$E_x =$ "UN INDIVIDUO (NEONATO) SOPRAVVIVE A ETÀ x" $\quad x \geq 0$

$0 \leq x < y$

$E_x \supset E_y$ (FAMIGLIA NON CRESCENTE PER INCLUSIONE)

$$ {}_tp_y = P(E_{y+t} \mid E_y) = 1 - {}_tq_y $$

$$ {}_{t|h}q_y = P(E_{y+t} \cap \overline{E_{y+t+h}} \mid E_y) $$

$${}_{t+s}p_x = {}_tp_x \cdot {}_sp_{x+t}$$

||

$$P(\underbrace{E_{x+t+s}}_{E_{x+t+s} \cap E_{x+t}} \mid E_x) = P(E_{x+t+s} \cap E_{x+t} \mid E_x)$$

THM PROB. COMPOSTE

$$= \underbrace{P(E_{x+t} \mid E_x)}_{= {}_tp_x} \cdot \underbrace{P(E_{x+t+s} \mid \cancel{E_x \cap} E_{x+t})}_{{}_sp_{x+t}}$$

$P(A) > 0 \quad P(B) > 0$

THM DI BAYES

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}$$

$$\frac{P(B|A)}{P(B)} = \frac{P(A|B)}{P(A)}$$

A, B INDIPENDENTI

$\Rightarrow \bar{A}, B$ INDIPENDENTI

$$P(B) = P(B \cap \Omega) = P(B \cap (A \cup \bar{A}))$$

$$= P((B \cap A) \cup (B \cap \bar{A})) =$$

$$= \underbrace{P(B \cap A)}_{P(A) \cdot P(B)} + P(B \cap \bar{A})$$

$*$

$*$

$$P(\bar{A} \cap B) = P(B) - P(A) \cdot P(B) = P(B)\underbrace{(1 - P(A))}_{P(\bar{A})}$$

$$= P(\bar{A})\, P(B)$$

P(B)

A, B INDIP. <=> A NON CORRELATO CON B

$$\Longrightarrow \quad P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)\,\cancel{P(B)}}{\cancel{P(B)}} = P(A)$$

$$\Longleftarrow \quad P(A\cap B) = P(B)\,\underbrace{P(A|B)}_{=P(A)}$$

A_1, A_2, A_3, A_4 $\qquad n=4$ EVENTI $\qquad 2^n - n - 1$

$$\left.\begin{aligned} &P(A_1 \cap A_2) = P(A_1)P(A_2) \\ &P(A_1 \cap A_3) = P(A_1)P(A_3) \\ &\dots\dots \end{aligned}\right\} \text{COPPIE}$$

$$\left.\begin{aligned} &P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3) \\ &P(A_1 \cap A_2 \cap A_4) = P(A_1)P(A_2)P(A_4) \\ &\dots\dots \end{aligned}\right\} \text{TERNE}$$

$$P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_1)P(A_2)P(A_3)P(A_4)$$

A_1, A_2, A_3

(A_1, A_2) INDIP.

(A_1, A_3) "

(A_2, A_3) "

(A_1, A_2, A_3) NON SONO INDIP.

