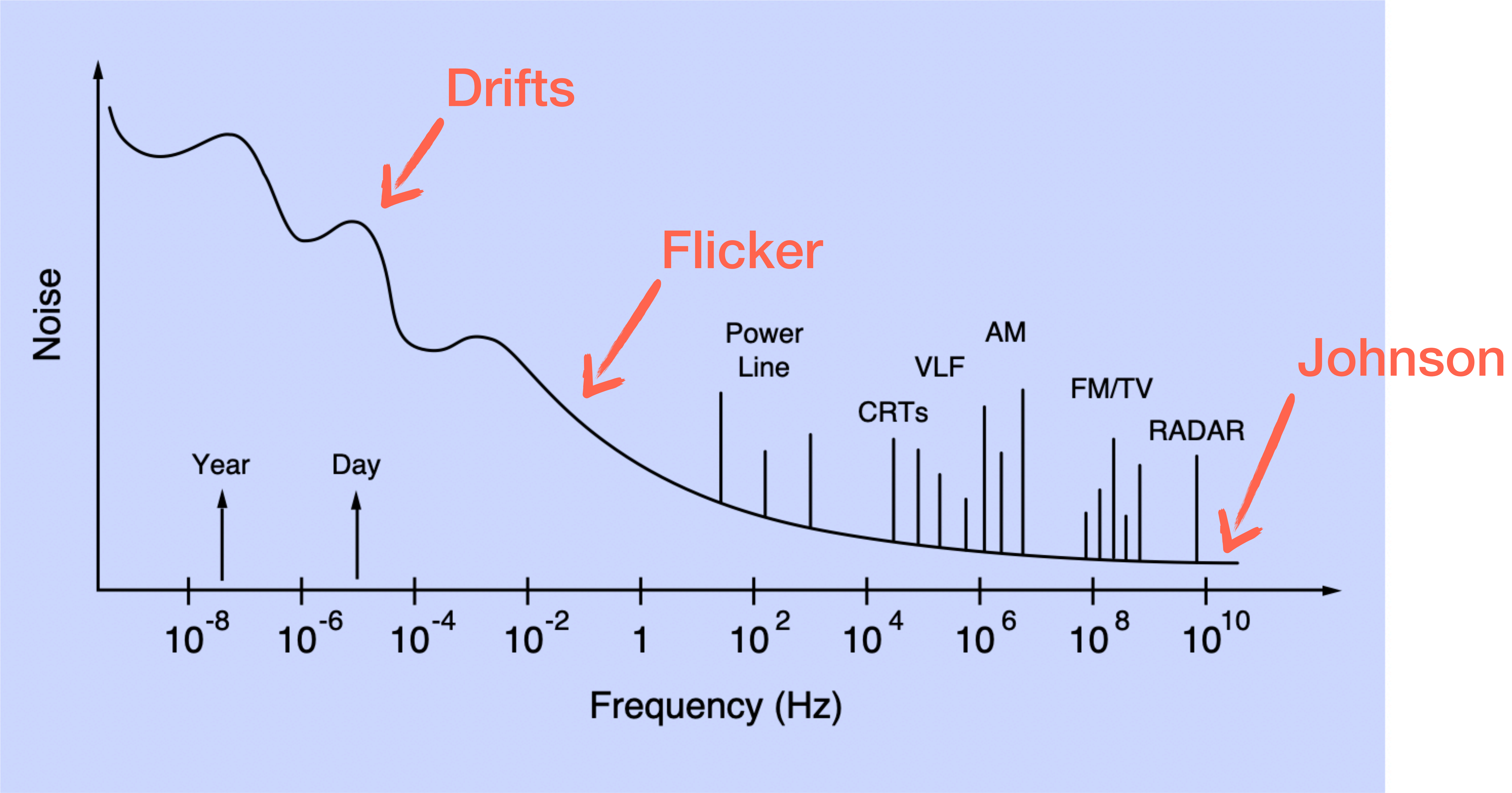
Flicker noise (or 1/f noise)

* The voltage across a resistor carrying a constant current will fluctuate because the resistance of the material used in the resistor fluctuates, giving rise to a frequency-dependent noise. However, there there is no general accepted theory that explains it in all the cases where the 1/f noise is present. It occurs in almost all electronic devices:

