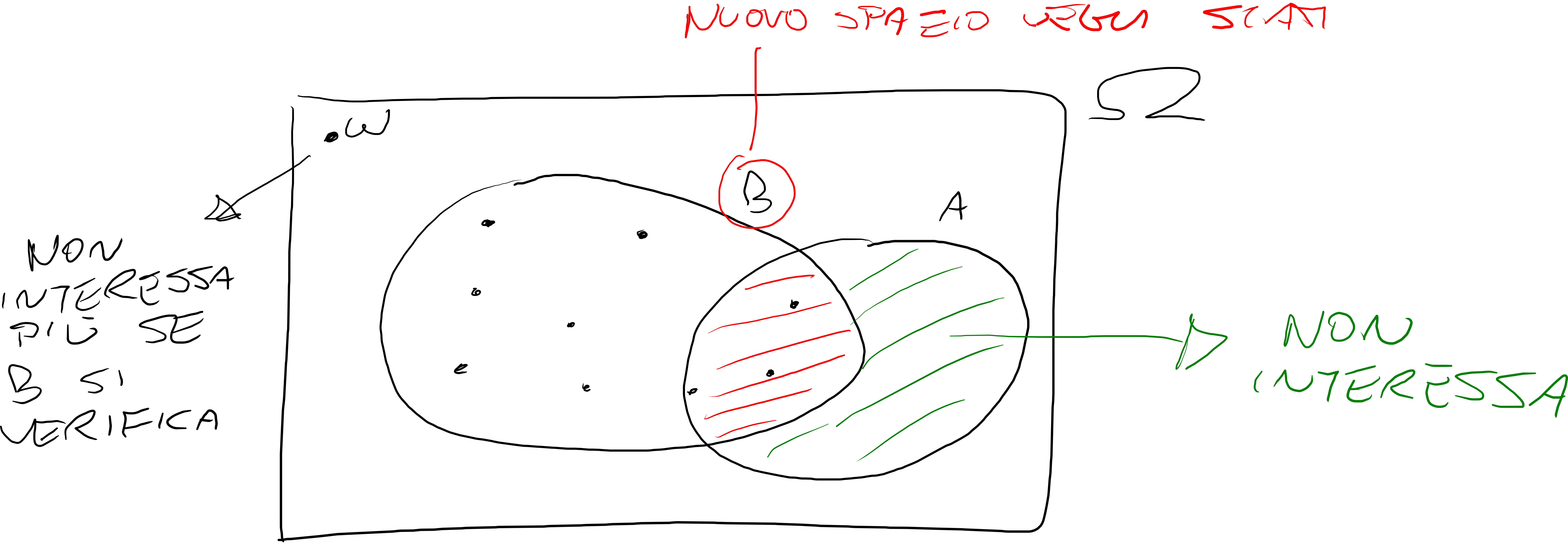

NUOVO SPAZIO DEGLI STATI
Ω
ω
B
A
NON INTERESSA PIÙ SE B SI VERIFICA
NON INTERESSA

A = " SOMMA È 7 " B = " MAX È 5 "

C = " DIFF. È 1 "

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{9}{36}} = \frac{2}{9} > P(A) = \frac{1}{6}$$

$$P(B|A) > P(B)$$

$$P(C|B) = \frac{P(C \cap B)}{P(B)} = \frac{P(\{(5,4),(4,5)\})}{\frac{9}{36}} = \frac{\frac{2}{36}}{\frac{9}{36}}$$

$$= \frac{2}{9} < P(C)$$

$$P(C) = P\left(\left\{ (1,2), (2,3) \text{ --- --- }, (5,6), (2,1), (3,2) \text{ ——— }, (6,5) \right\}\right) = \frac{10}{36}$$

$$P(B|C) < P(B)$$

$$P(\underbrace{A_1 \cap A_2 \cap \dots \cap A_{n-1}}_{B} \cap \underbrace{A_n}_{A}) =$$

$$= P(A \cap B) = P(A|B)P(B) =$$

$$= P(A_n | A_1 \cap \text{---} \cap A_{n-1}) P(\underbrace{A_1 \cap \text{---} \cap}_{} \underbrace{A_{n-1}}_{})$$

