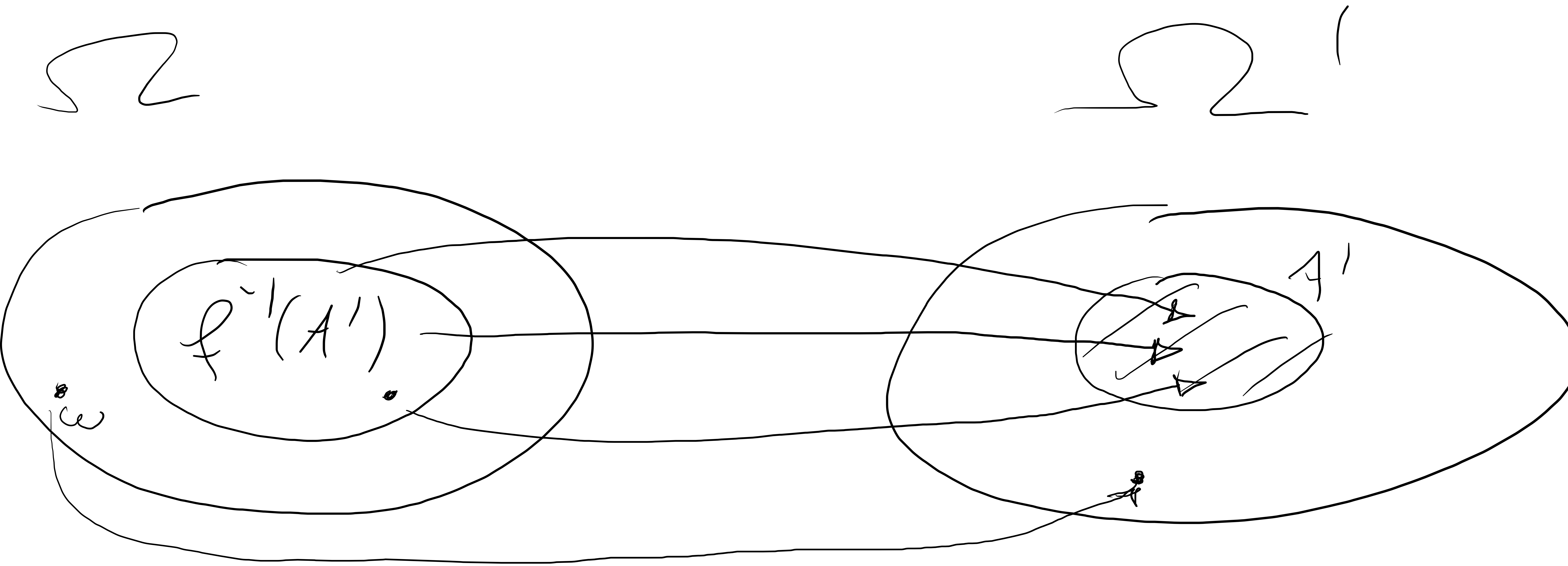
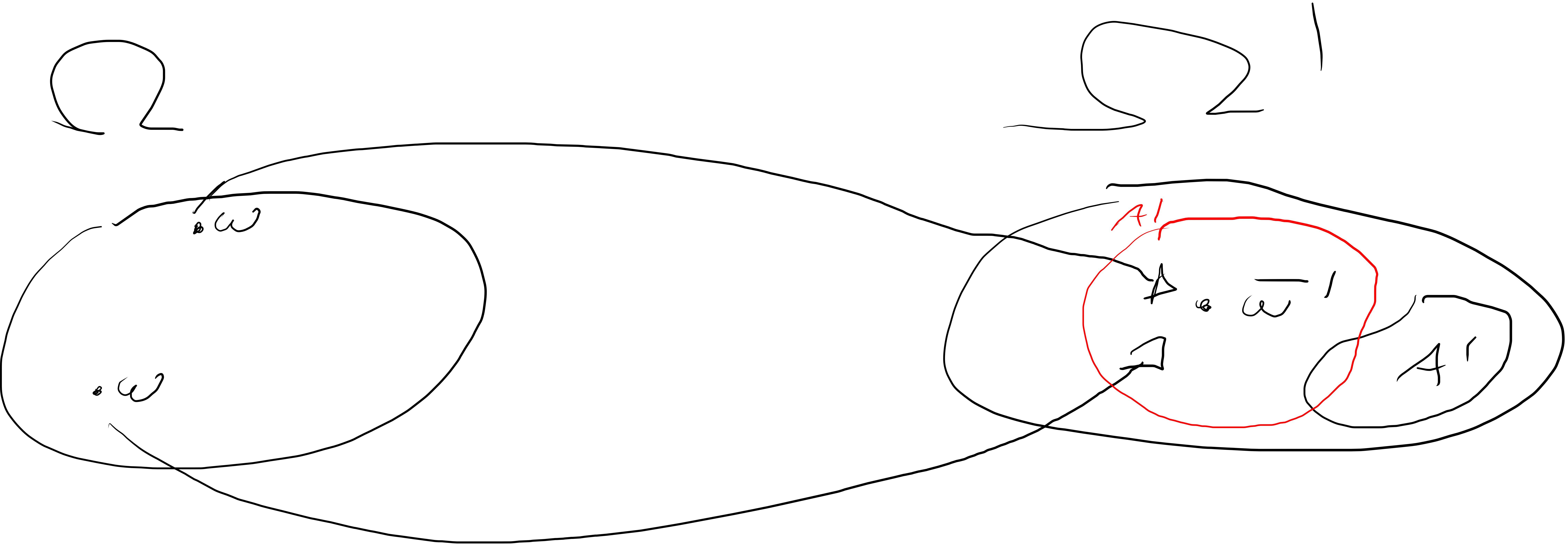