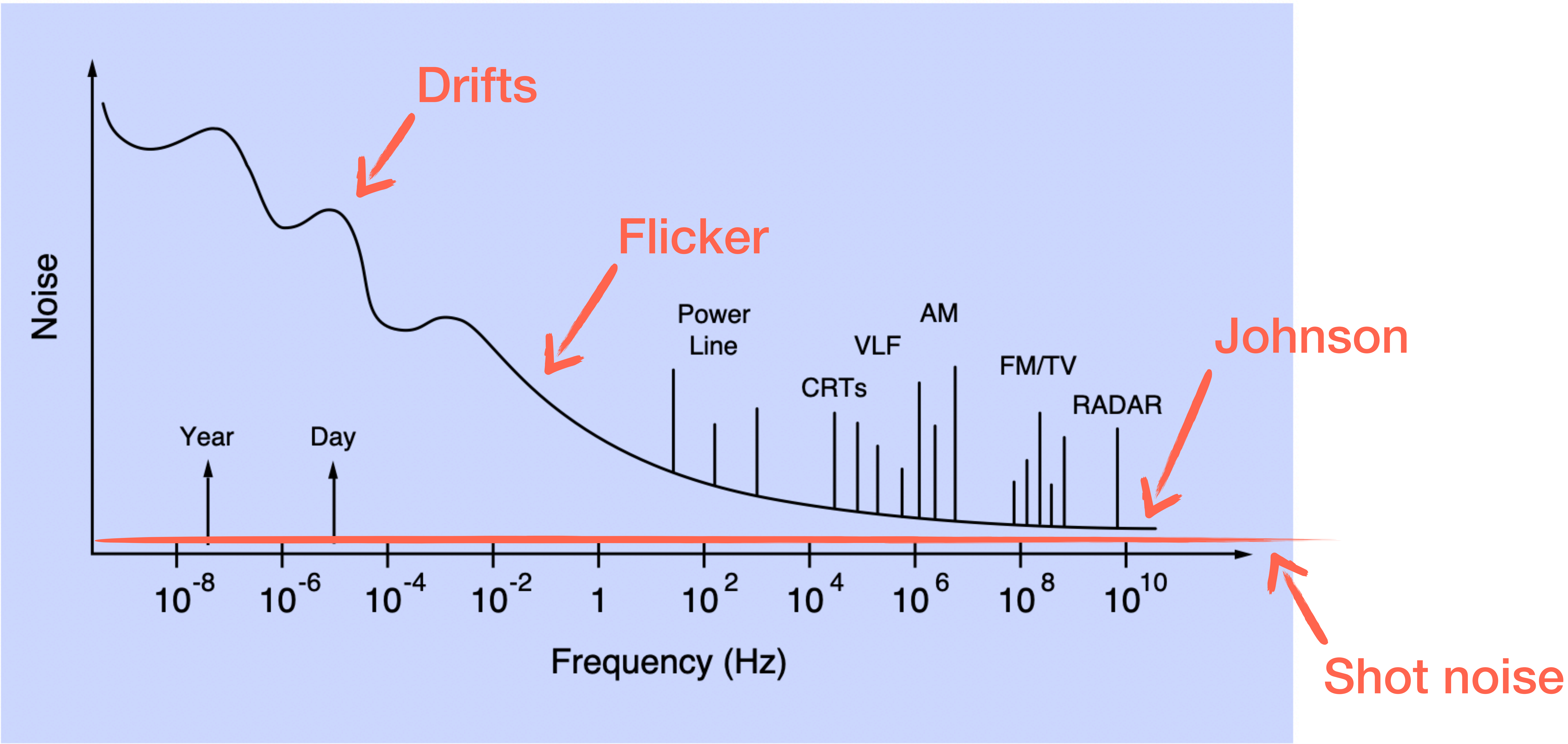
$$V^2_{\mathrm{pink,rms}} \propto \Delta f / f$$

* This is a *pink* noise, and it will impact mostly the low-frequency part of the noise spectral density

Examples of noise sources



Examples of noise sources



Measuring small signals: amplifiers

* Several considerations are involved in choosing the correct amplifier for a particular application: bandwidth, gain, impedance, noise characteristics…

* General technique: perform AC measurements to avoid noise close to DC frequencies ➙ Modulate the source and analyse at the modulation frequency

* When the source is modulated, one may choose from gated integrators, boxcar averagers, transient digitizers, lock-in amplifiers, spectrum analyzers…We will only use the **lock-in amplifier**

➙ Lock-in amplifiers are used to detect and measure very small AC signals… all the way down to a few nV!

Let's see an example…

- Suppose the signal is a 10 nV sine wave at 10 kHz. Clearly some amplification is required to bring the signal above the noise. A very good low-noise amplifier will have about 5 nV/√Hz of input noise.

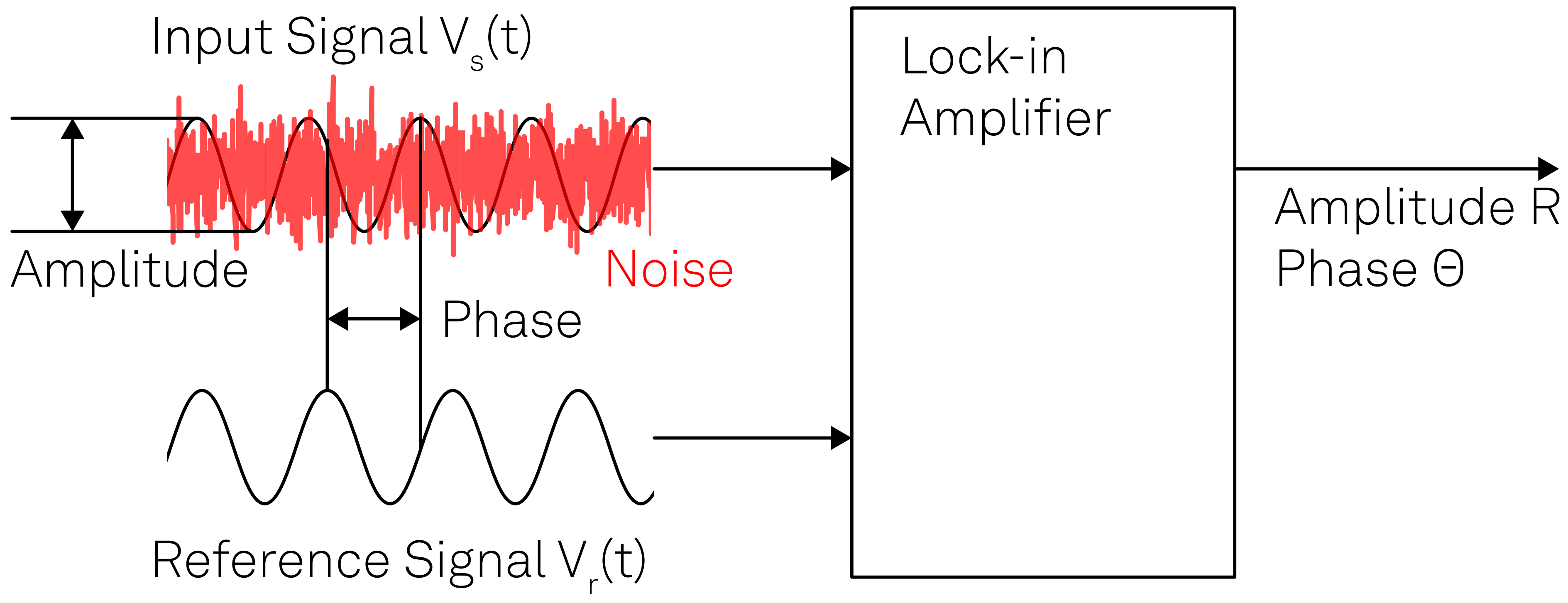
- If the amplifier bandwidth is 100 kHz and the gain is 1000, we can expect our output to be 10 μV of signal (10 nV × 1000) and 1.6 mV of broadband noise (5 nV/√Hz × √100 kHz × 1000) ☹️