$$= P(A_n | A_1 \cap \text{---} \cap A_{n-1}) P(A_{n-1} | A_1 \cap \text{---} \cap A_{n-2}) \cdot$$

$$\cdot P(A_1 \cap \text{---} \cap A_{n-2})$$

$$= P(A_n | A_1 \cap \text{---} \cap A_{n-1}) P(A_{n-1} | A_1 \cap \text{---} \cap A_{n-2}) \cdot \dots$$

$$\dots P(A_2 | A_1) P(A_1)$$

ESTRAZIONI DA UN URNA CON COMPOSIZIONE NOTA

b PALLINE BIANCHE

r ROSSE

$b + r = n$

ESTRAZIONI SUCCESSIVE DI PALLINE DALL' URNA

DOPO OGNI ESTRAZIONE VENGONO REINSERITE NELL' URNA d PALLINE DELLO STESSO COLORE DI QUELLA ESTRATTA

$d = 1$ CON REINSERIMENTO

$d = 0$ SENZA //

$d > 1$ CONTAGIO POSITIVO

$d < 0$ // NEGATIVO

E_i = "ESCE BIANCA ALL' i-ESIMA ESTRAZIONE"

PROB. CHE NELLE PRIME 4 ESTRAZIONI SI ALTERNINO PALLINE BIANCHE E ROSSE

$\underline{\underline{d=0}}$ SENZA REINSERIMENTO $\underline{n \geq 4}$

$$P\left(\left(E_1 \cap \overline{E_2} \cap E_3 \cap \overline{E_4}\right) \cup \left(\overline{E_1} \cap E_2 \cap \overline{E_3} \cap E_4\right)\right) =$$

DISGIUNTI

$$= P\left(E_1 \cap \overline{E_2} \cap E_3 \cap \overline{E_4}\right) + P\left(\overline{E_1} \cap E_2 \cap \overline{E_3} \cap E_4\right)$$

$$P(E_1 \cap \overline{E_2} \cap E_3 \cap \overline{E_4}) =$$

$$= P(E_1)\, P(\overline{E_2} \mid E_1)\, P(E_3 \mid E_1 \cap \overline{E_2})\, P(\overline{E_4} \mid E_1 \cap \overline{E_2} \cap E_3)$$

$$= \frac{b}{n} \quad \frac{r}{n-1} \quad \frac{b-1}{n-2} \quad \frac{r-1}{n-3}$$

$$P(\overline{E_1} \cap E_2 \cap \overline{E_3} \cap E_4) = \frac{r}{n} \quad \frac{b}{n-1} \quad \frac{r-1}{n-2} \quad \frac{b-1}{n-3}$$

(B_i) PARTIZIONE DI Ω $B_i \in \mathcal{F}$
(DISCRETA)

$A \in \mathcal{F}$

$$P(A) = \sum_{i : P(B_i) > 0} P(B_i) P(A | B_i)$$

$$A = A \cap \Omega = A \cap \left(\bigcup_{i \geq 1} B_i \right) = \bigcup_{i \geq 1} \underbrace{(A \cap B_i)}_{\text{A DUE A DUE DISGIUNTI}}$$

$$P(A) = P\left(\bigcup_{i \geq 1} (A \cap B_i) \right) = \sum_{i \geq 1} P(A \cap B_i) =$$

$$= \sum_{i : P(B_i) > 0} P(A \cap B_i) =$$

THM PROB.
COMPOSTE

$$= \sum_{i : P(B_i) > 0} P(B_i) P(A | B_i)$$

THM DI BAYES

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A|B)\,P(B)}{P(A)}$$

DEF. DI PROB. CONDIZIONATA

THM PROB. COMPOSTE

$$P_B(A) = P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P_B : \mathcal{F} \to [0, 1]$$

[P1] $P_B(\emptyset) = 0$

$P_B(\Omega) = 1$

$$P_B(\emptyset) = P(\emptyset \mid B) = \frac{P(\emptyset \cap B)}{P(B)} = \frac{P(\emptyset)}{P(B)} = 0$$

