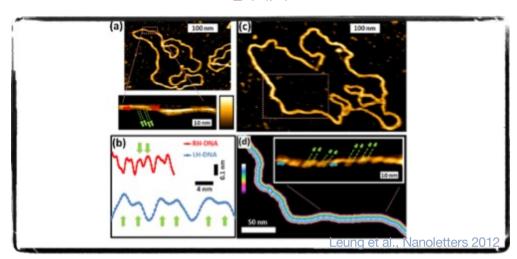
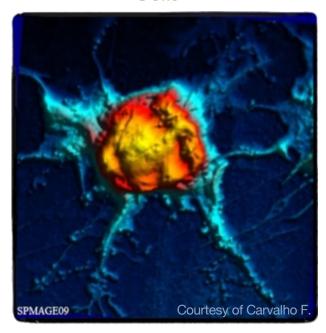
## High Res/High Speed AFM

#### AFM in Biology

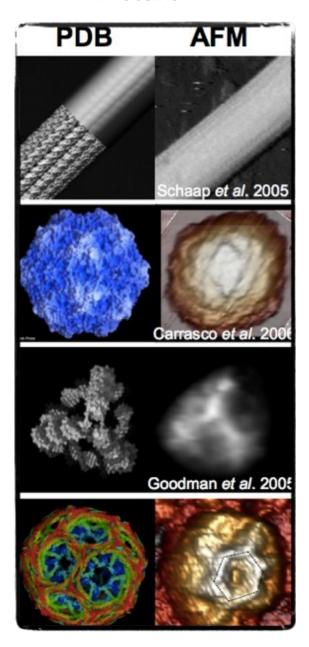
#### DNA



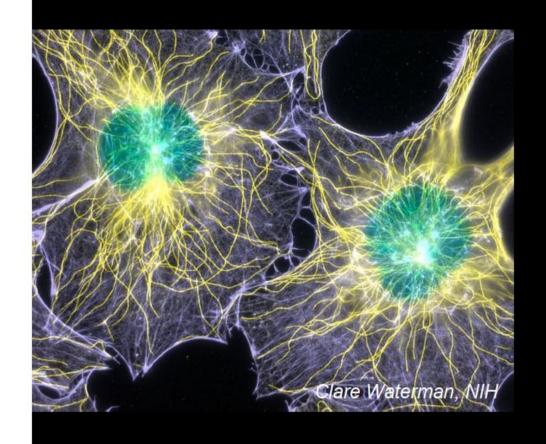
Cells

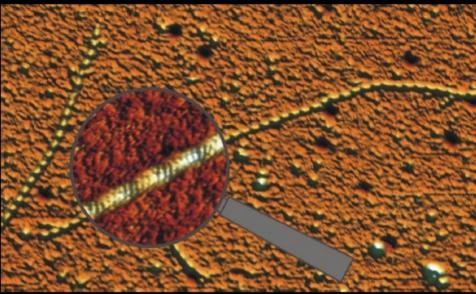


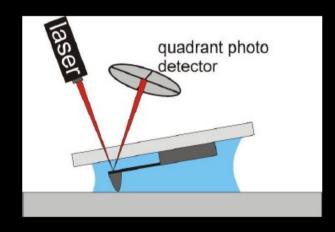
#### **Proteins**



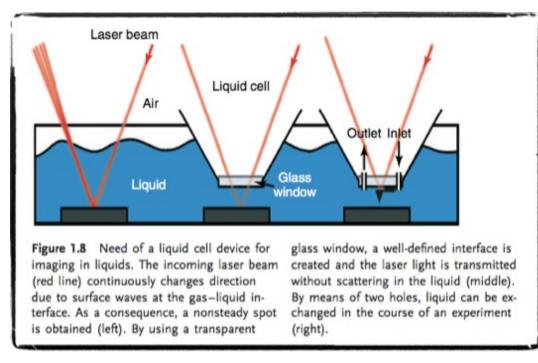
#### bio-molecules: working in liquid



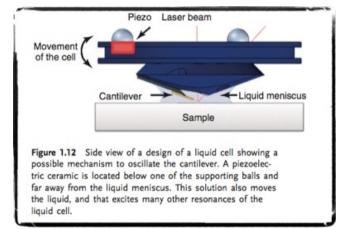




## AFM in liquid



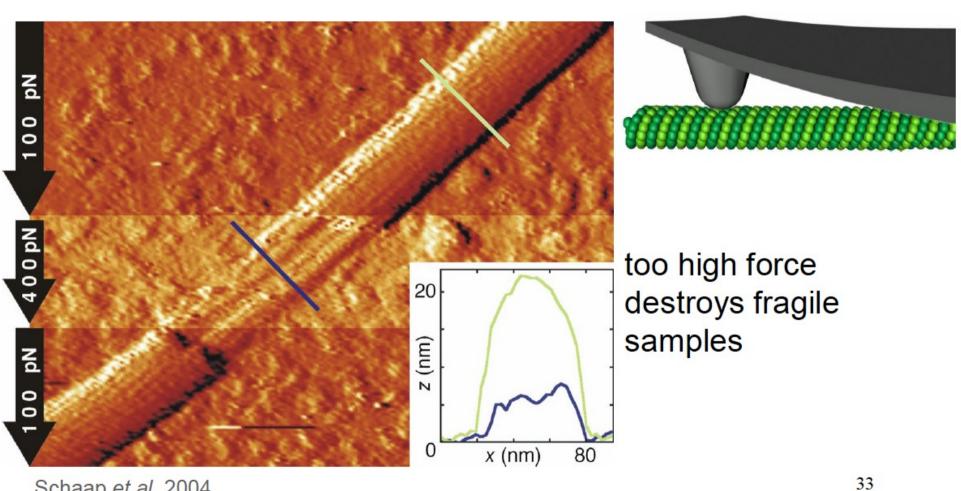
An important step of development, enabling biological AFM, was the development of a liquid cell in which cantilever, tip and sample are immersed in buffer solution.



In the case of soft biological samples, non-destructive imaging at nanometer-resolution in physiological conditions (i.e. in aqueous buffer, at ambient temperature and pressure) requires the ability to control forces  $< 100 \, \mathrm{pN}$ .

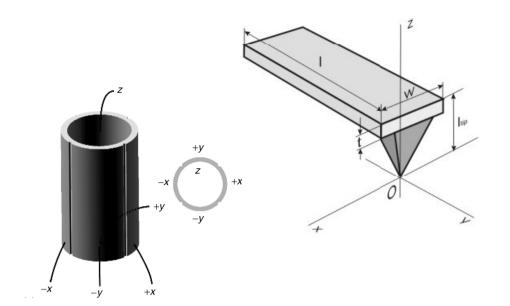
With a high optical lever sensitivity, one is able to control normal forces applied by the cantilever down to about 50 pN in liquid, enabling contact mode imaging of biological samples. Although lateral scanning forces impose a restriction on the sample type to be imaged in contact mode, it has yielded fascinating insights into the organization of proteins in biological membranes

#### sample deformation



Schaap et al. 2004

#### AFM in liquid



E= Young Modulus  $\rho$  = density

$$k = \frac{LVW}{4l^3}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{4\pi} \frac{t}{l^2} \sqrt{\frac{E}{\rho}}$$

Main concern: electrical isolation of the piezo to avoid any shortcut due to wetting For imaging in liquids: recommended to move the tip relative to the sample to minimize the added effective mass  $m^*$  (only the tip and parts of the tip holder are immersed in the buffer container)

A cantilever in thermodynamic equilibrium with the thermal bath at temperature T has a thermal energy  $k_BT$  that increases the elastic energy stored in the cantilever, giving a **thermal** noise amplitude A which can be of the order of surface corrugation for k = 0.1 Nm<sup>-1</sup> (A = 0.2 nm)

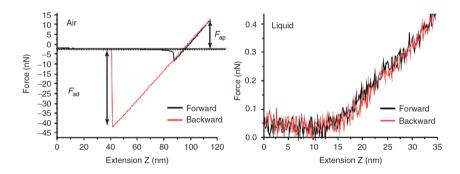
## Operation modes in liquid

#### **Contact Mode**

Feedback set point chosen depending on exp. Conditions. Drawbacks: drift of free cantilever deflection photodiode signal; friction

#### **Jumping Mode**

Developed to minimize shear forces. At each pixel, a force-extension curve is performed. Accurate control of force; lateral motion performed out of contact. Not applicable in air due to capillary forces. Drawback: bit slow



#### **Dynamic Mode**

Tapping mode: AM-AFM mode. Linear decrease of amplitude with tip-sample distance due to interaction with the surface. Reduction of amplitude in the tapping region gives the force. Comparable results, in liquid, to jumping mode, but much faster.

#### Jumping mode or "peak force" AFM

JM is particularly suitable for scanning in liquids, where the low adhesion forces allow using small Z displacement at each point. Because of the oscillation damping in liquid in amplitude modulation modes (NC, Tapping), JM although slow can give better performance.

N.B.: Dynamic modes use as feedback the oscillation amplitude. JM the cantilever deflection! Like in contact mode...easier. Is like a CM with the fine tuning of the zero interaction force

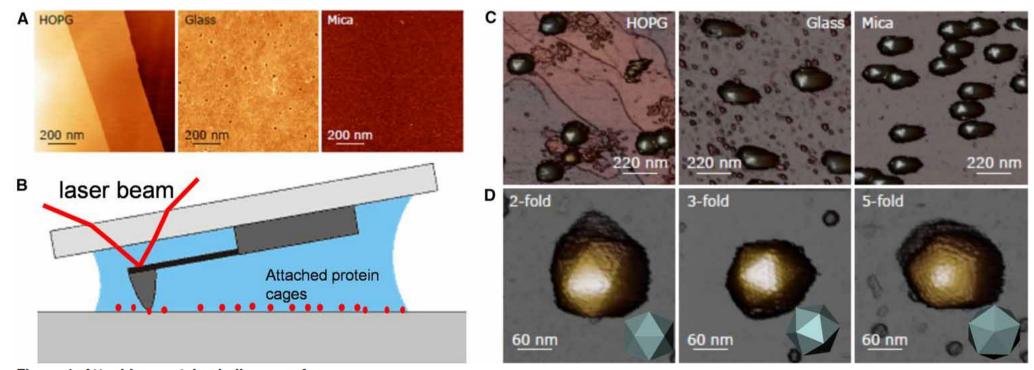


Figure 1. Attaching protein shells on surfaces.

(A) HOPG, glass, and mica bare substrates before attaching the samples. (B) Cartoon of the experimental system. Protein cages and cantilever are not in scale. (C) HAdv on HOPG, glass, and mica. (D) Individual HAdv particles showing 2-fold, 3-fold, and 5-fold symmetry axis orientations after adsorption on the surface. HAdV has a 95 nm diameter icosahedral, non enveloped capsid enclosing a ds DNA genome

#### Imaging of single virus particles on different surfaces

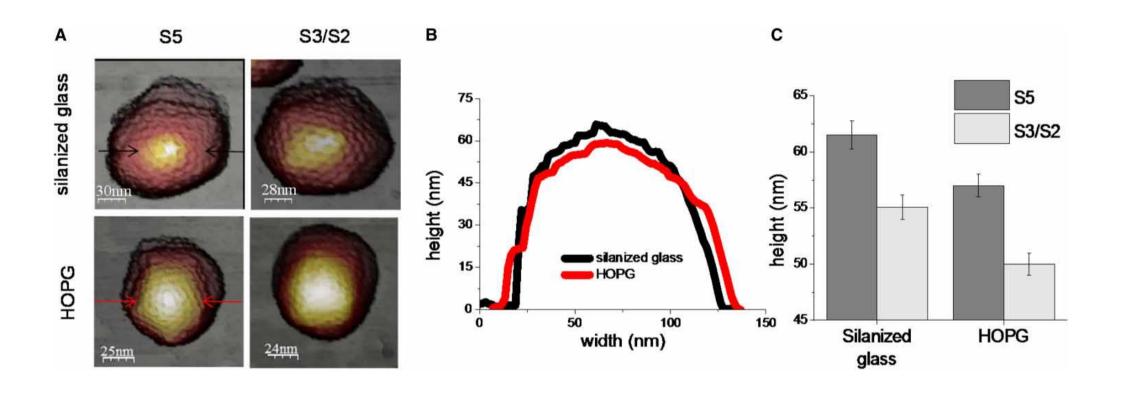


Figure 2. Protein shells collapse on the surface.

(A) P22 bacteriophage particles on glass and HOPG oriented to 5-fold and 3-/2- fold symmetry axes. (B) Comparison of topographical profiles obtained on two particles adsorbed on glass (black) and HOPG (red) obtained from A. (C) Comparison of average height of particles adsorbed at different orientations and substrates.