$A' \in \mathcal{F}'$

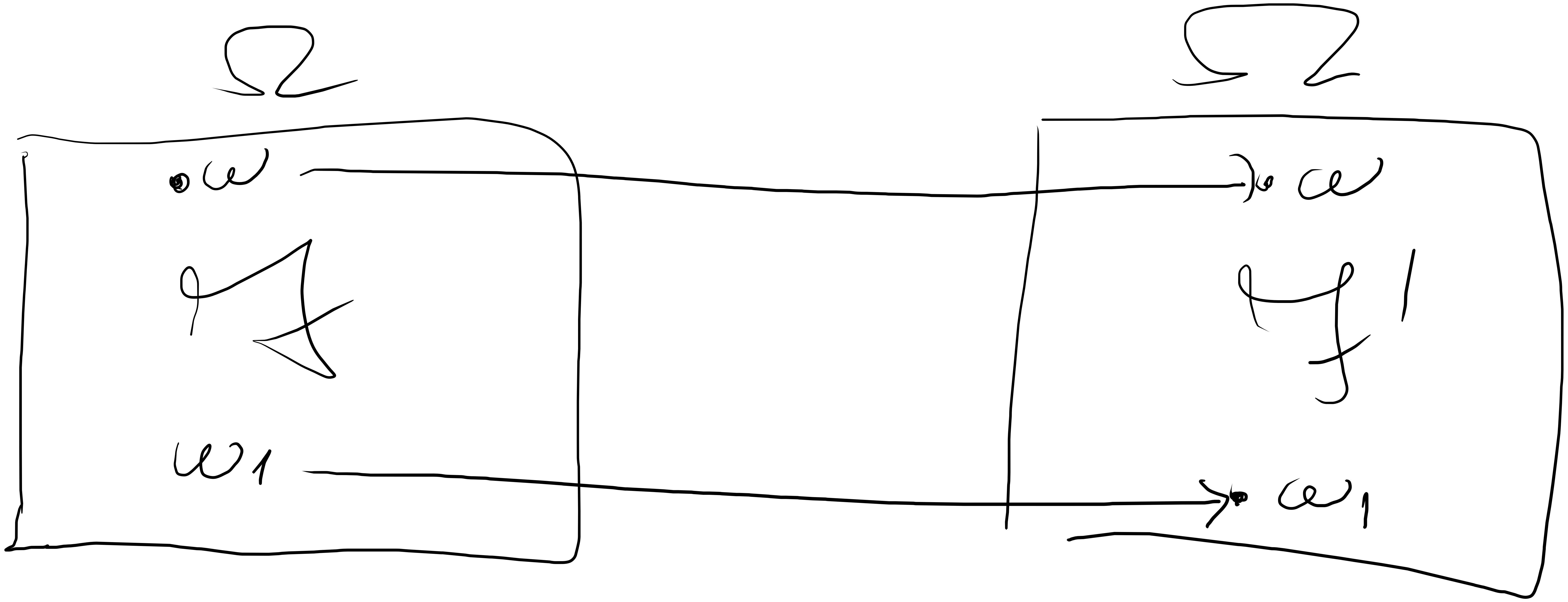


$$f^{-1}(A') = \{\omega \in \Omega \mid f(\omega) \in A'\}$$

$$f^{-1}(A') = \phi \in \mathcal{F} \quad \text{se} \quad A' \not\ni \bar{\omega}'$$

$$f^{-1}(A') = \Omega \in \mathcal{F} \quad \text{se} \quad A' \ni \bar{\omega}'$$

$\Omega' = \Omega$