[P2] σ - ADDITIVITÀ

$(A_n)_{n\geq 1}$ $A_n \in \mathcal{F}$ A DUE A DUE DISGIUNTI

$$P_B\left(\bigcup_{n\geq 1} A_n\right) = \sum_{n\geq 1} P_B(A_n)$$

$$P_B\left(\bigcup_{n\geq 1} A_n\right) = P\left(\bigcup_{n\geq 1} A_n \,\middle|\, B\right) = \frac{P\left(B\cap\left(\bigcup_{n\geq 1} A_n\right)\right)}{P(B)}$$

A DUE A DUE DISGIUNTI

$$= \frac{P\left(\bigcup_{n\geq 1} (B\cap A_n)\right)}{P(B)} = \frac{\sum_{n\geq 1} P(B\cap A_n)}{P(B)} =$$

$$= \sum_{n \geq 1} \frac{P(B \cap A_n)}{P(B)} = \sum_{n \geq 1} P(A_n | B) = \sum_{n \geq 1} P_B(A_n)$$

$$P(A_1 \cup A_2 | B) = P(A_1 | B) + P(A_2 | B) - P(A_1 \cap A_2 | B)$$

$$A_n \uparrow A = \bigcup_{n \geq 1} A_n \qquad P(\lim A_n | B) =$$

$$= \lim P(A_n | B)$$

$$P(A_1 \cap A_2 \cap \dots \cap A_n \mid B) =$$

$$= P_B(A_1 \cap \text{———} \cap A_n)$$

$$= \underbrace{P_B(A_1)}_{P(A_1 \mid B)} \; \underbrace{P_B(A_2 \mid A_1)}_{\underbrace{(P_B)_{A_1}(A_2)}_{\underbrace{P_{B \cap A_1}(A_2)}_{P(A_2 \mid A_1 \cap B)}}} \; \underbrace{P_B(A_3 \mid A_1 \cap A_2)}_{\underbrace{\text{- - - -}}_{P(A_3 \mid B \cap A_1 \cap A_2)}} \cdot \ldots \cdot \underbrace{P_B(A_n \mid A_1 \cap \dots \cap A_{n-1})}_{\underbrace{\text{- ——}}_{P(A_n \mid B \cap A_1 \cap \dots \cap A_{n-1})}}$$

DISINTEGRABILITÀ CONDIZIONATA ?

(B_i) PARTIZIONE A, C EVENTI

$$P(A|C) = ?$$

THM DI BAYES CONDIZIONATO ?

A, B, C

$$P(B|A \cap C) = \frac{P(A|B \cap C) \;---}{---}$$

B, N, M

B = "IL RISCHIO È BUONO"

N NORMALE

M MEDIORE

$P(B) = 20\%$ $P(N) = 45\%$ $P(M) = 35\%$

S = "IL RISCHIO GENERA UN SINISTRO"

$P(S|B) = 0.02$ $P(S|N) = 0.05$ $P(S|M) = 0.15$

$$P(S) = P(B)\,P(S|B) + P(N)\,P(S|N) + P(M)\,P(S|M)$$

DISINTEGRABILITÀ

SU $\{B, M, N\}$

$$= \frac{20}{100}\frac{2}{100} + \frac{45}{100}\frac{5}{100} + \frac{35}{100}\frac{15}{100}$$

$$= \frac{40 + 225 + 525}{100 \cdot 100} = \frac{79\cancel{0}}{1000\cancel{0}} = 7.9\%$$