#### Jumping mode AFM

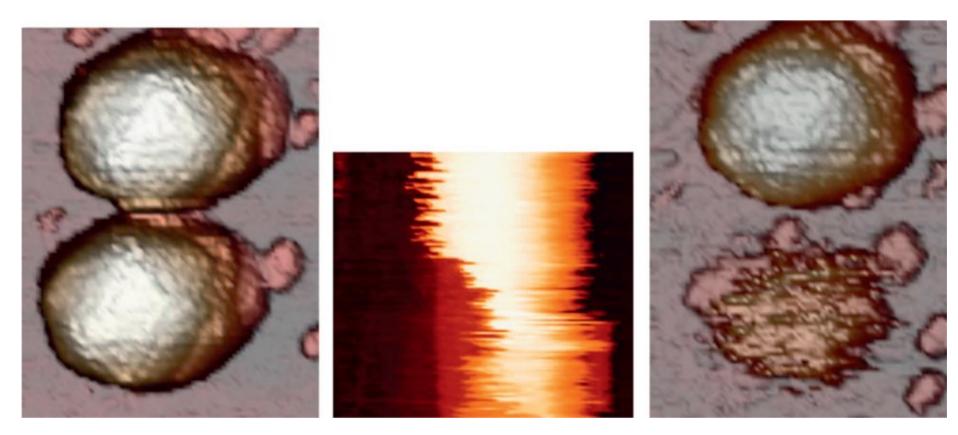
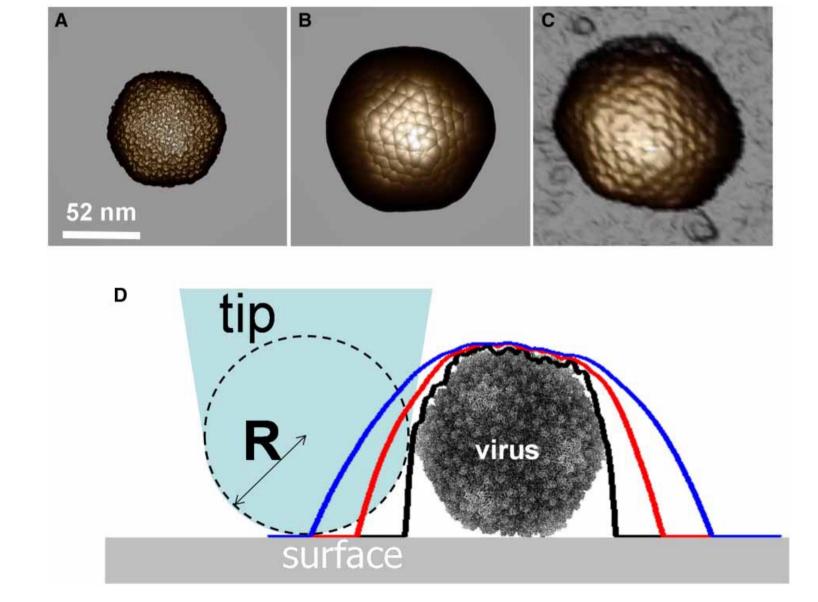


Fig. 4. (a)  $250 \text{ nm} \times 330 \text{ nm}$  AFM topography image of two HAdV particles acquired with JM+. (b) Time evolution (downwards) of the topography at the dashed line of figure (a). At the horizontal solid line the new algorithm is switched off, reverting to conventional JM. Notice the sudden change in the scanning profile. (c) Subsequent imaging of the same area with JM+ procedure demonstrates total destruction of the scanned virus while the other particle remains unmodified. Set point 150 pN.

under the new JM procedure, HAdV particles can be scanned for a long time without significant damage.

Figure 5. Dilation effects in the protein shell of bacteriophage P22.

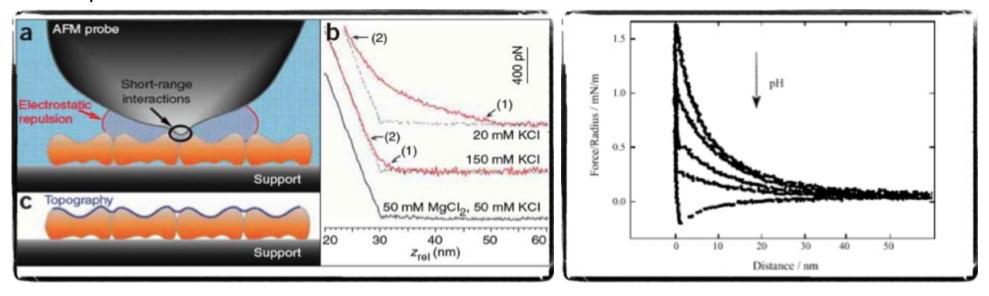
(A) The EM-1826 model of P22 bacteriophage oriented to the 2-fold symmetry axis. (B) Dilated data of (A) obtained with a tip of 10 nm in diameter by using the dilation algorithm of the WSxM software. (C) AFM image of a single P22 bacteriophage oriented to the 2-fold symmetry axis. The cartoon of (D) indicates the dilation as a function of the tip size: black, red, and blue curves are the topographical profiles obtained with tips of 0.5, 10, and 15 nm in diameter, respectively.



## Balancing Electrostatic Forces

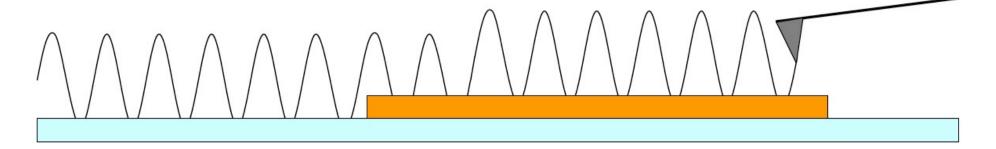
#### **Main Forces in Liquid:**

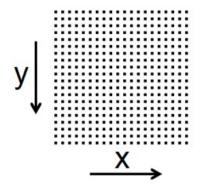
- 1) Van der Waals, globally attractive, short range.
- 2) Double layer forces: Repulsive, long range, dependent on pH and ionic strength. Due to ionic atmosphere over the surfaces of the tip and the surface. The 2 layers create the repulsive force.



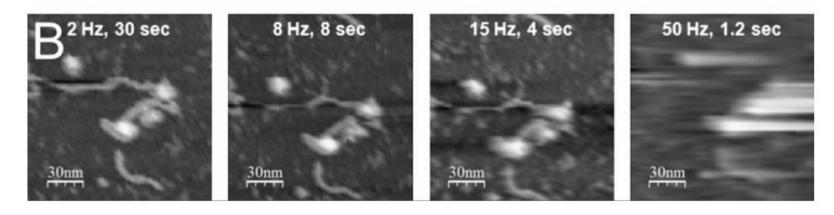
Fine tuning of electrolyte concentration is always required to minimize the distance of tip-sample electrostatic interactions and achieve high-resolution.

#### SPEED LIMITS OF AFM IMAGING



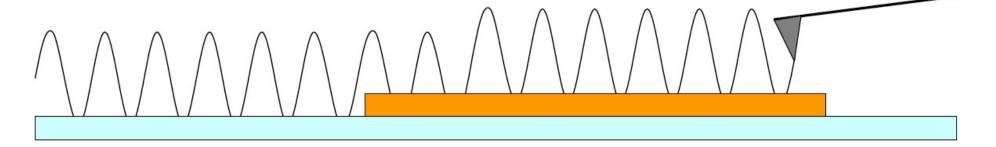


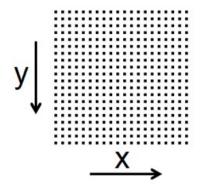
64 x 64 = 4096 px need a few oscillations per pixel (~5) fast bio-cantilever ~ 25 kHz → 0.04 ms \* 5 \* 4096 = 1 s



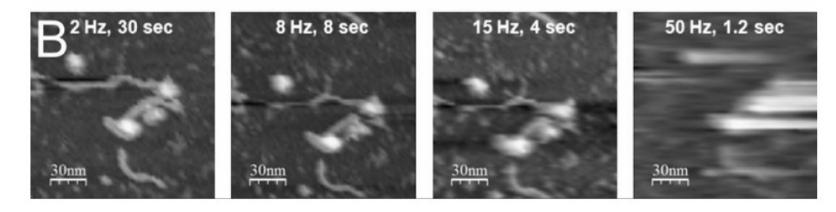
(there are other limiting factors (z-piezo, feed-back loop)

#### SPEED LIMITS OF AFM IMAGING





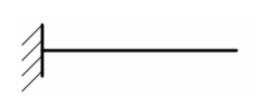
64 x 64 = 4096 px need a few oscillations per pixel (~5) fast bio-cantilever ~ 25 kHz → 0.04 ms \* 5 \* 4096 = 1 s



(there are other limiting factors (z-piezo, feed-back loop)

#### small cantilevers are faster

	<i>l</i> (μm)	w (μ <b>m</b> )	t (μm)	$\omega_o(kHz)$	k (N/m)	
rc800	200	20	0.8	3	0.05	8 s
bl150	60	30	0.18	8	0.03	3 s
ac40	38	16	0.2	25	0.1	1 s
ac10	9	2	0.13	500	0.1	50 ms



$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{Et^2}{l^4 \rho}}$$

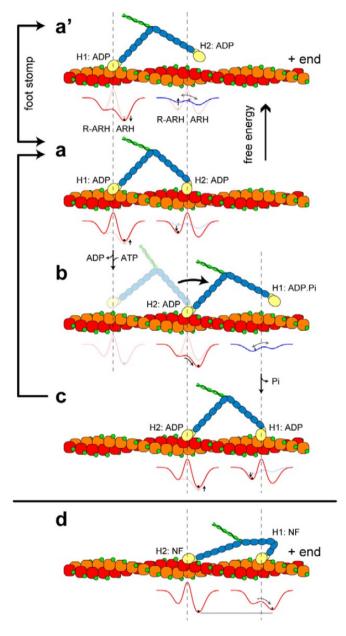
$$k = \frac{F}{d} = \frac{Ewt^3}{4l^3}$$

make cantilevers short to increase  $\omega_0$  and thinner to restore k



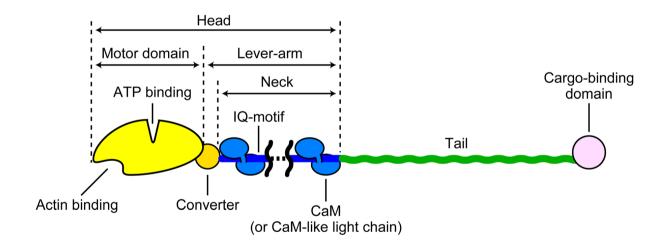
# ATOMIC FORCE MICROSCOPY Imaging in Biology\_2 Examples

## Single molecule imaging 1



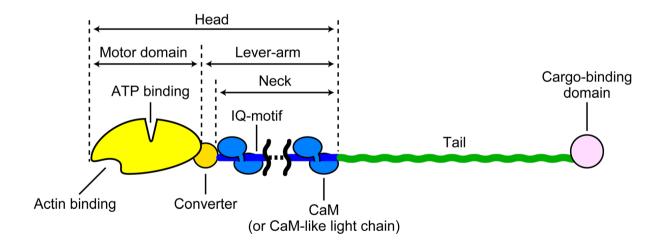
In dynamic HS-AFM the molecule itself is visualized while working and moving on its biological track, providing concomitant structural and dynamic data: not only did the observation confirm the hand-over-hand walking mechanism of myosin-V, it did reveal that the power stroke of this motor is driven by intramolecular mechanical tension

Each head of the double-headed myosin hydrolyzes ATP into ADP and inorganic phosphate (Pi). The ATPase rate is very low when myosin is alone but is markedly accelerated by its interaction with actin, where the chemical energy liberated by ATP hydrolysis is converted into mechanical work.



Using single-molecule fluorescence microscopy and optical-trap nanometry it has been shown that M5 moves along actin filaments toward the plus end in a "hand-over-hand" manner, advancing 36 nm per ATP hydrolysis cycle.

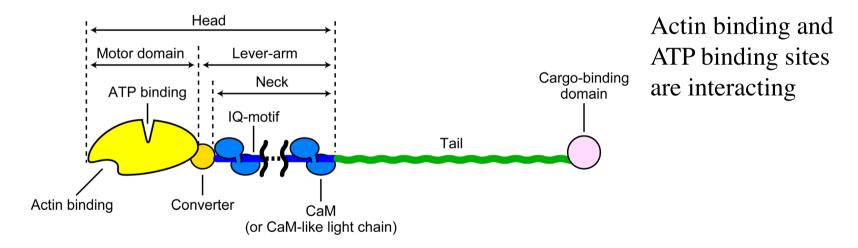
The 36 nm stride corresponds to a half pitch of the right-handed, double-helical structure of an actin filament, and, therefore, M5 moves approximately on a plane.



The mechanism underlying the alternate steps was suggested to arise from asymmetric kinetics of ADP dissociation from the two heads; ADP dissociation at the trailing head is more accelerated than at the leading head and/or ADP dissociation at the leading head is decelerated.