$A \in \mathcal{F}' \qquad f^{-1}(A) = \{\omega \in \Omega \mid \underbrace{f(\omega)}_{=\omega} \in A\} = \underset{\substack{\Uparrow \\ \mathcal{F}}}{A}$

QUINDI f MISURABILE $\iff \mathcal{F}' \subset \mathcal{F}$

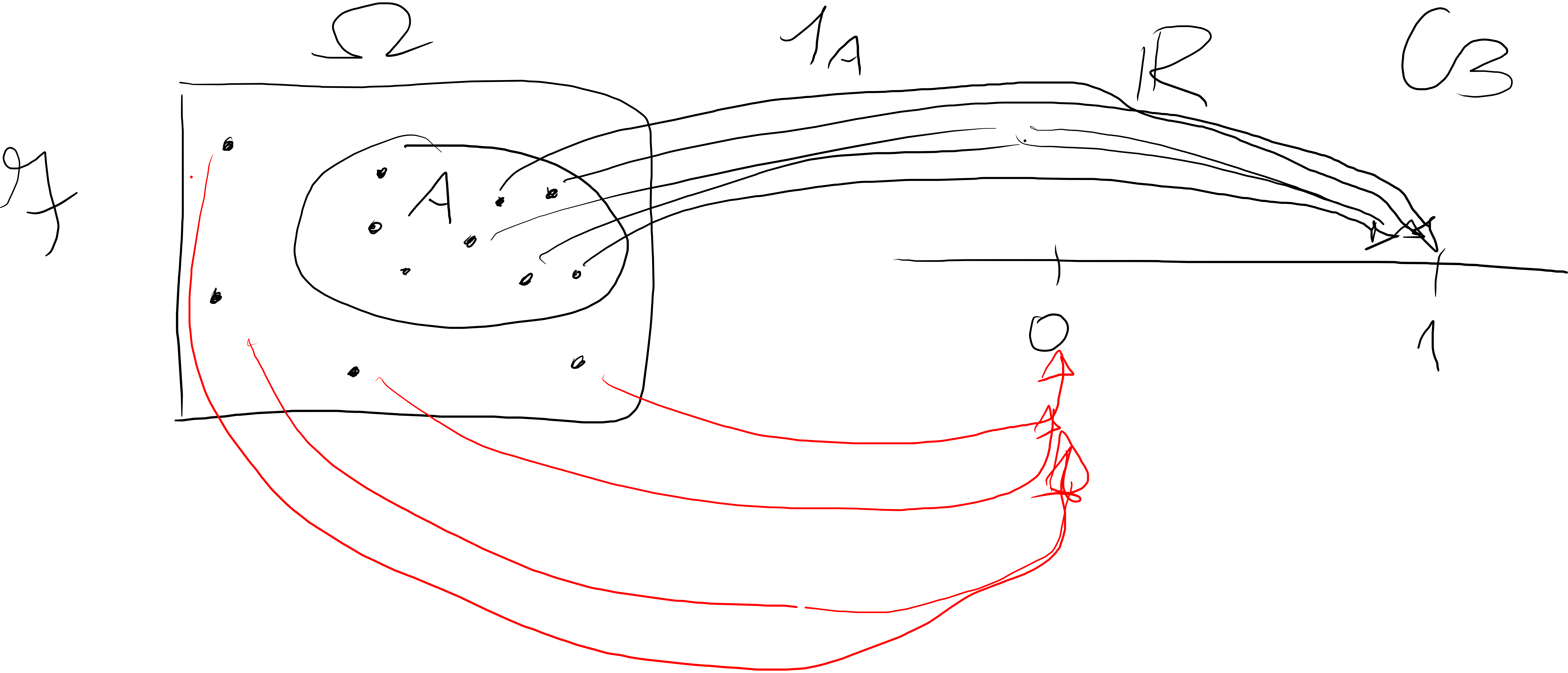
$$X : \underset{\mathcal{F}}{\Omega} \longrightarrow \underset{\mathcal{B}}{\mathbb{R}}$$