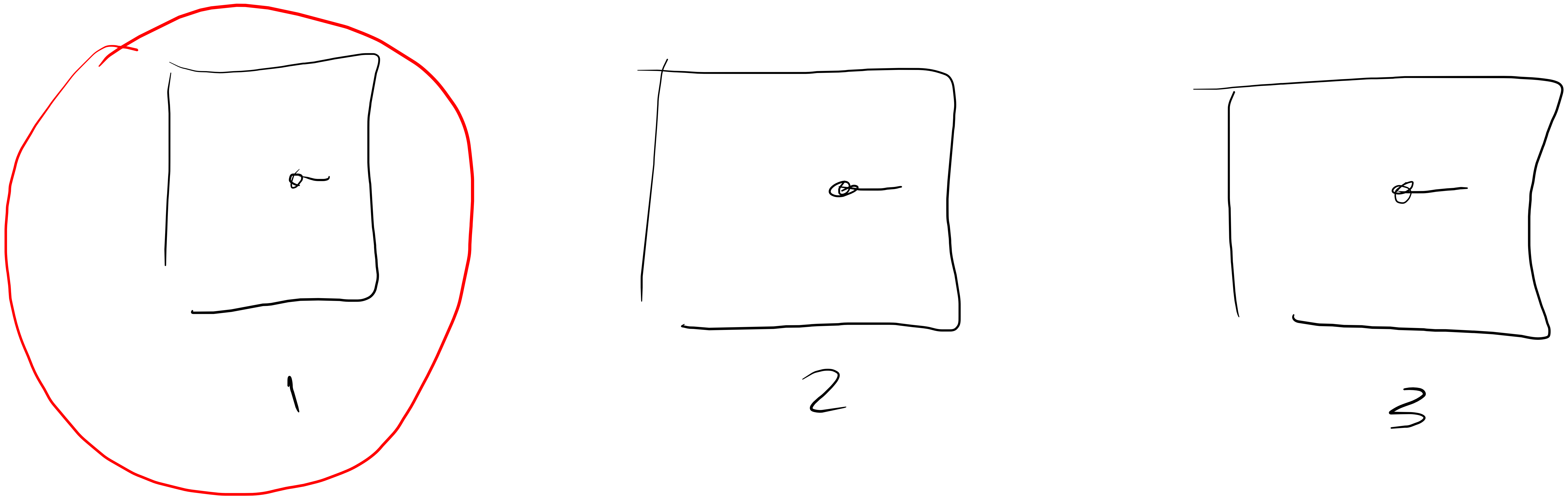
$$P(B|S) < P(B)$$

$$P(M|S) > P(M)$$

$$P(N|S) \; ? \; P(N)$$

$$P(B|S) = \frac{P(S|B)P(B)}{P(S)} = \frac{\frac{2\cancel{0}}{100} \cdot \frac{2}{\cancel{100}}}{\frac{79\cancel{0}}{\cancel{10000}}} = \frac{4}{79}$$

BAYES



E_i = "IL PREMIO È DIETRO LA PORTA i"

$$P(E_i) = \frac{1}{3} \qquad i = 1, 2, 3$$

D_i = "IL PRESENTATORE APRE LA PORTA i" $i = 1, 2, 3$

$P(E_1 | D_2)$? $P(E_2 | D_2) = P(E_3 | D_2)$

$P(E_1 | D_3)$ $= 0$

$D_1 = \emptyset$

$$P(E_1|D_2) = \frac{P(D_2|E_1)\,P(E_1)}{P(D_2)} = \frac{\frac{1}{3}\cancel{\frac{1}{2}}}{\cancel{\frac{1}{2}}} = \frac{1}{3}$$

BAYES

$$P(D_2) = P(E_1)\,P(D_2|E_1) + P(E_2)\,P(D_2|E_2) + P(E_3)\,P(D_2|E_3)$$

$$= \frac{1}{3}\left\{\frac{1}{2} + 0 + 1\right\} = \frac{1}{3}\,\frac{3}{2} = \frac{1}{2}$$

$$P(E_2 | D_2) = 0 \qquad P(E_3 | D_2) = \frac{2}{3}$$

$$(P_B)_C = P_{B\cap C}$$

$$(P_B)_C(A) = P_{B\cap C}(A) \quad \text{PER OGNI } A \in \mathcal{F}$$

$$(P_B)_C(A) \overset{\text{DEF. DI } (P_B)_C}{=} P_B(A|C) \overset{\text{DEF DI PROB. COND.}}{=} \frac{P_B(A\cap C)}{P_B(C)} \overset{\text{DEF. DI } P_B}{=}$$

$$= \frac{P(A\cap C|B)}{P(C|B)} = \frac{\frac{P(A\cap C\cap B)}{\cancel{P(B)}}}{\frac{P(C\cap B)}{\cancel{P(B)}}} =$$

DEF DI PROB. COND.

$$= \frac{P(A \cap C \cap B)}{P(C \cap B)} = P(A \mid C \cap B) = P_{C \cap B}(A)$$