- Supposing we know the frequency of our signal, we follow the amplifier with a very good band-pass filter with a Q=100 centered at 10 kHz. Any signal in a 100 Hz bandwidth will be detected (10 kHz/Q). The noise in the filter pass band will be 50μV (5 nV/√Hz × √100Hz ×1000), and the signal will still be 10 μV.
  - ➝ The output noise is much greater than the signal! ☹️

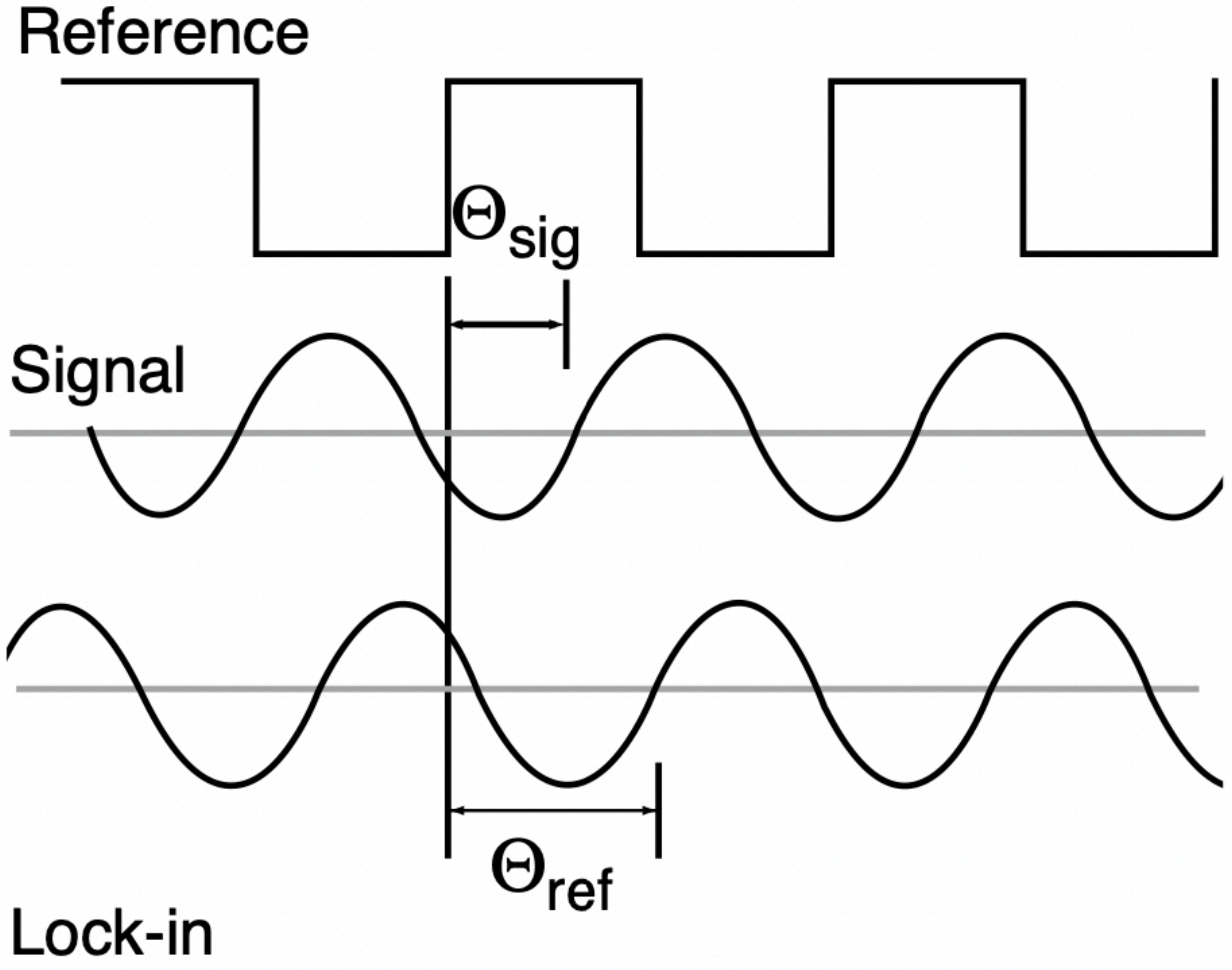
Phase-sensitive detection

* An amplifier with a phase-sensitive detector can detect the signal at 10 kHz with a bandwidth as narrow as 0.01 Hz: in our previous example, the noise in the detection bandwidth will be 0.5 μV (5 nV/√Hz × √0.01 Hz × 1000), while the signal is still 10 μV! 😀

* How to achieve this? Lock-in measurements require a **frequency reference**. Typically, an experiment is excited at a fixed frequency, and the lock-in detects the response from the experiment only at the reference frequency.