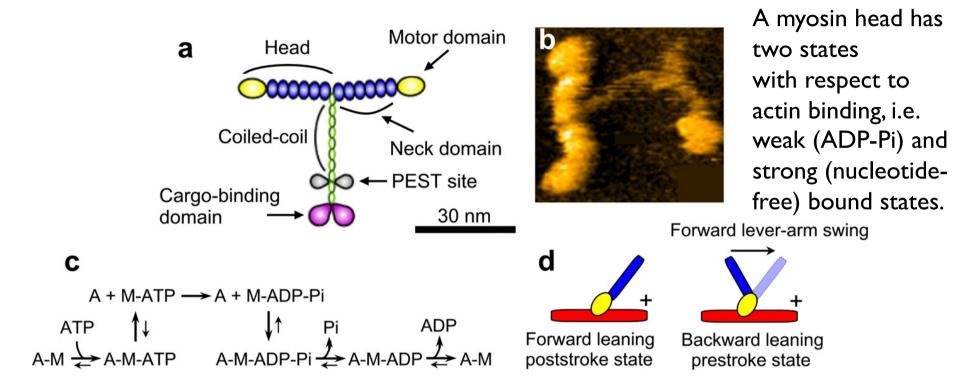
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The 36 nm stride corresponds to a half pitch of the right-handed, double-helical structure of an actin filament, and, therefore, M5 moves approximately on a plane.



However, despite numerous and extensive studies, the heart of the motor mechanism, that is, how the tension for the forward step is generated in the molecule, coupled with the ATPase reaction, and how the energy liberated by ATP hydrolysis is used, has remained elusive.

dx.doi.org/10.1021/cr4003837 | Chem. Rev. 2014, 114, 3120-3188



The nucleotide-free head tightly bound to actin detaches from the actin immediately after binding to ATP, quickly followed by hydrolysis of the bound ATP to ADP—Pi. When the ADP—Pi bound head is attached to actin, the bound Pi dissociates from the head, which is followed by the formation of a strongly bound tertiary complex A—M—ADP (A and M denote actin and myosin, respectively) and then by ADP dissociation, completing one ATPase cycle.

The main role of actin in the ATPase reaction is to accelerate the otherwise very slow Pi and ADP dissociation from a myosin head.

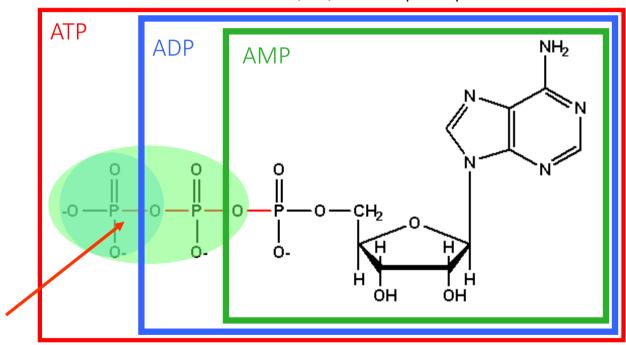
#### Energy in the cell: ATP

In the cell reactions that require energy are associated with ATP hydrolysis (hydrolysis= breaking down). ATP hydrolysis is an exothermic reaction, and the energy generated can be used to drive a non-spontaneous reaction.

ATP 
$$\mapsto$$
 ADP + P  
( $\Delta$ H = -30 kJ/mol)

$$ATP \mapsto AMP + PP_i$$
  
( $\Delta H = -30 \text{ kJ/mol}$ )

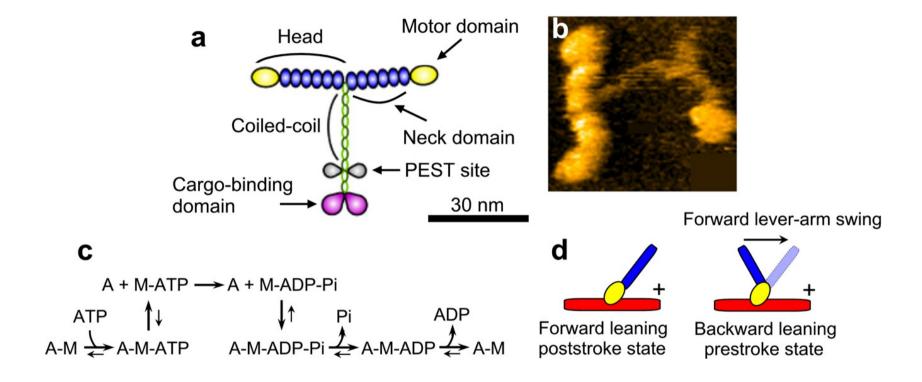
phosphodiester bonds have a large energy of hydrolysis (about 30 kJ/mol) Adenosine Tri/Di/Mono phosphate



Energy production: accumulation of ATP

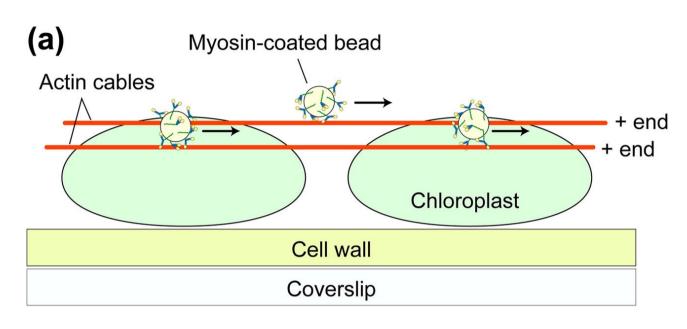
Energy consumption: breaking down (hydrolysis) of ATP  $\mapsto$  ADP or AMP

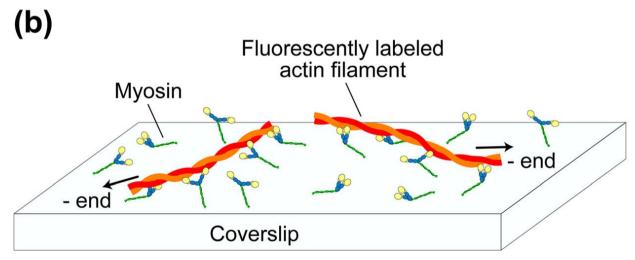
dx.doi.org/10.1021/cr4003837 | Chem. Rev. 2014, 114, 3120-3188



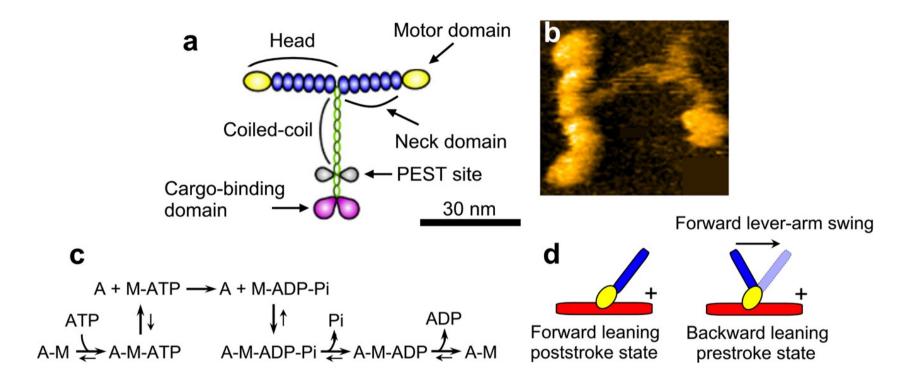
The key idea in the prevailing view on the chemo-mechanical coupling in myosin motility, which has been mainly derived from muscle myosin studies, is that the myosin head is supposed to take two different conformations, prestroke and poststroke conformations corresponding to different angles between the motor domain and the neck domain (often called "lever-arm"), depending on the nucleotide states

Fig. 4 Schematics showing in vitro motility assay systems for actomyosin. a Myosin-coated bead assay. The myosin-coated fluorescent beads are subjected to the polar arrays of actin cables naturally formed on chloroplastd of the alga Nitella, and movement of the beads are observed under a florescent microscope. b Actin filament gliding assay. Myosin molecules are attached to the surface of a nitrocellulose-coated coverslip and gliding motion of the fluorescently labeled actin filaments are observed under a fluorescence microscope





dx.doi.org/10.1021/cr4003837 | Chem. Rev. 2014, 114, 3120-3188

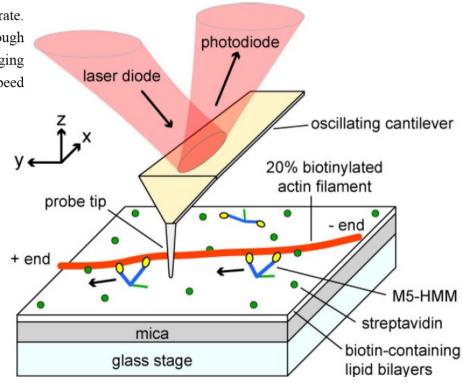


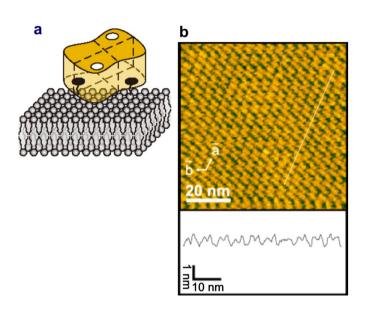
High-speed atomic force microscopy (HS-AFM), allow video-recording the structure and dynamics of functioning biomolecules at single-nanometer resolution, without disturbing their function. It helped to discover that the tension responsible for forward movement can be generated without any chemical transition, meaning that no chemical energy input is required for the tension generation. Moreover, the lever-arm swing (powerstroke) by the leading head spontaneously occurs when the trailing head detaches, thus demonstrating that no chemical energy input is required for the lever-arm swing either.

Video imaging by high-speed AFM has been applied to capture the dynamic behaviour of myosin V (two headed motor that functions as cargo transporter in cells) translocating along an acting filament. Moves hand-over-hand, 36 nm per ATP hydrolysis

**Supplementary Figure 1** | **Schematic of assay system for HS-AFM imaging (not scaled).** A mica surface was fully covered with biotin-containing lipid bilayers. Streptavidin molecules (green circles) were partially deposited on the substrate. Biotinylated actin filaments were immobilised on the bilayer surface through streptavidin molecules. M5-HMM was deposited on the lipid bilayers. All imaging experiments were performed in the tapping mode using a laboratory-built high-speed AFM apparatus<sup>5,6</sup>.

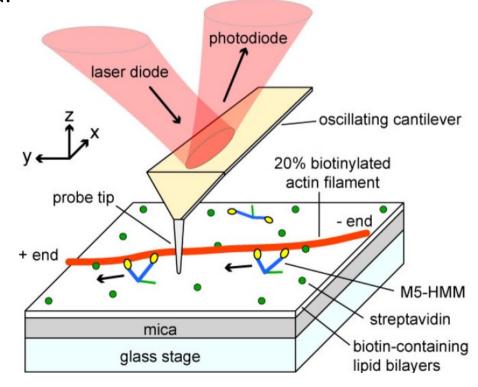
A positively charged lipid in the mixed lipid bilayer was necessary to assure weak interaction with Myosine and translocation along the actin filament

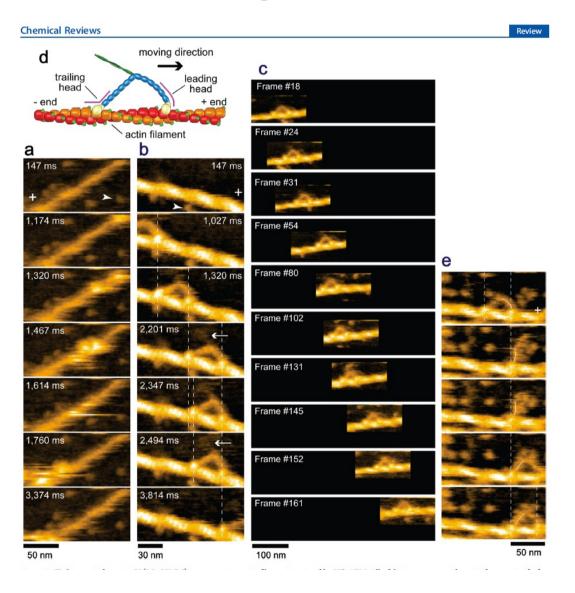




Streptavidin is a homo tetramer with dihedral D2 symmetry. Importantly, it is not favorable to nonspecific binding of many proteins, while each subunit has a high affinity biotin binding site. Streptavidin 2D crystals are easily formed onmthe surface of a fluid SLB containing biotin-lipid.