$\Omega = \{abc, acb, cab, cba, bca, bac,$
$aaa, bbb, ccc\}$

$A_1 = \{abc, acb, aaa\}$ $A_2 = \{cab, bac, aaa\}$

$A_3 = \{cba, bca, aaa\}$ $P(A_k) = \frac{1}{3}$

$$P(A_1 \cap A_2) = P(\{aaa\}) = \frac{1}{9}$$

$$= P(A_1)P(A_2)$$

STESSO PER $P(A_1 \cap A_3)$, $P(A_2 \cap A_3)$

$$P(A_1 \cap A_2 \cap A_3) = P(\{aaa\}) = \frac{1}{9}$$

$$\neq P(A_1)P(A_2)P(A_3) = \frac{1}{27}$$

$\Omega = \{1C, \dots, 10C, JC, QC, KC, 1F, \text{---}, KF, \text{---}\}$

$\mathcal{F} = 2^{\Omega}$ $\qquad P(\{\omega\}) = \frac{1}{52}$

A_i = "IL RANGO È i" $\qquad i = 1$ —— $10, J, Q, K$

B_J = "IL SEME È J" $\qquad J = C, Q, F, P$

FISSATI i, J $\qquad P(A_i) = \frac{4}{52} = \frac{1}{13}$

A_i, B_J INDIPENDENTI $\qquad P(B_J) = \frac{13}{52} = \frac{1}{4}$

$$P(A_i \cap B_J) = P(\{iJ\}) = \frac{1}{52}$$

$$= \underbrace{P(A_i)}_{\frac{1}{13}} \underbrace{P(B_J)}_{\frac{1}{4}}$$

LANCIO DI DUE DADI $\Omega = \{ (i,j) \mid 1 \leq i,j \leq 6 \}$

A = "SOMMA È 7"

B_i = "IL PRIMO DADO È i"

FISSATO i,

A, B_i INDIPENDENTI

$P(A) = \frac{6}{36} = \frac{1}{6}$

$P(B_i) = \frac{1}{6}$

$P(A \cap B_i) = P(\{(i, 7-i)\}) = \frac{1}{36} = P(A)P(B_i)$

ESEMPIO DI ESTRAZIONE DI UNA CARTA DA UN MAZZO DI 52

A_i

B_J

$$\{A_1, A_2, \ldots, A_{10}, A_J, A_Q, A_K\} = \mathcal{A}_1$$

$$\{B_C, B_Q, B_F, B_P\} = \mathcal{A}_2$$

$\mathcal{A}_1, \mathcal{A}_2$ SONO INDIPENDENTI

$E_1, E_2, \ldots, E_{10}$ INDIPENDENTI

$$P\left(\left(E_1 \cup (E_3 \cap \overline{E_5}) \cup E_9\right) \,\middle|\, \cancel{\overline{E_2} \cap (E_4 \cup E_8)}\right)$$

$$E_2 \cap (E_4 \cup E_9)$$

$\{E_1, E_3, E_5, E_9\}$ $\{E_2, E_4, E_8\}$

$\sigma(\{E_1, E_3, E_5, E_9\})$ $\sigma(\{E_2, E_4, E_8\})$

$A_1, A_2, \ldots, A_{10}$ INSIEMI DI EVENTI INDIPENDENTI

$I = \{1 \ldots 10\}$

$= \underbrace{\{1,3\}}_{I_1} \cup \underbrace{\{2,4,10\}}_{I_2} \cup \underbrace{\{5,6,7,8,9\}}_{I_3}$

I_1, I_2, I_3 PARTIZIONE DI I

$\sigma(A_1 \cup A_3)$ $\quad$ $\sigma(A_2^{3} \cup A_4 \cup A_{10})$

$\sigma(A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9)$ INSIEMI DI EVENTI INDIPENDENTI

INFINITI LANCI DI UNA MONETA

$$P(\underbrace{E'_1}_{\Pi} \cap \text{———} \cap E'_n) = P(E'_1) \cdot \ldots \cdot P(E'_n)$$

$\sigma(\{E_1\})$

$$= \frac{1}{2} \cdot \ldots \cdot \frac{1}{2}$$

$$= \frac{1}{2^n}$$

$$\omega = E_1 \overline{E_2} E_3 \overline{E_4} E_5 \overline{E_6} \ldots$$

$\{\omega\}$ $\quad A_1 = E_1 \quad A_2 = E_1 \cap \overline{E_2} \quad A_3 = E_1 \cap \overline{E_2} \cap E_3$

$\ldots$

NON CRESCENTE

$$\lim_{n} A_n = \bigcap_{n=1}^{\infty} A_n = \{\omega\} = \{\bar{E}_1 \cap \overline{E_2} \cap \bar{E}_3 \cap \overline{E_4} \cap \dots\}$$

$$P(\{\omega\}) = \lim_{n} \underbrace{P(A_n)}_{\frac{1}{2^n}} = 0$$

"T APPARE PRIMA O POI" = "ESCE SEMPRE C"

$$= \{ CCC \ldots C \ldots C \ldots \}$$

"T APPARE PRIMA O POI" =

$$= E_1 \cup (\overline{E_1} \cap E_2) \cup (\overline{E_1} \cap \overline{E_2} \cap E_3) \cup \ldots$$