Phase-sensitive detection



$$V_{sig}\sin(\omega_r t + \theta_{sig})$$

$$V_L\sin(\omega_L t + \theta_{ref})$$

Phase-sensitive detection

- The lock-in amplifies the signal and then multiplies it by the lock-in reference using a phase-sensitive multiplier (a mixer). The output is simply the product of two sine waves:

$$V_{out} = V_{sig}V_L\sin(\omega_r t + \theta_{sig})\sin(\omega_L t + \theta_{ref})$$

$$= \tfrac{1}{2}V_{sig}V_L\cos([\omega_r - \omega_L]t + \theta_{sig} - \theta_{ref}) - \tfrac{1}{2}V_{sig}V_L\cos([\omega_r + \omega_L]t + \theta_{sig} + \theta_{ref})$$

- The output is composed by two AC signals, one at the difference-frequency $(\omega_r - \omega_L)$ and the other at the sum-frequency $(\omega_r + \omega_L)$. If the output is further passed through a low-pass filter, the AC signals will be removed.
- What will be left? In the general case, nothing. However…

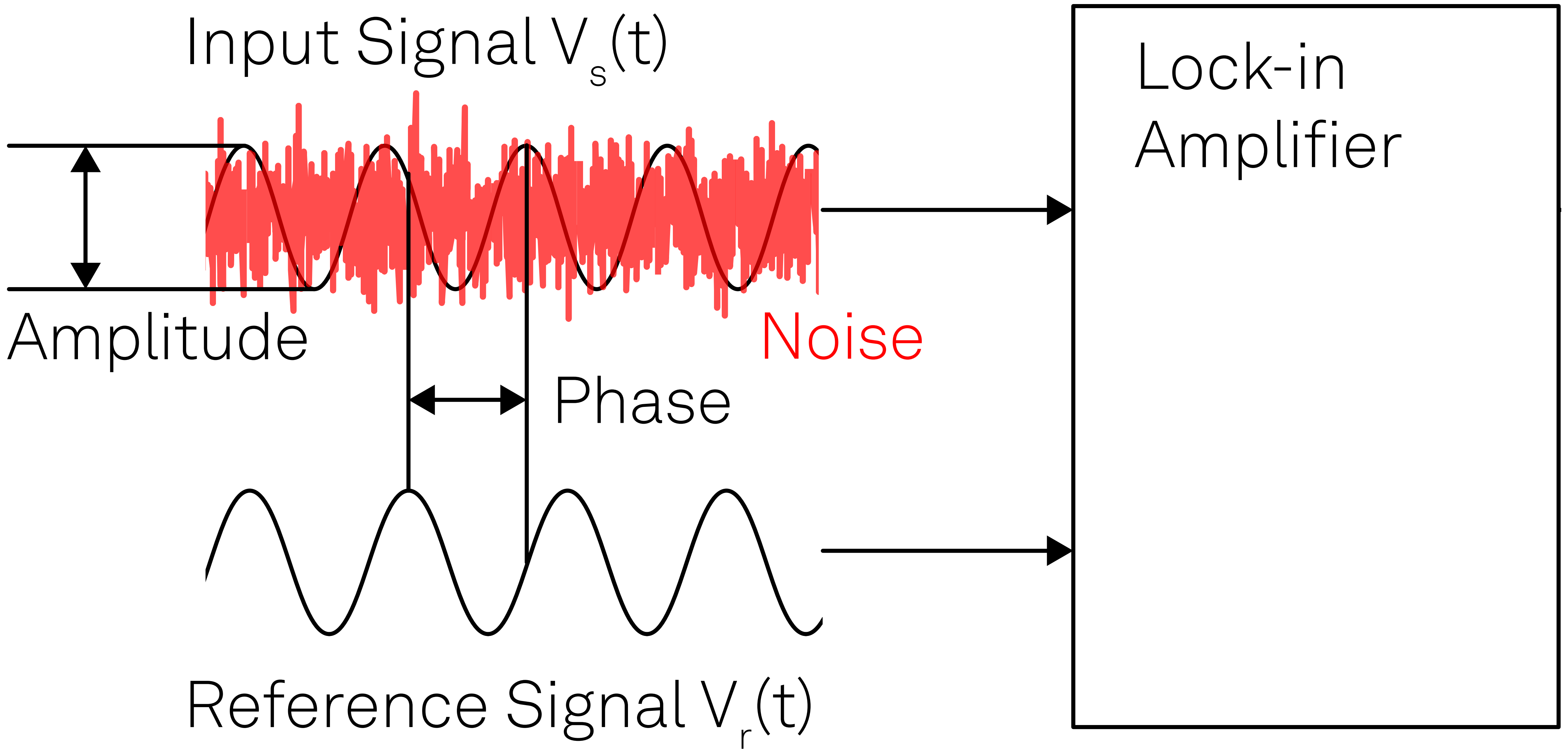
Lock-in amplification

* However, if ω_L equals ω_r, the difference-frequency component will be a DC signal! In this case, the filtered output will be:

$$V_{out} = \tfrac{1}{2} V_{sig} V_L \cos(\theta_{sig} - \theta_{ref})$$

* This is a very nice output — it is a **DC signal** proportional to the signal amplitude! We have converted the signal at the modulation frequency ω_L into a DC signal, while signals at any other frequency are attenuated by the filtering.