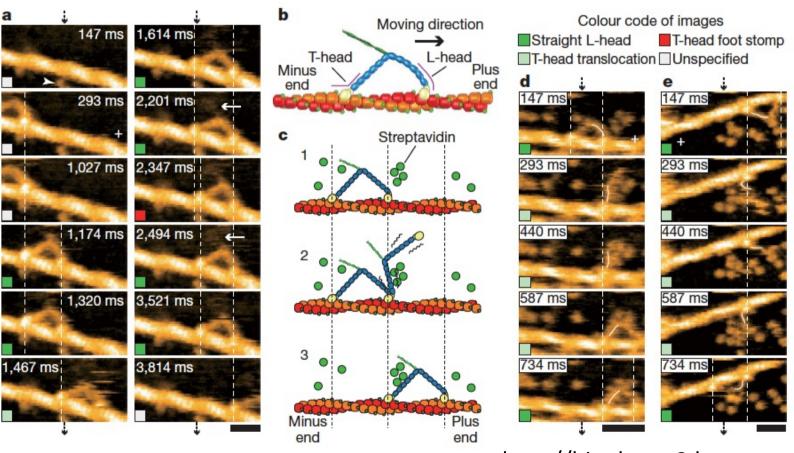
On the SLBs, two of the four biotin binding sites of streptavidin face the lipid bilayer and are occupied by biotin, whereas the other two are exposed to the aqueous environment and accessible. Therefore, biotinylated samples can be specifically immobilized on the surface of streptavidin 2D crystals





N. Kodera, D. Yamamoto, R. Ishikawa, T. Ando *Nature* **468**, 72 (2010) dx.doi.org/10.1021/cr4003837 | Chem. Rev. 2014, 114, 3120–3188

AFM images demonstrate a hand-over-hand movement, with swinging leverarm motion: the detached T-head rotationally diffused around the advancing neck-neck junction. Extra STV needed as an "obstacle" to slow down the motion to be visualized (100 ms/frame)

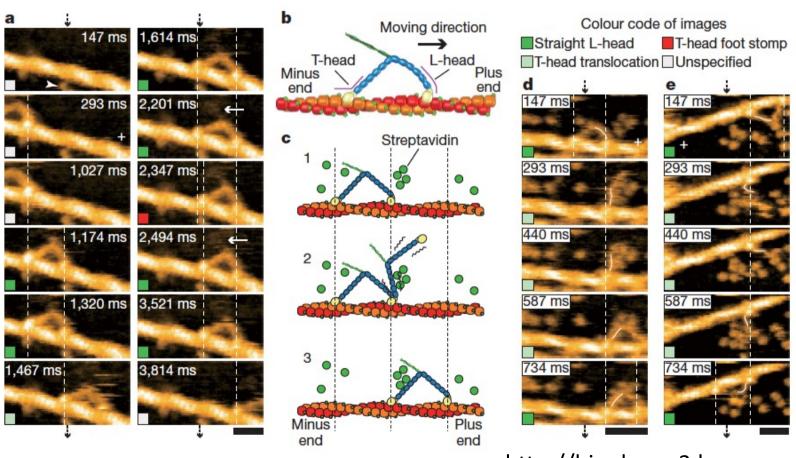


N. Kodera, D. Yamamoto, R. Ishikawa, T. Ando Nature 468, 72 (2010)

http://biophys.w3.kanazawau.ac.jp/M5\_movies.htm

The neck-motor domain junction appears smooth in the leading head (L-head) but is V-shaped in the trailing head (T-head) without exception.

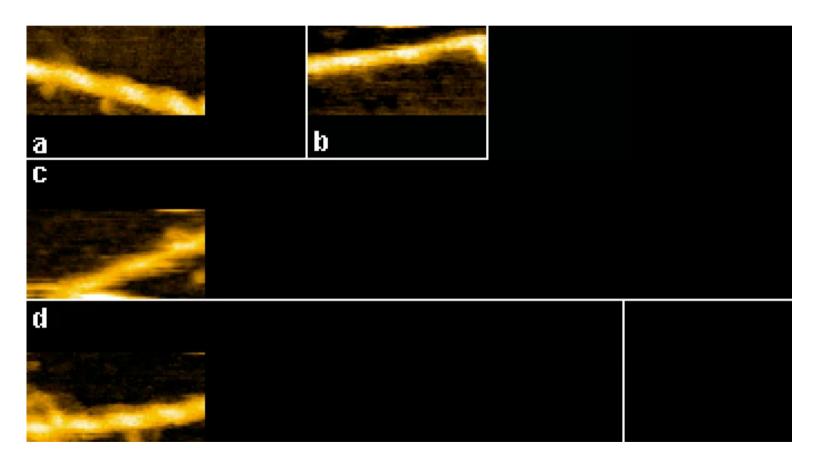
The short coiled coil tail was mostly tilted towards the minus end of actin



http://biophys.w3.kanazawau.ac.jp/M5\_movies.htm

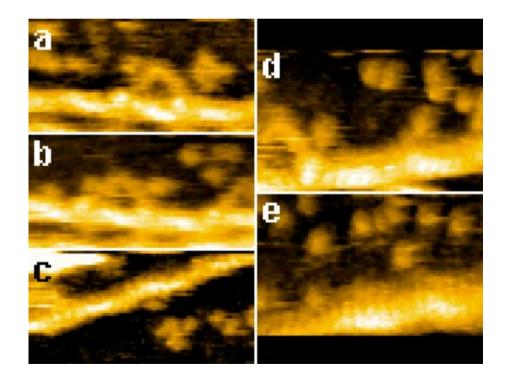
The neck-motor domain junction appears smooth in the leading head (L-head) but is V-shaped in the trailing head (T-head) without exception.

The short coiled coil tail was mostly tilted towards the minus end of actin



http://biophys.w3.kanazawau.ac.jp/M5\_movies.htm

After T-head detachment, the nearly straight leading neck swung from the reverse arrowhead (R-ARH) orientation to the arrowhead (ARH) orientation confirming the swinging lever-arm motion initially proposed for muscle myosin. The detached T-head rotationally diffused around the advancing neck—neck junction (no translational diffusion on the actin occurs) and then bound to a forward site on the actin filament, completing one step.

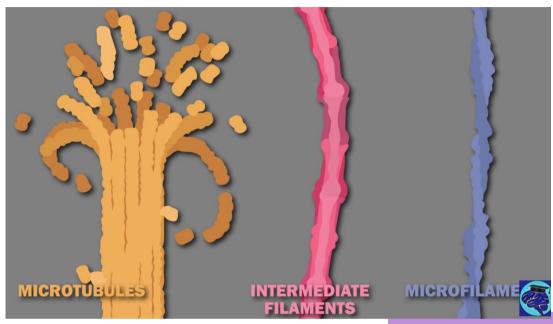


The captured images show that the forward movement is driven not by bending but by rotation of the L-head. The rotation seems to occur spontaneously after T-head detachment, suggesting that intramolecular tension driving the L-head swing exists in the two-headed bound molecules.

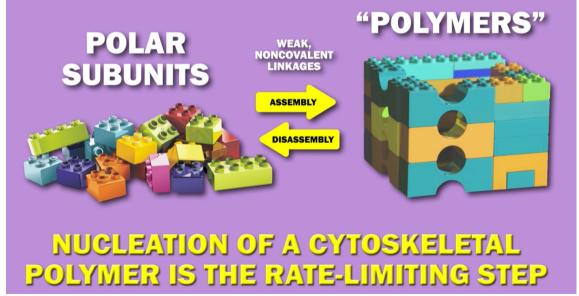
Moreover, it was observed that the leading head of the two-headed bound M5-HMM was often sharply bent in the nucleotide-free condition while it was mostly straight in ADP and ATP. Therefore, just by looking at the shape of the leading head, we can judge whether or not the leading head contains nucleotides. ADP dissociation rate constant at the leading head is 0.1 s-1. This means that ADP is released from the leading head every 10 s, on average. M5-HMM walks many steps for 10 s. Thus, we can conclude that during walking ADP does not dissociate from the leading head. ADP dissociation, and the subsequent ATP binding, and the resulting detachment from actin solely occurs at the trailing head.

Just before foot stomping at the leading head, the head never showed the sharply bent conformation which is unique to the nucleotide-free leading head. This fact reinforces our conclusion that the leading head performing foot stomp carries ADP and thus the brief detachment from actin (i.e. the initial stage of foot stomp process) is not caused by binding of new ATP to the leading head.

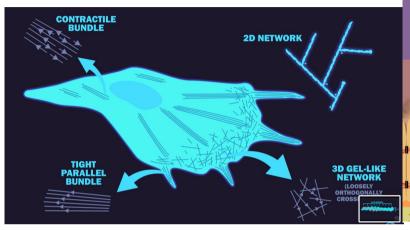
#### Polymeric fibers: cytoskeleton

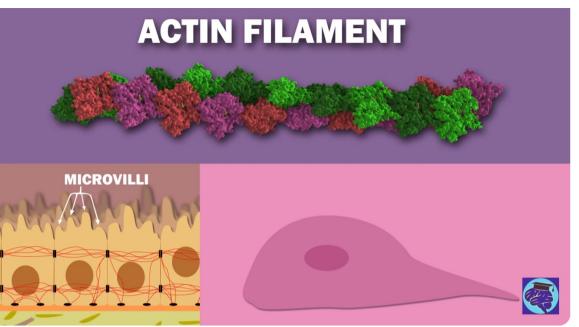


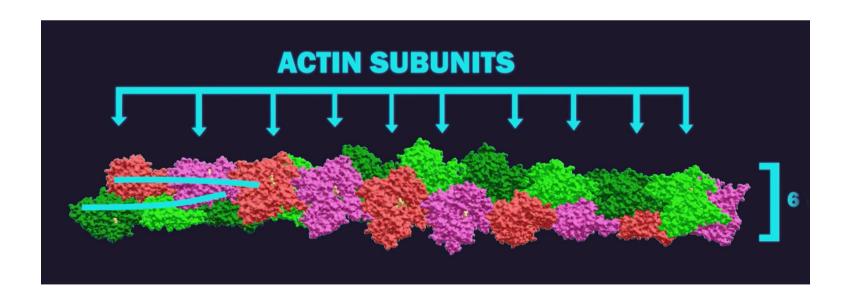
Weak, non covalent interactions More filaments bind together



#### Actin filaments







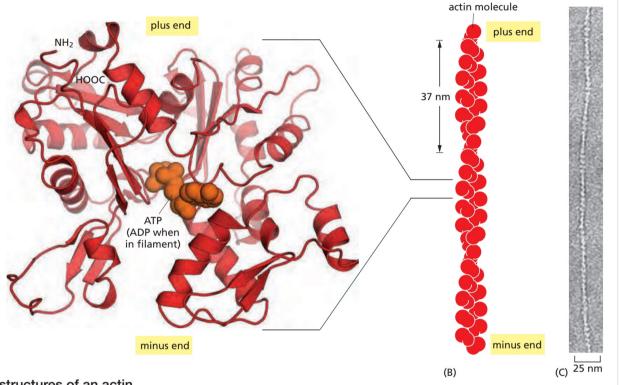
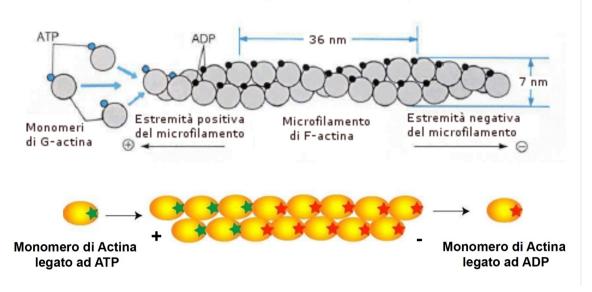


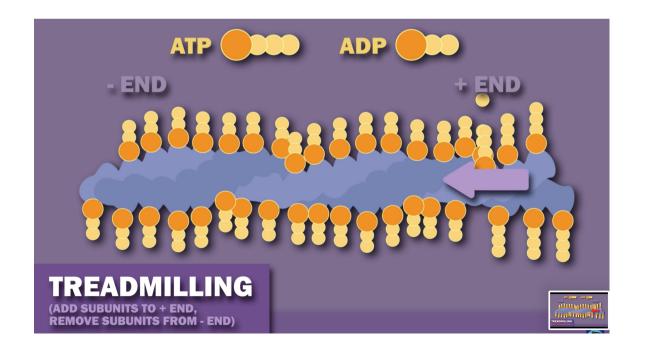
Figure 16–11 The structures of an actin monomer and actin filament. (A) The actin monomer has a nucleotide (either ATP or ADP) bound in a deep cleft in the center of the molecule. (B) Arrangement of monomers in a filament consisting of two protofilaments, held together by lateral contacts, which wind around each other as two parallel strands of a helix, with a twist repeating every 37 nm. All the subunits within the filament have the same orientation. (C) Electron micrograph of negatively stained actin filament. (C, courtesy of Roger Craig.)