B_k = "UNA FISSATA SEQUENZA DI T E C DI LUNGHEZZA K SI VERIFICA PRIMA O POI"

$$P(B_k) = 1$$

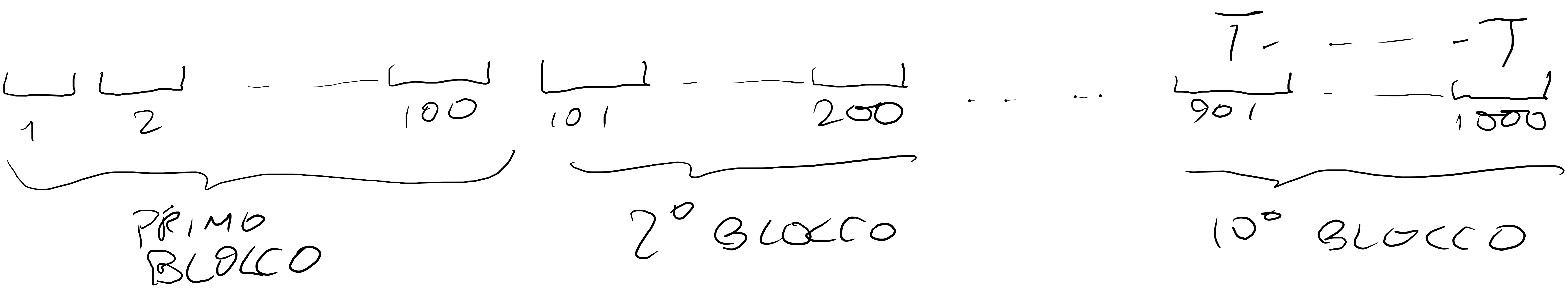
$k = 100$ TESTE CONSECUTIVE

$$P(\overline{B_k}) = 0$$

F_n = "LA DATA SEQUENZA NON SI VERIFICA NEI PRIMI $n \cdot k$ LANCI"

$\overline{B_k} \subset F_n$ PER OGNI n

D_n = "LA DATA SEQUENZA NON APPARE IN NESSUNO DEI PRIMI n BLOCCHI CONSECUTIVI DI LUNGHEZZA K"



$$F_n \subset D_n$$

$$P(D_n) = P\Big(\underbrace{\text{"NON APPAIONO 100 TESTE NEI PRIMI K LANCI"}}_{\in \sigma(\{E_1 \ldots E_k\})\ \text{PER OGNI } n} \cap \underbrace{\text{"NON APPAIONO 100 T NEI SECONDI K LANCI"}}_{\sigma(\{E_{k+1} \ldots E_{2k}\})}$$

$$\ldots \cap \underbrace{\text{"NON APPAIONO 100 T NEGLI } n\text{-ESIMI K LANCI"}}_{\sigma(\{E_{(n-1)k+1}, \ldots, E_{nk}\})}\Big)$$

$$= \overbrace{P(\quad)\cdot P(\text{—})\ldots P(\text{—})}^{n \text{ VOLTE}}$$

$$= \left(1-\frac{1}{2^k}\right)\left(1-\frac{1}{2^k}\right)\text{—}\left(1-\frac{1}{2^k}\right) = \left(1-\frac{1}{2^k}\right)^n$$

$$P(\overline{B}_k) \leq P(D_n) = \underbrace{\left(\underbrace{1 - \frac{1}{2^k}}\right)}_{<1}^{n} \quad \text{PER OGNI } n$$

$$\longrightarrow 0$$

QUANDO $n \to +\infty$

$$(\mathcal{A}_\alpha)_{\alpha \in I}$$

$\forall n \geq 1 \quad \forall \alpha_1, \dots, \alpha_n \in I$ (DISTINTI)

$\forall A_1 \in \mathcal{A}_{\alpha_1}, \dots, A_n \in \mathcal{A}_{\alpha_n}$

$$P(A_1 \cap A_2, \dots \cap A_n) = P(A_1) \dots P(A_n)$$