Lock-in amplification



- Let's now take the input signal as composed of signal + noise
- The lock-in and the low-pass filter only detect signals whose frequencies are very close to the lock-in reference frequency
- Noise signals, at frequencies far from the reference, are attenuated by the low pass filter, since $\omega_{\text{noise}} - \omega_r$ and $\omega_{\text{noise}} + \omega_r$ are not close to 0.

- A narrower filter will remove noise sources very close to the reference frequency; a wider bandwidth allows some signals to pass → The low-pass filter bandwidth determines the remaining noise, at the expenses of longer integration

The phase

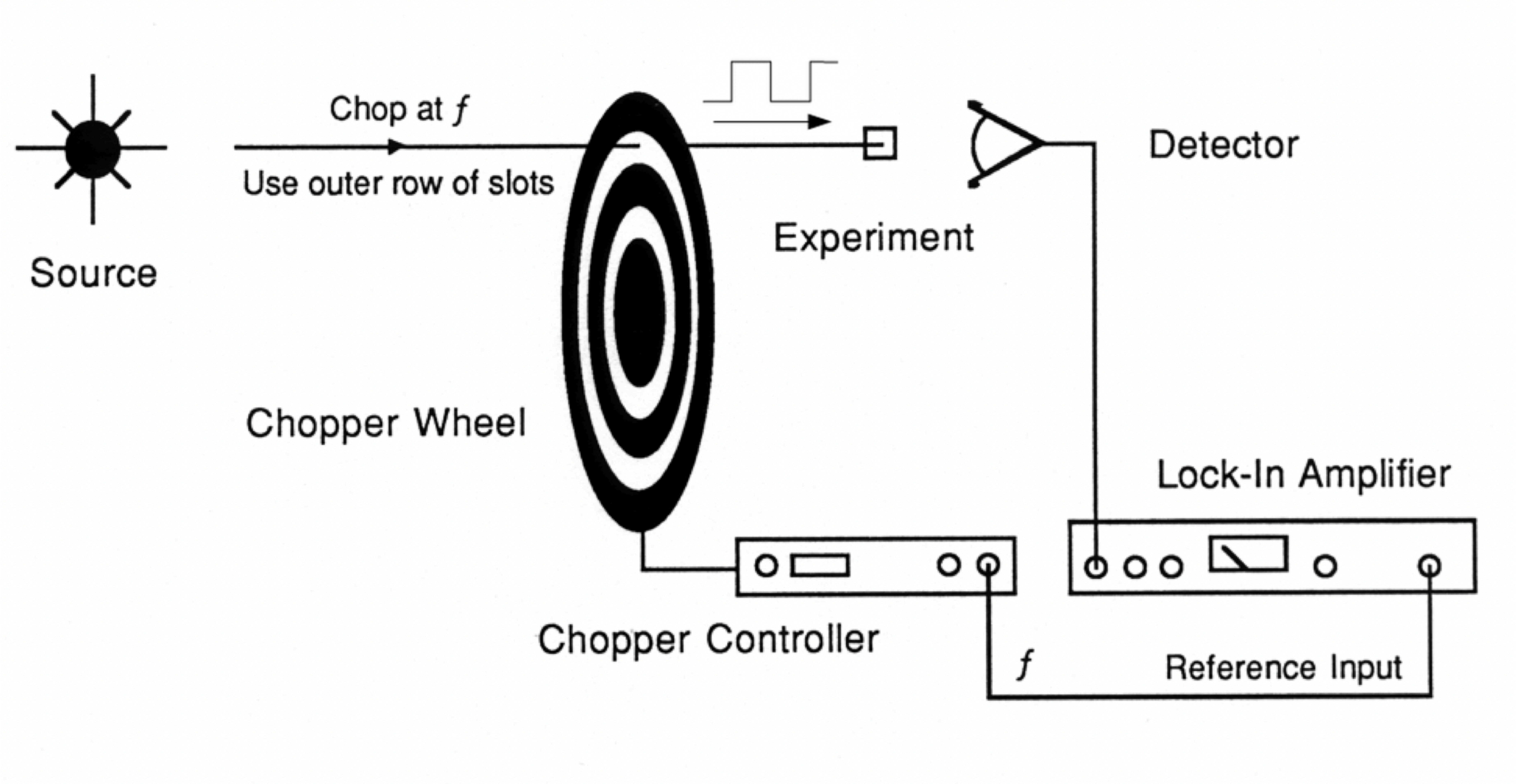
* We need to make the lock-in reference the same as the signal frequency, i.e. $\omega_L = \omega_r$ Not only the frequencies need to be the same, also **the phase between the signals can not change over time**. Otherwise, $\cos(\theta_{\text{sig}} - \theta_{\text{ref}})$ will change and the output will not be a DC signal.

  → In other words, the lock-in reference needs to be phase-locked to the signal reference.