Individual actin filaments are quite flexible. The stiffness of a filament can be characterized by its persistence length, the minimum filament length at which random thermal fluctuations are likely to cause it to bend. The persistence length of an actin filament is only a few tens of micrometers. In a living cell, accessory proteins bundle filament together—more rigid

Alberts, Bruce; Johnson, Alexander; Lewis, Julian; Morgan, David; Raff, Martin; Roberts, Keith; Walter, Peter. Molecular Biology of the Cell (p. 898). W. W. Norton & Company. Kindle Edition.

I filamenti di actina hanno una polarità: per convenzione l'estremità "-" cresce lentamente mentre l'estremità "+" è quella che si accresce più velocemente.





#### Actin filament dynamics:

actin can catalyze the hydrolysis of the nucleoside triphosphate ATP. For free actin subunits, this hydrolysis proceeds very slowly;

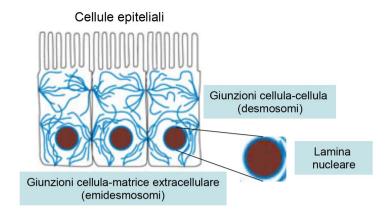
however, it is accelerated when the subunits are incorporated into filaments.

Shortly after ATP hydrolysis occurs, the free phosphate group is released from each subunit, but the ADP remains trapped in the filament structure.

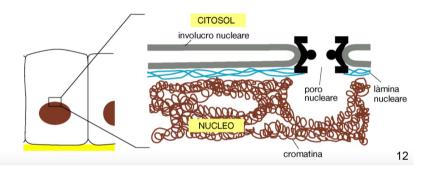
Thus, two different types of filament structures can exist, one with the "T form" of the nucleotide bound (ATP), and one with the "D form" bound (ADP). When the nucleotide is hydrolyzed, much of the free energy released by cleavage of the phosphate—phosphate bond is stored in the polymer. This makes the free-energy change for dissociation of a subunit from the D-form polymer more negative than the free-energy change for dissociation of a subunit from the T-form polymer. Consequently, the ratio of k off/k on for the D-form polymer, which is numerically equal to its critical concentration [Cc(D)], is larger than the corresponding ratio for the T-form polymer. Thus, Cc(D) is greater than Cc(T). At certain concentrations of free subunits, D-form polymers will therefore shrink while T-form polymers grow.

# Intermidiate filaments

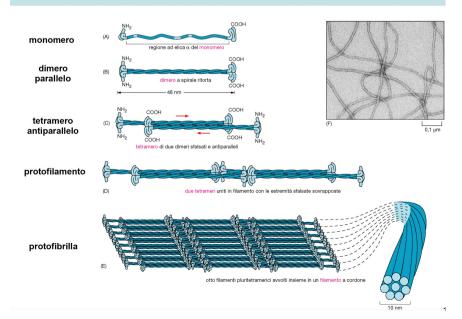
Attraversano il citoplasma da una giunzione all'altra, sostenendo la membrana plasmatica



Regolano struttura e funzione dell'involucro nucleare e della cromatina (lamine nucleari)

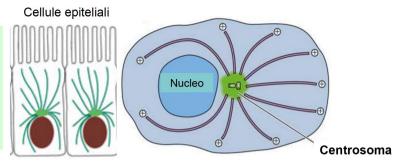


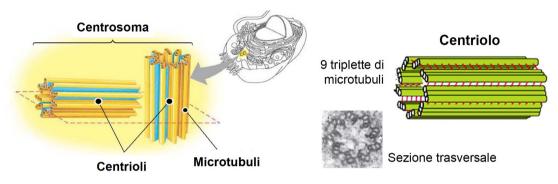
#### FILAMENTI INTERMEDI - ASSEMBLAGGIO



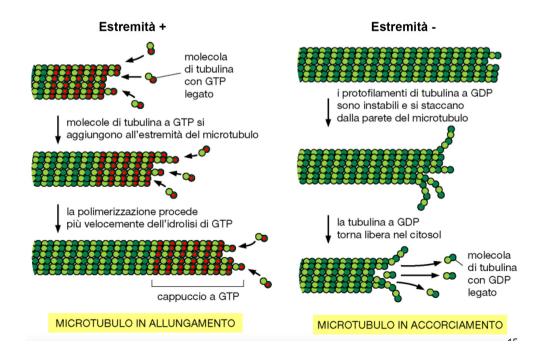
# Microtubules

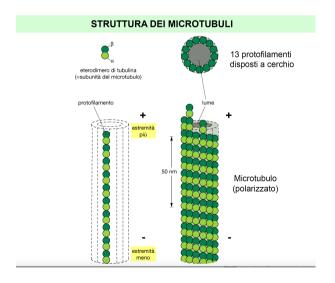
Disposti come cavi in maniera radiale polaizzata a partire dal Centro Organizzatore dei MicroTubuli (detto anche Centrosoma), cosituito da due centrioli perpendicolari





#### **MICROTUBULI - LUNGHEZZA DINAMICA**





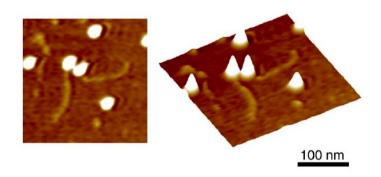
# Single molecule imaging 2

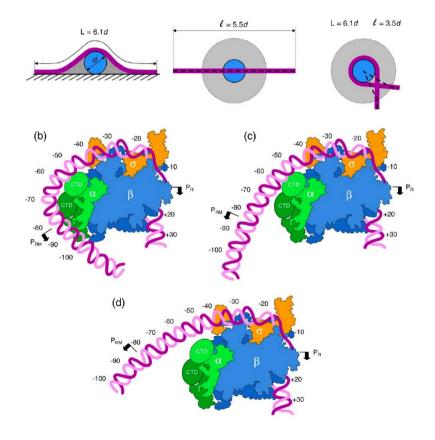
# HR-AFM imaging: DNA

Also, high-resolution AFM imaging has been recently employed to study topological details of DNA/RNA – enzymes interaction. For example, the upstream interaction of Escherichia coli RNA polymerase (RNAP) in an open promoter complex (RPo) formed at

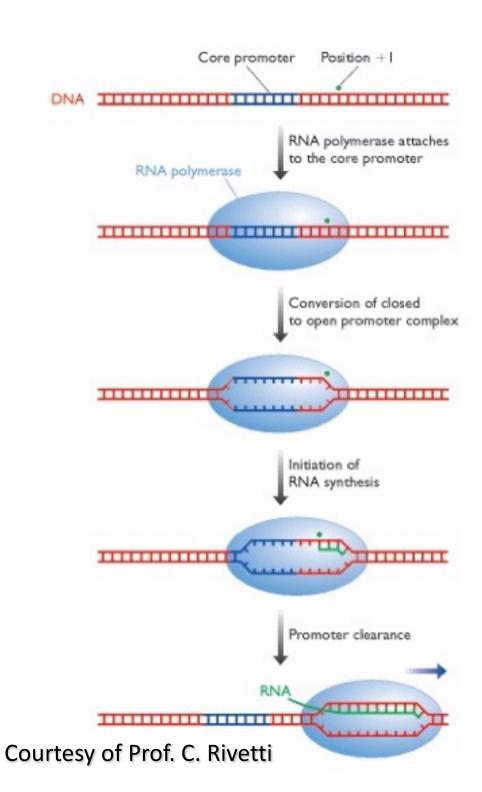
the PR and PRM promoters of

bacteriophage  $\lambda$ .