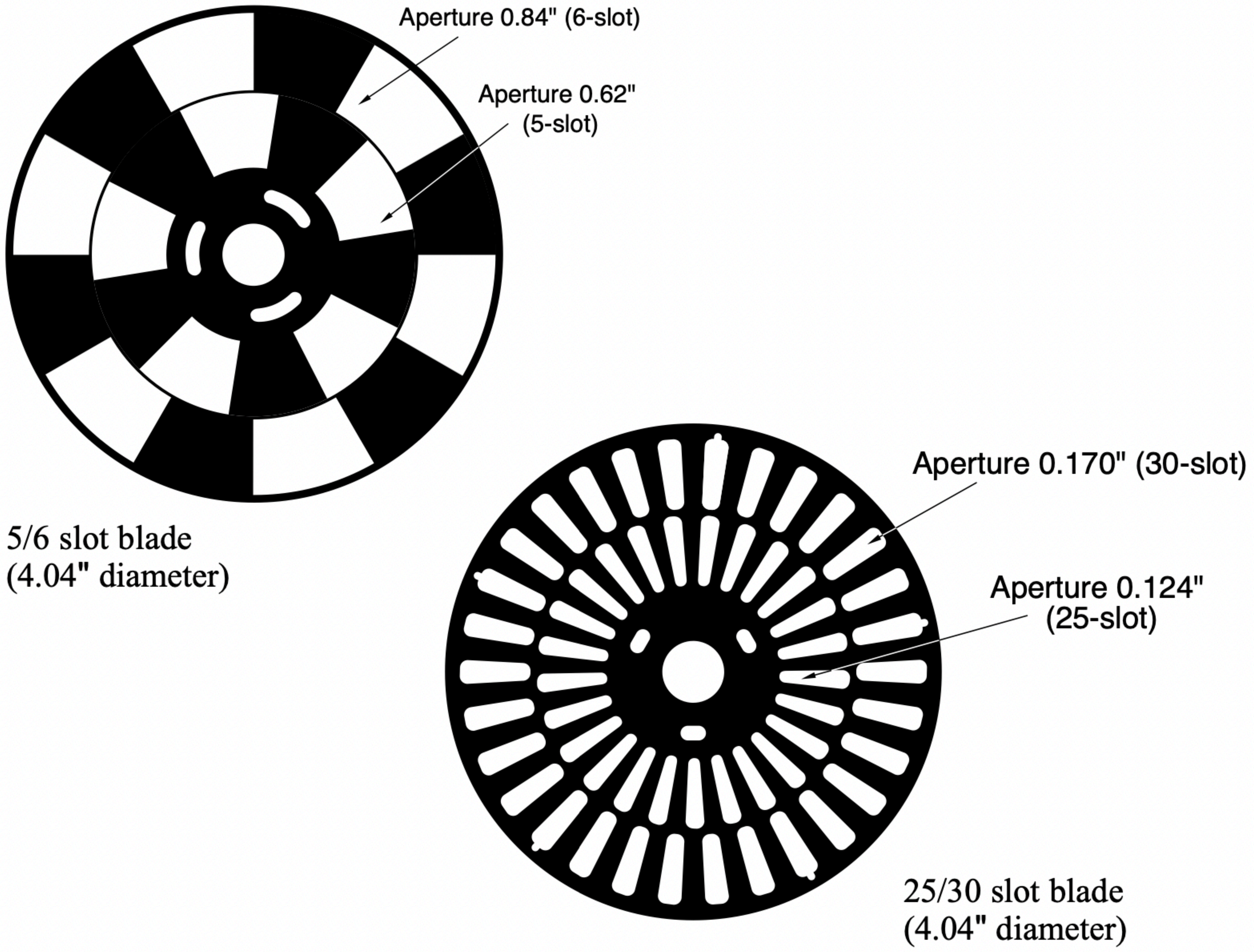
* The lock-in amplifier generates a signal internally, in phase with the frequency reference wave. Let's call θ the phase difference between the signal and the lock-in reference oscillator. By adjusting θ_{ref} we can have $\theta = 0$, in which case we can measure V_{sig}, since $\cos\theta = 1$. Conversely, if θ is 90°, there will be no output at all.

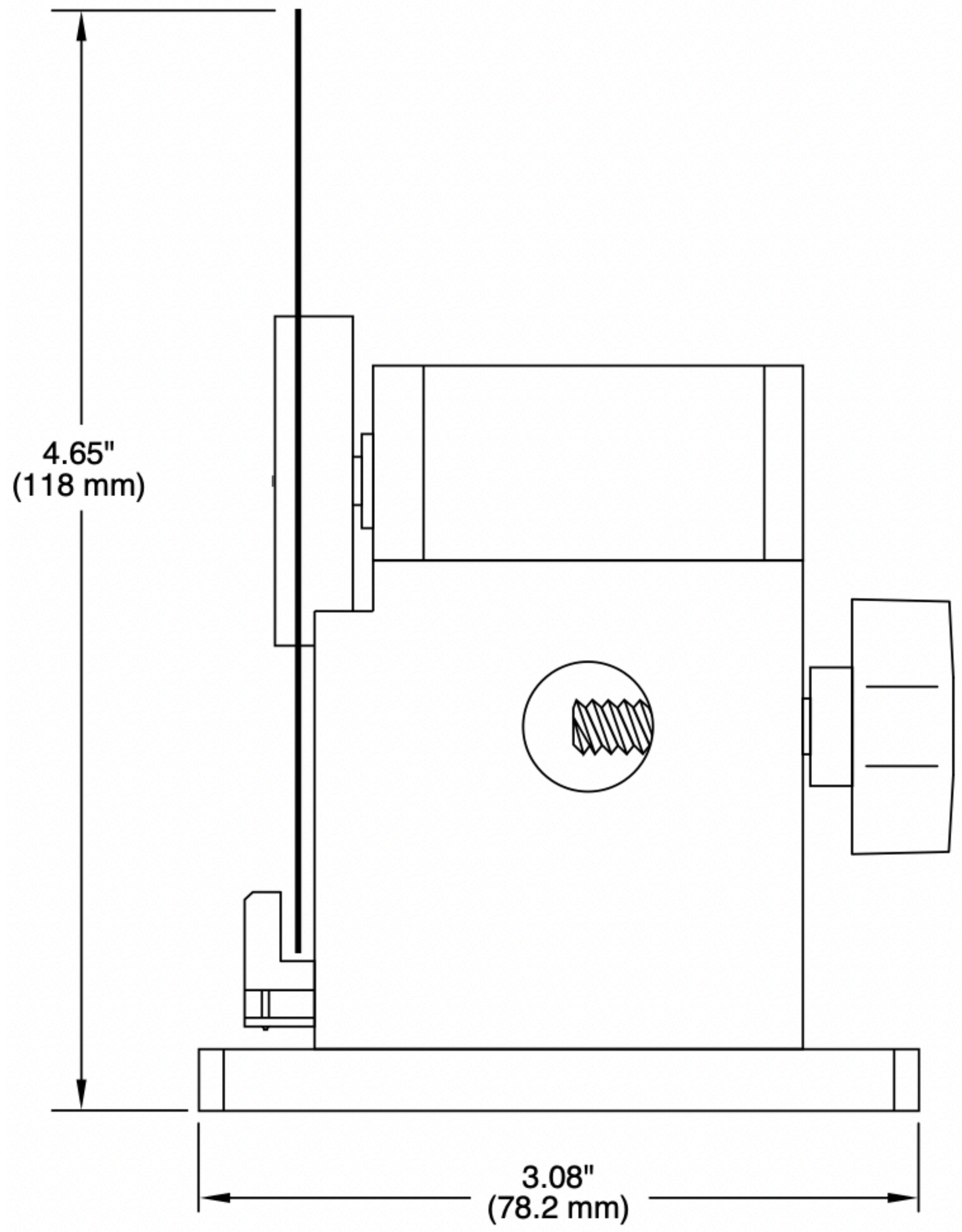
  → This fact can be used in practice to tune the lock-in phase.