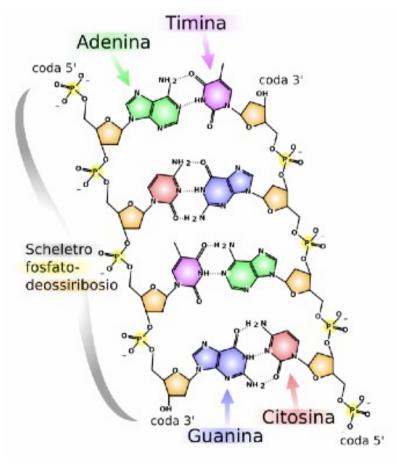


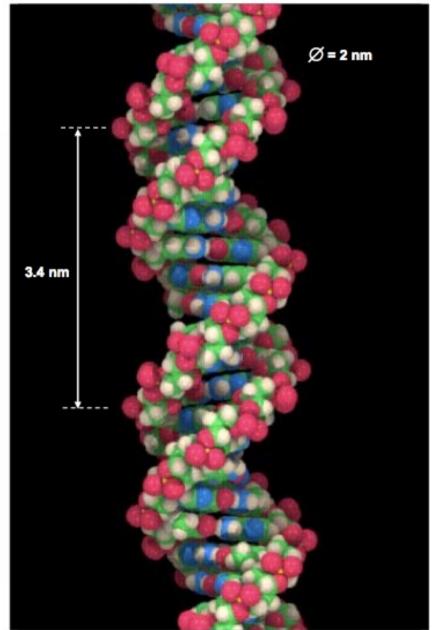


# Prokaryotic transcription

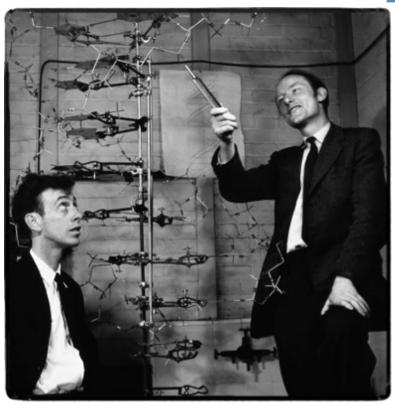


# Struttura del DNA

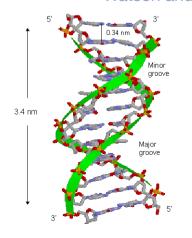


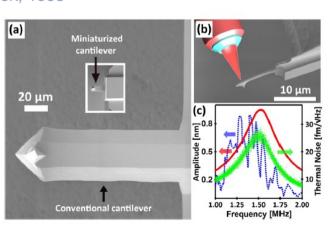


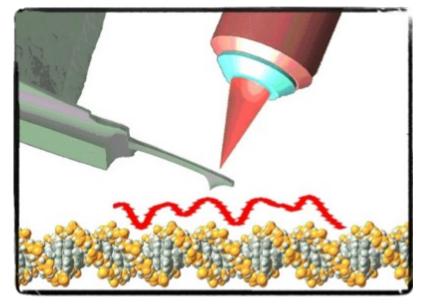
# DNA

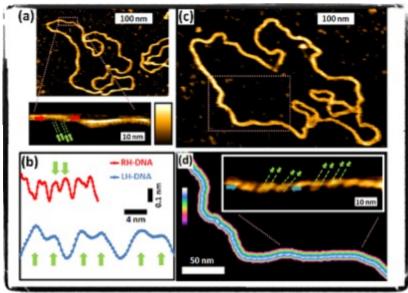


Watson and Crick, 1953









Leung et al., Nanoletters 2012

# AFM image simulation of DNA

Rc = 7.0 nm

Rc = 5.0 nm

Rc = 4.0 nm

Rc = 2.0 nm

Rc = 1.0 nm

\*\*\*\*\*

Rc = 0.5 nm

\*\*\*\*\*\*\*\*\*

Rc = 0.1 nm

\*\*\*\*\*

20 nm

Rc = 0.1 nm

3.0 nm

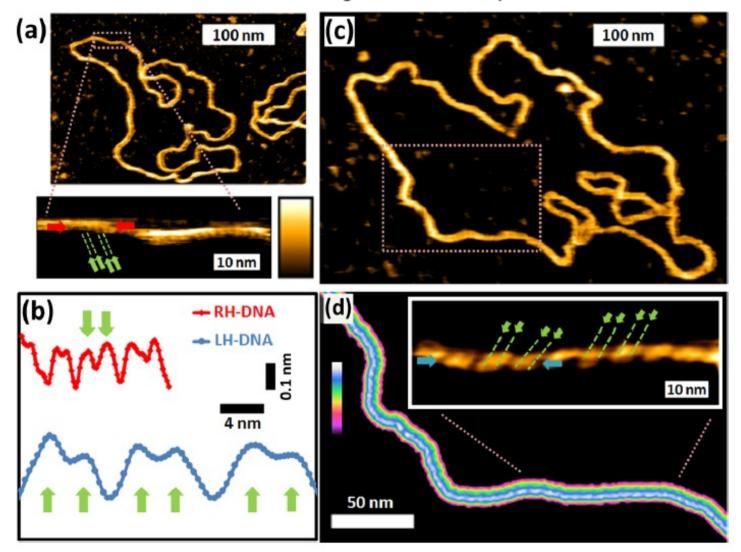
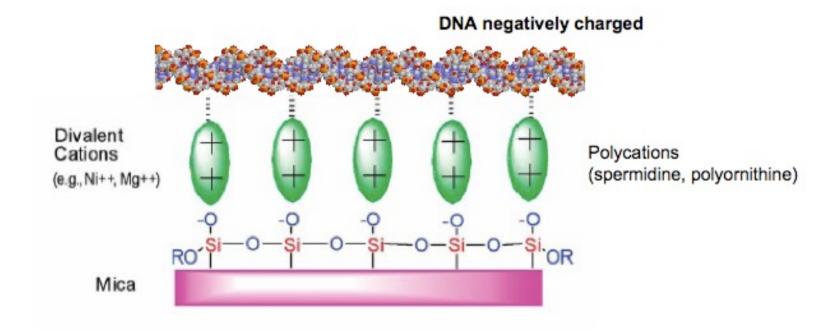
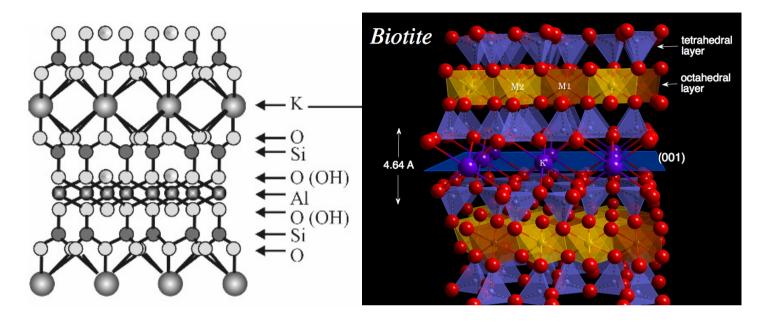
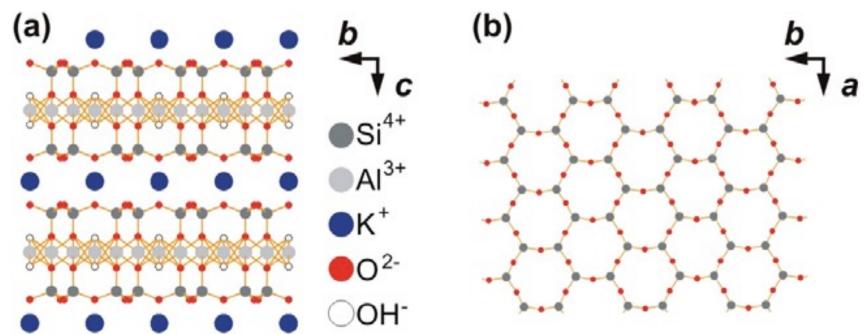


Figure 4. High-resolution AFM on plasmid DNA. (a) 3486 bp plasmid DNA, which on magnification shows a right-handed double helix, superposed to substantial height differences on and along the DNA. (b) Profiles along right-handed (RH) and left-handed (LH) DNA, acquired along the lines marked by the red and blue arrows in the insets of A and D. (c) Elongated configuration of the 3486 bp plasmid DNA displayed at the same scale as part a. (d) On subsequent magnifications of the dashed rectangle in c, an elongated left-handed double helix is resolved. Color scale: 1.5 nm (a); 1.1 nm (a); 0.7 nm (d, inset). Green arrows indicate the two strands of the double helix, separated by the minor groove (depth ≤0.1 nm). The major groove (depth ~0.2 nm) separates the subsequent turns of the double helix.

# DNA adhesion on mica







# DNA deposition methods

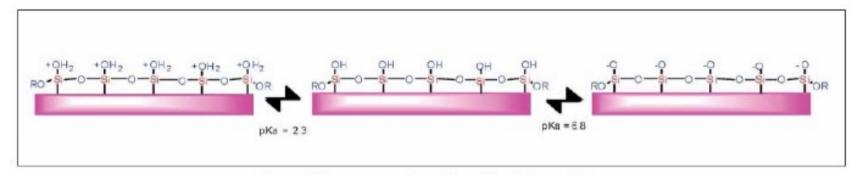


Figure 1. Siloxy groups on the surface of freshly cleaved mica.

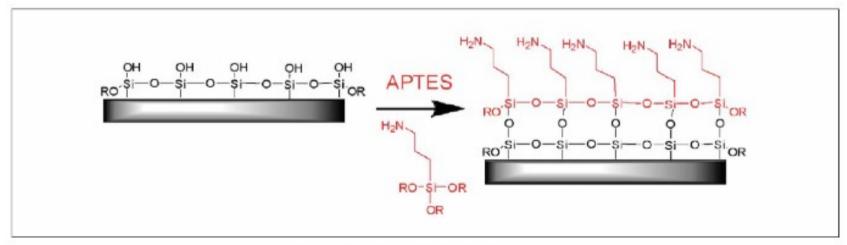
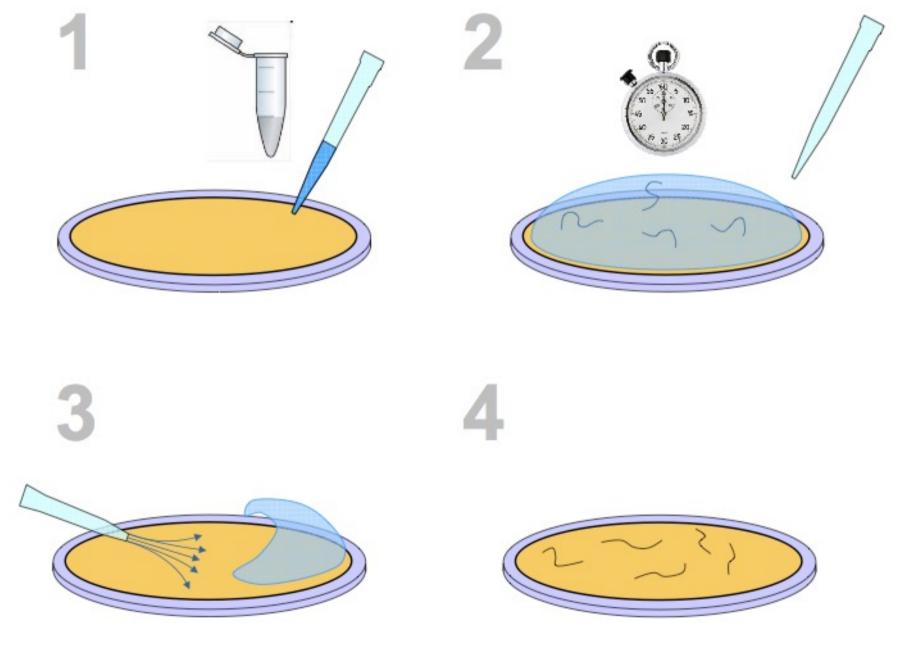


Figure 2. APTES treated mica substrate.

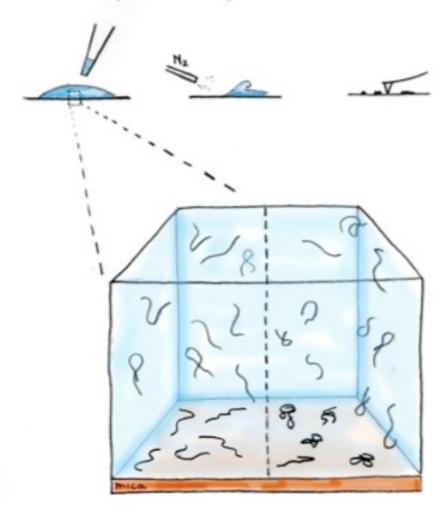
### Courtesy of C. Rivetti



Courtesy of C. Rivetti

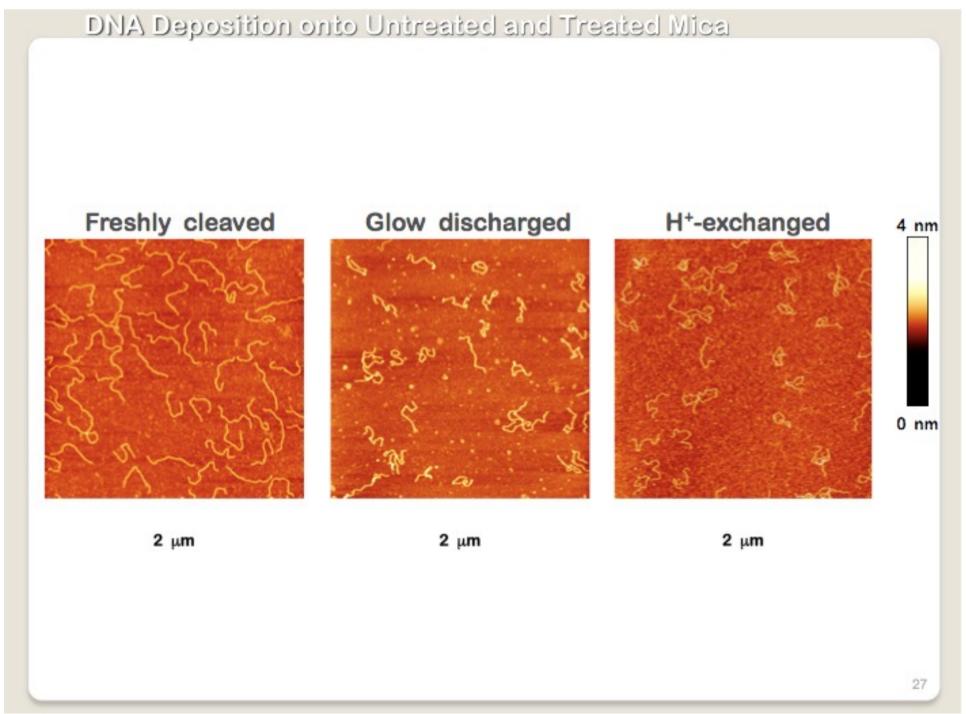
# Imaging DNA molecules onto a surface

#### DNA deposition steps



- How do DNA molecules go from solution to the surface?
- Once bound to the surface can they go back into solution?
- What happens to the molecules on the surface before removing the buffer? Can they move in 2D or are they trapped in a single conformation?

Can we quantitatively distinguish between the different cases?



Courtesy of Prof. C. Bustamante

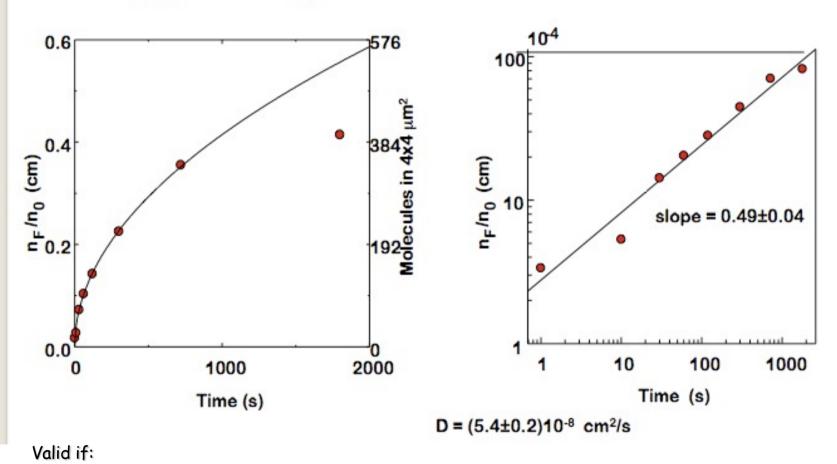
# Number of DNA Molecules on the Surface vs. Time On cleaved mica, transfer of DNA from solution to the surface is solely governed by diffusion:

$$n_F = \frac{N. \text{ of Molecules}}{Area} = \frac{2}{\sqrt{\pi}} n_0 \sqrt{Dt}$$

n<sub>0</sub> initial DNA concentration in molecules/cm<sup>3</sup>

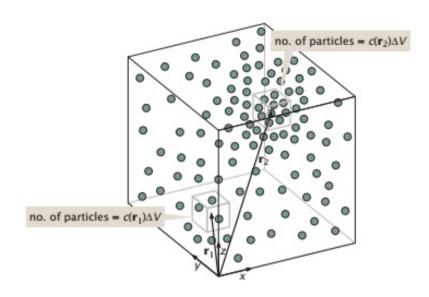
D diffusion coefficient

time of deposition



- The molecules are irreversibly adsorbed to the surface
- Convection currents do not contribute to the transport of the molecules to the surface
- The solution is not significantly depleted of DNA molecules and the surface is not saturated during the time of deposition
   Courtesy of Prof. C. Bustamante

# Diffusive dynamics

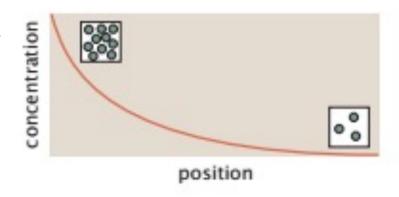


we divide space up into a bunch of small boxes, large enough to include many molecules, but small enough so that the density is nearly uniform over the scale of the box.