Experimental scheme

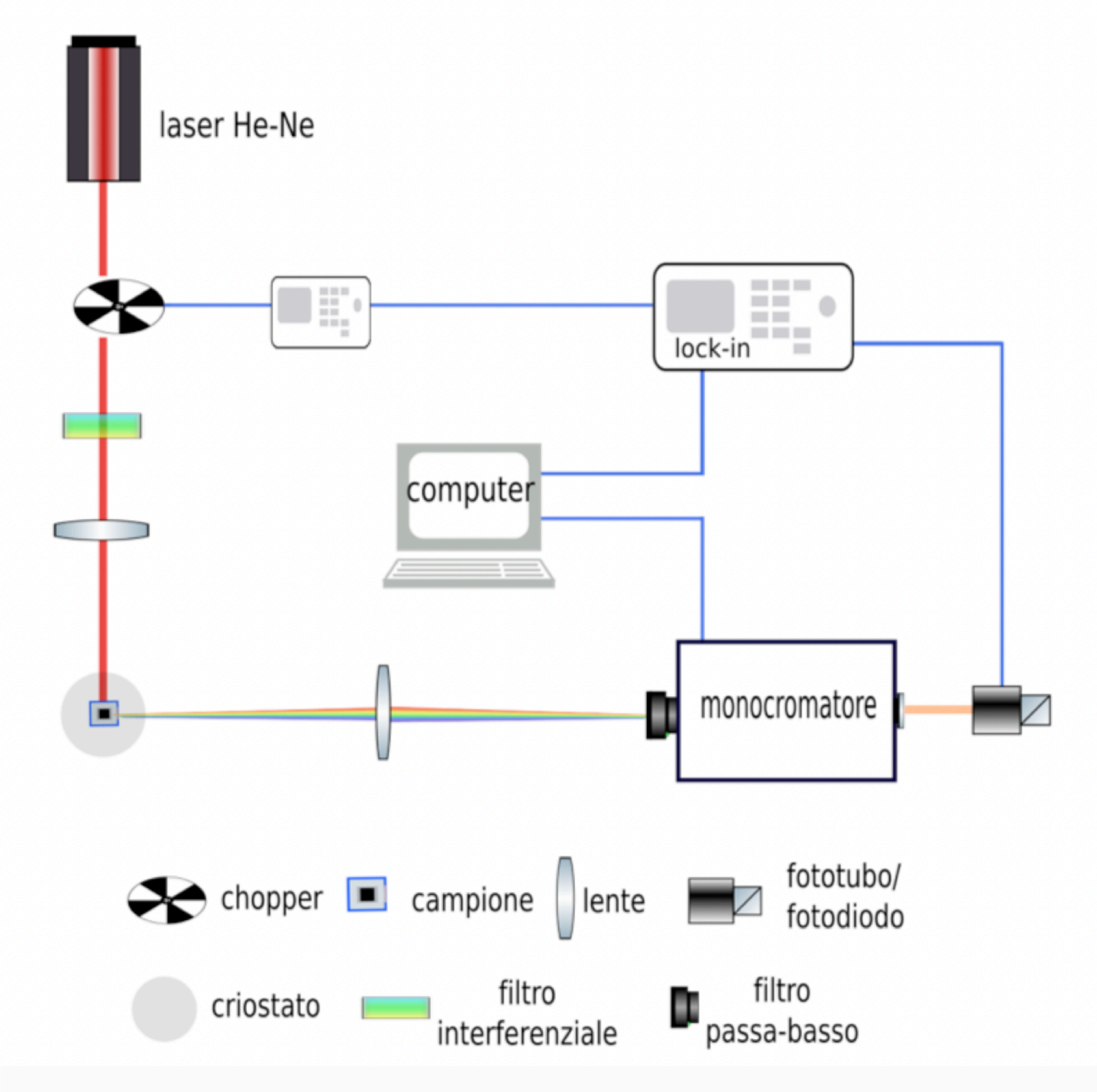


Optical choppers

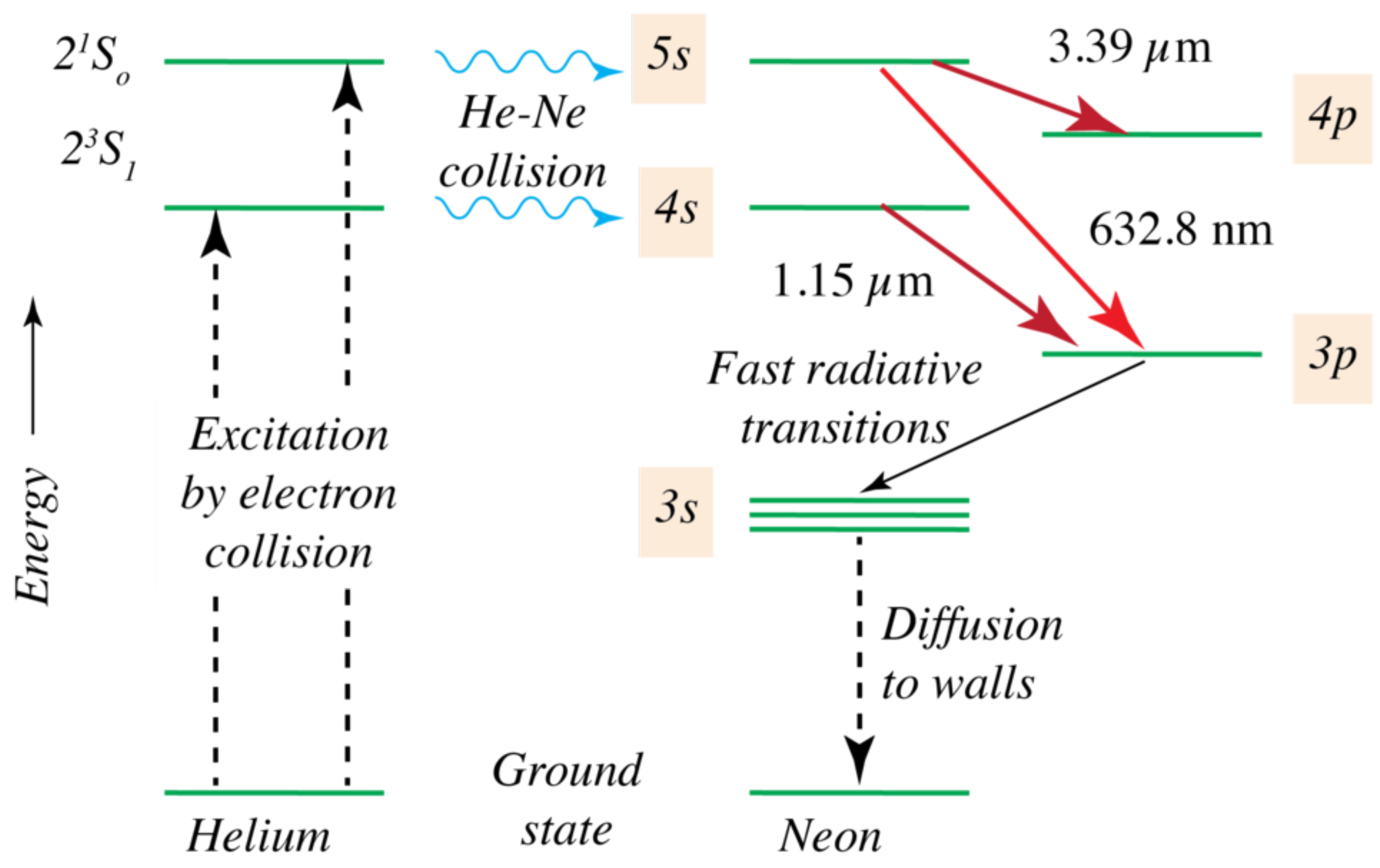




Experimental apparatus



He-Ne laser



Laser Elio-Neon

- La **scarica elettrica** nel gas eccita gli atomi di He per bombardamento elettronico. Gli stati eccitati decadono nei livelli 2^1S_0 e 2^3S_1 che hanno una vita media molto lunga. Per coincidenza, il Ne ha i livelli eccitati 5s e 4s che entro pochi meV hanno la stessa energia dei livelli metastabili dell'He. Per urto quindi l'He può diseccitarsi eccitando il Ne nei livelli 5s e 4s, mentre i livelli 4p e 3p rimangono vuoti. Si ha quindi inversione di popolazione tra i livelli 5s e 4s e i livelli 4p e 3p (i primi sono più popolati pur stando ad energia più alta) e ci può essere amplificazione per emissione stimolata tra un livello del primo e uno del secondo gruppo se la cavità risonante del laser è accordata su questa transizione.

- La transizione usata nel laser del laboratorio per ottenere radiazione coerente è a 632.8 nm.