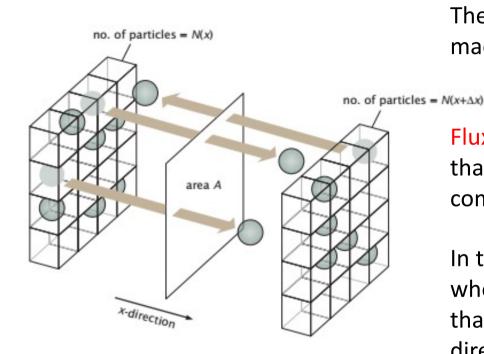
We use the notation  $c(\mathbf{r},t)$  to signify the concentration in a box centered at position  $\mathbf{r}$  in three-dimensional space (with units of number of particles per unit volume) and c(x,t) to signify the concentration field in one-dimensional problems (with units of number of particles per unit length).

"Concentration gradient" is a spatial variation in the concentration field.

simple concentration profile where on the lefthand side of the domain of interest, the concentration of the molecule of interest is high, while on the right-hand side of the domain of interest, the concentration is low



# Diffusive dynamics



The other key quantity of interest for our macroscopic description of diffusion is the flux.

Flux can be seen as the net number of molecules that cross area A per unit time. That is the component of the flux vector in that direction.

In three dimensions, the flux is actually a vector whose components give the flux across planes that are perpendicular to the x-, y-, and z directions.

The goal of our thinking is to determine what amounts to an "equation of motion" that tells how the concentration field changes in both space and time.

# Diffusive dynamics

in one dimension, flux is linearly related to concentration gradient:

$$j = -D\frac{\partial c}{\partial x},\tag{13.1}$$

J = current density, number of particles crossing unit area/ unit time D = diffusion coefficient

$$[j] = \frac{1}{\text{length}^2 \times \text{time}},$$

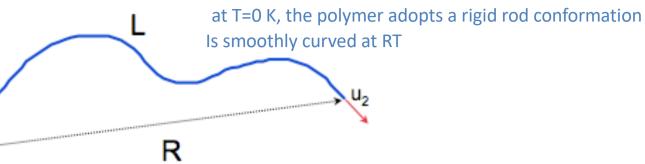
$$\left[\frac{\partial c}{\partial x}\right] = \frac{\text{number of particles/length}^3}{\text{length}} = \frac{\text{number of particles}}{\text{length}^4}.$$

[D] = length<sup>2</sup>/time, indipendent on dimensionality of space!

### Equilibrium Statistic of a Worm-like Chain

Isotropic rod continuously flexible.

The worm-like chain model is particularly suited for describing stiffer polymers, with successive segments displaying a sort of cooperativity: all pointing in roughly the same **u**<sub>1</sub> direction



$$\left\langle \overrightarrow{u_1} \cdot \overrightarrow{u_2} \right\rangle = e^{-\frac{L}{P}}$$

The persistence length of the molecule, P, is the decay length through which the initial orientation of the molecule persist. It is a measure of the stiffness of a polymer chain.

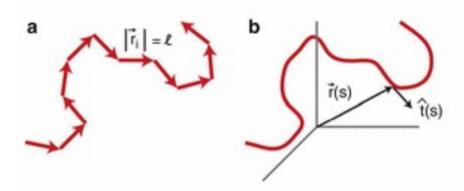
In 2D the mean square end-to-end distance of a worm-like chain of length L, and persistence length P, is:

$$\left\langle R^{2}\right\rangle _{2D}=4PL\left(1-\frac{2P}{L}\left(1-e^{-\frac{L}{2P}}\right)\right)$$
 For  $L\rightarrow\infty$   $\left\langle R^{2}\right\rangle _{2D}=4PL$ 

WLC is for semi-flexible polymers.

Assumes polymers are inextensible, has a linear elastic bending energy and is subjected to thermal fluctuations.

#### a) L=NI segments, freely rotating



Worm-Like Chain (WLC) Model, Fig. 1 (a) Discrete model of a polymer composed of a chain of segments  $\vec{r_i}$  each of length  $\ell$ . (b) Continuous WLC polymer parameterized by the tangent vector  $\vec{t}(s) = \frac{\partial \vec{r}(s)}{\partial s}$  along the contour

Adding energy cost associated to the bending:

$$H = -\varepsilon \sum_{i=1}^{N-1} \vec{r}_{i+1} \cdot \vec{r}_i, \qquad (1)$$

which imposes an energy cost of  $\varepsilon \ell^2$  times the cosine of the angle between neighboring segments. The right-hand side of (1) can be rewritten using the relation  $\vec{r}_{i+1} \cdot \vec{r}_i = \frac{2\ell^2 - (\vec{r}_{i+1} - \vec{r}_i)^2}{2}$ . Moving from a discrete model to a continuous model requires taking the limits  $N \to \infty$  and  $\ell \to 0$ . Likewise:

$$\lim_{\ell \to 0} \left( \frac{\vec{r}_{i+1} - \vec{r}_i}{\ell} \right) = \frac{\partial \hat{t}(s)}{\partial s}, \tag{2}$$

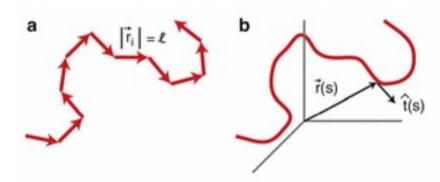
where  $\bar{t}(s)$  is a tangent vector at location s along the contour of the polymer (see Fig. 1b). Finally, converting the sum to an integral yields:

$$\frac{H}{k_B T} = \frac{\xi}{2} \int_0^L \left( \frac{\partial \bar{t}(s)}{\partial s} \right)^2 ds, \tag{3}$$

WLC is for semi-flexible polymers.

Assumes polymers are inextensible, has a linear elastic bending energy and is subjected to thermal fluctuations.

a) L=NI segments, freely rotating

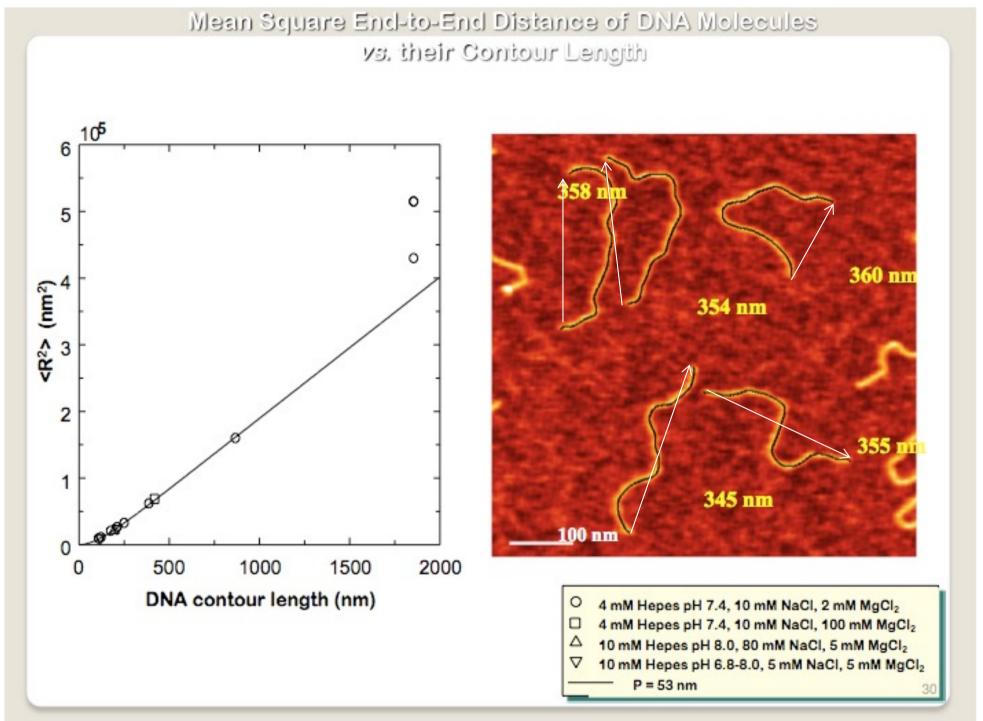


Worm-Like Chain (WLC) Model, Fig. 1 (a) Discrete model of a polymer composed of a chain of segments  $\vec{r_i}$  each of length  $\ell$ . (b) Continuous WLC polymer parameterized by the tangent vector  $\vec{t}(s) = \frac{\partial \vec{r}(s)}{\partial t}$  along the contour

which is the Hamiltonian for the WLC model assuming that the contour length L is very large (Doi and Edwards 1988). Note the introduction of the persistence length  $\xi = \frac{\varepsilon \ell}{k_B T}$ . The persistence length serves as a characteristic length scale over which two tangent vectors along the polymer remain correlated, i.e.:

$$\langle \hat{t}(s) \cdot \hat{t}(s') \rangle = e^{-\frac{|t-s'|}{\zeta}}.$$
 (4)

The WLC describes a polymer that is stiff over short distances and flexible at longer ones, with the persistence length setting the length scale of this transition in characteristic behavior. Moreover, while



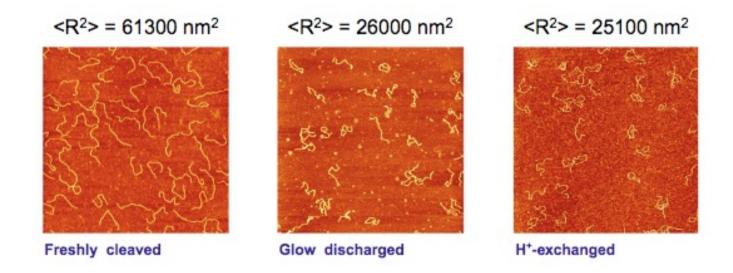
Courtesy of Prof. C. Bustamante and C. Rivetti

$$\langle R^2 \rangle_{3D} = 2PL \left( 1 - \frac{P}{r} \left( 1 - e^{-\frac{L}{P}} \right) \right)$$

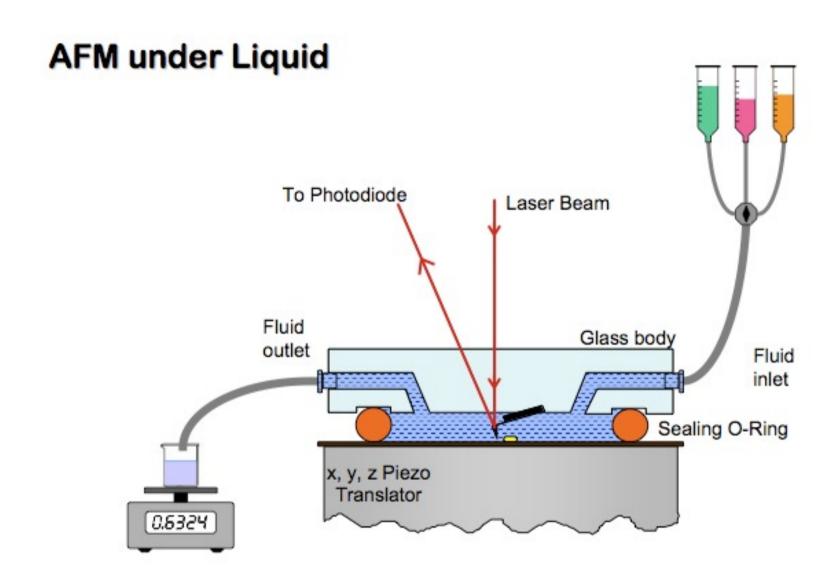
$$\langle R^2 \rangle_{2D} = 4PL \left( 1 - \frac{2P}{L} \left( 1 - e^{-\frac{L}{2P}} \right) \right)$$

$$\langle R^2 \rangle_{proj} = \langle R_x^2 \rangle + \langle R_y^2 \rangle = \frac{2}{3} \langle R^2 \rangle_{3D}$$

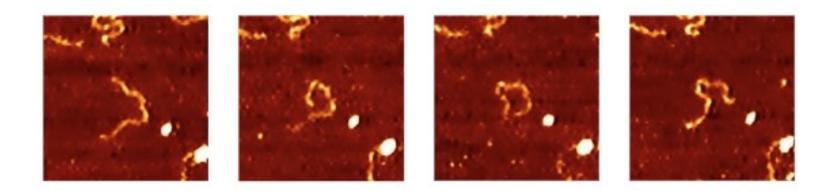
Theoretical model for a 1258 bp DNA	<r2> nm2</r2>
Ideal worm-like chain in 3D	35600
Ideal worm-like chain in 2D	60500
Orthogonal 3D → 2D projection	23700

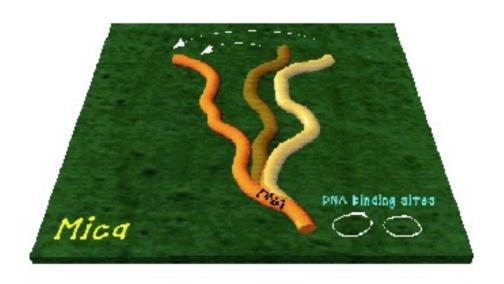


Courtesy of Prof. C. Rivetti



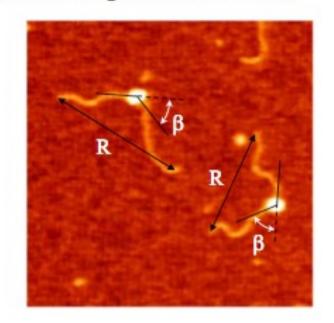
# **DNA** imaged in liquid

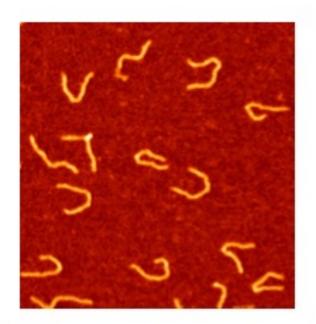


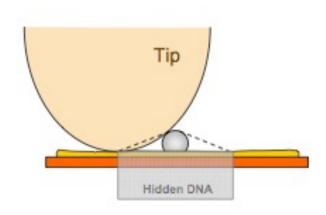


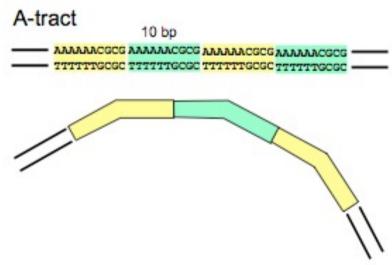
Courtesy of Prof. C. Rivetti

# DNA bend angle measurements



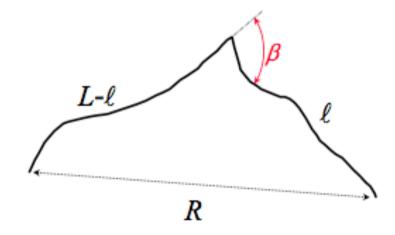






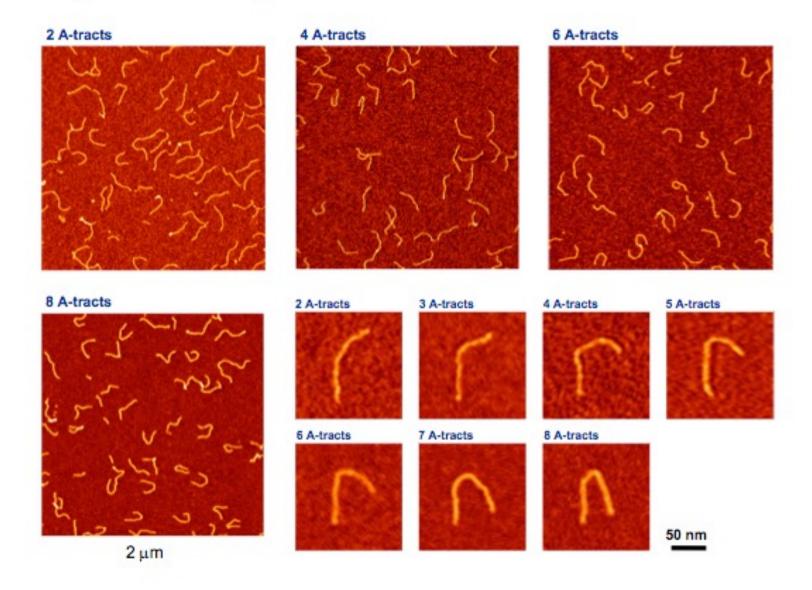
#### Using the end-to-end Distance to Determine Bend Angles

For a polymer molecule that is bent at any location along the chain, the mean square end-to-end distance is given by:

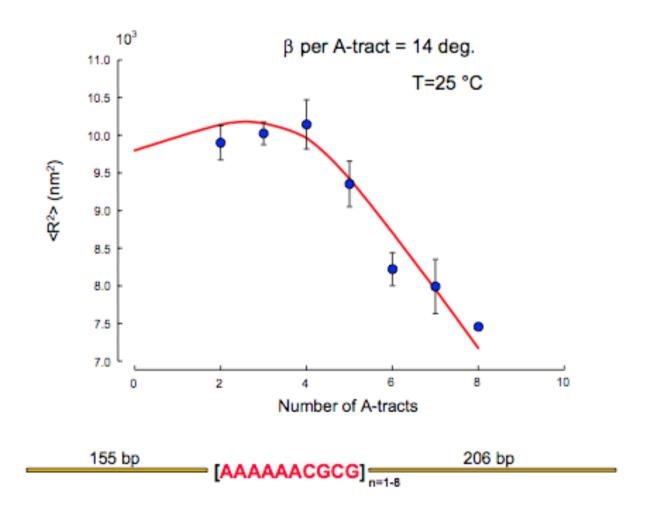


$$\left\langle R_{\beta}^{2}\right\rangle _{2D}=4PL\left\{ 1-\frac{2P}{L}\left[\left(1-e^{-\ell/2P}\right)+\left(1-e^{-(L-\ell)/2P}\right)-\cos(\beta)\left(1-e^{-\ell/2P}\right)\left(1-e^{-(L-\ell)/2P}\right)\right]\right\}$$

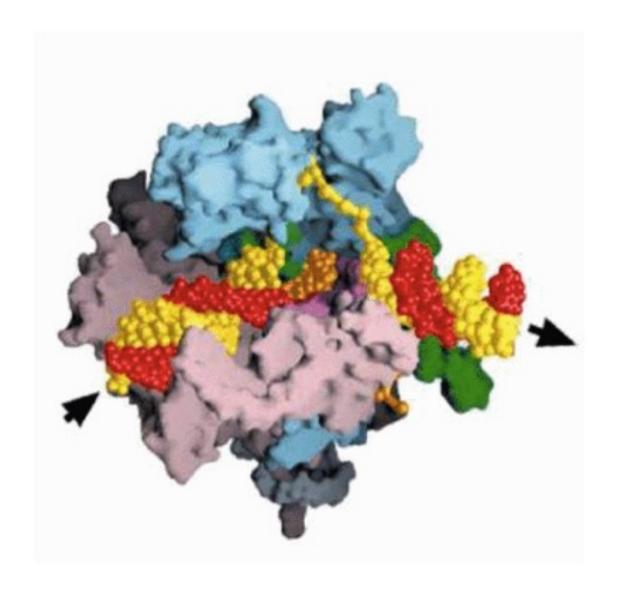
## **DNA fragments containing A-tracts**



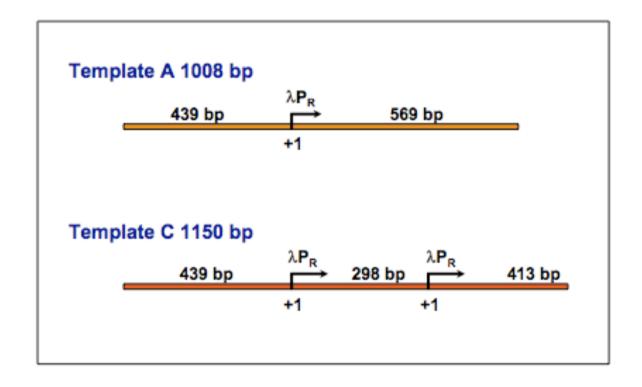
#### Mean square end-to-end distance as a function of the number of A-tracts



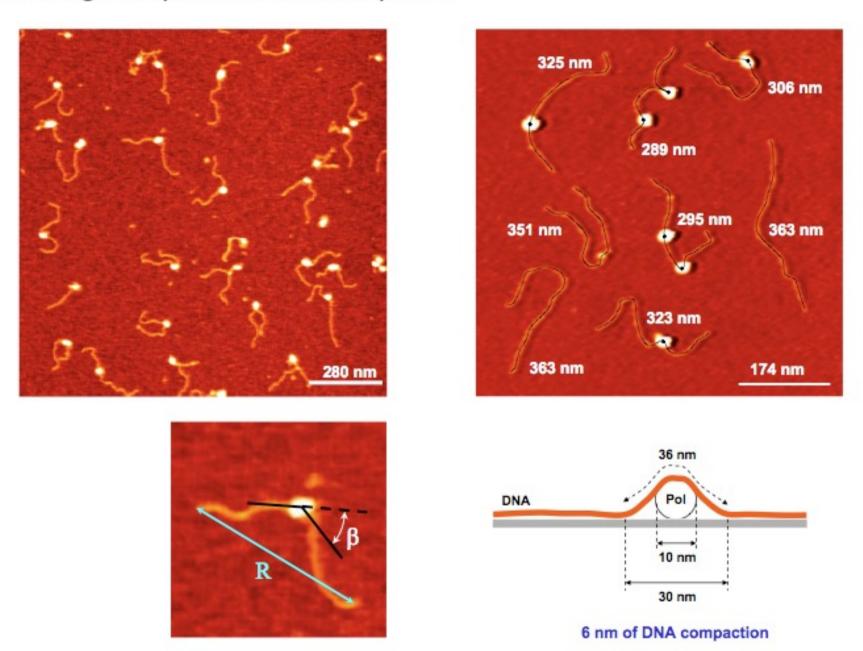
# Structure of the E. coli RNA Polymerase



Courtesy of Prof. C. Rivetti

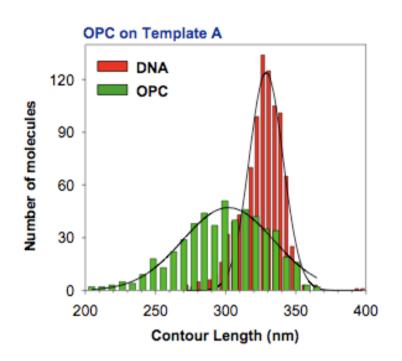


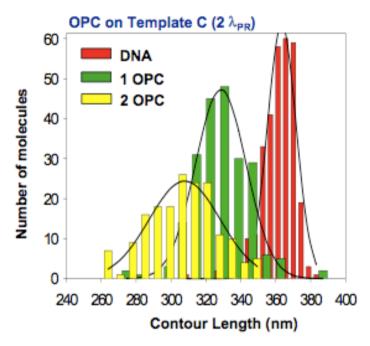
# **AFM image of Open Promoter Complexes**



Courtesy of Prof. C. Rivetti

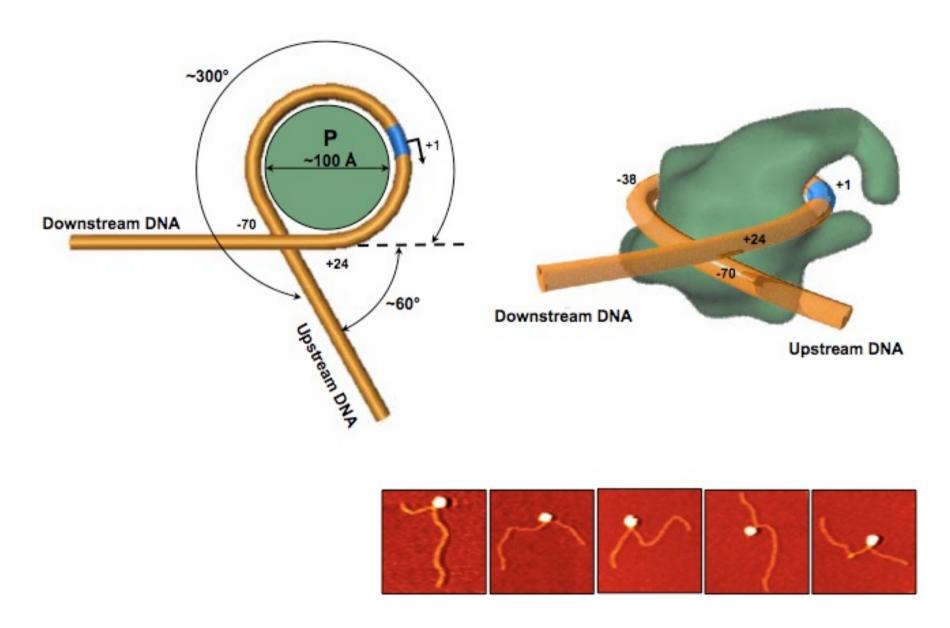
#### **DNA Contour Length Measurements of Open Promoter Complexes**





	Contour length (nm	) Compaction (nm)	N. of molecules
DNA (A) OPC (A)	329 ± 12 297 ± 34	32	947 514
DNA (C) One OPC (C) Two OPC (C)	363 ± 8 332 ±14 308 ± 20	31 55	317 157 173
	(a) NO wrapping (side view)	NO wrapping (top view) Wrapping (top view) $\ell = 5.5d \qquad \qquad L = 6.1d  \ell = 3$	3.5 <i>d</i>

# Proposed model for the open promoter complex at $\lambda_{PR}$



Courtesy of Prof. C. Rivetti