$$B = (-\infty, a]$$

$$X^{-1}(B) = X^{-1}((-\infty, a]) = \{\omega \in \Omega \mid X(\omega) \in (-\infty, a]\}$$

$$= \{\omega \in \Omega \mid X(\omega) \leq a\} = \text{"}\{X \leq a\}\text{"}$$

$$X^{-1}(B) = \{X \in B\}$$



Ω
$\mathcal{F}$
A
1_A
$\mathbb{R}$
$\mathcal{B}$
0
1

$B \in \mathcal{B}$

$$\mathbb{1}_A^{-1}(B) = \{\mathbb{1}_A \in B\}$$

$$= \{\omega \in \Omega \mid \mathbb{1}_A(\omega) \in B\}$$

$= \emptyset \in \mathcal{F}$ $\qquad 0 \notin B,\ 1 \notin B \qquad B = [2, 3]$

$= \Omega \in \mathcal{F}$ $\qquad 0 \in B,\ 1 \in B \qquad B = (-1, 5)$

$= \bar{A}$ $\qquad 0 \in B,\ 1 \notin B \qquad B = \left(-\frac{1}{2}, \frac{1}{2}\right)$

$= A$ $\qquad 0 \notin B,\ 1 \in B \qquad B = [1, +\infty)$

DEVONO STATE IN $\mathcal{F}$ PERCHÉ $\mathbb{1}_A$ SIA $\mathcal{F}$-MISURABILE

$$1_{\bar{A}} = 1 - 1_A$$

$$1_{A \cap B} = 1_A \cdot 1_B$$

1 SE $A \cap B$ È V

0 SE $A \cap B$ È F

1 SE A V

0 SE A F

1 SE B V

0 SE B F

1 SE A V, B V

0 SE ALMENO UNO DEI DUE F

$$1_{A\cup B} = 1_A + 1_B - 1_{A\cap B}$$

A	B	
V	V	$1+1-1=1$
V	F	$1+0-0=1$
F	V	$0+1-0=1$
F	F	$0+0-0=0$

$A \cup B$ È V

$X \qquad x_1, \ldots, x_n$

$$X = \sum_{i=1}^{n} x_i \cdot 1_{\{X = x_i\}}$$

$f : \Omega \longrightarrow \Omega'$

$\mathcal{F} \qquad\qquad \mathcal{F}' = \sigma(\mathcal{E}')$

SE $f^{-1}(A') \in \mathcal{F}$ PER OGNI $\boxed{A' \in \mathcal{E}'}$

ALLORA f $\mathcal{F}/\mathcal{F}'$ MISURABILE

$$\mathcal{G}' = \{ A' \subset \Omega' \mid f^{-1}(A') \in \mathcal{F} \}$$

PER IPOTESI $\mathcal{E}' \subset \mathcal{G}'$

$\mathcal{G}'$ È UNA σ - ALGEBRA ?

$\phi \in \mathcal{G}'$? $\quad f^{-1}(\phi) = \phi \in \mathcal{F}$

$\Omega' \in \mathcal{G}'$? $\quad f^{-1}(\Omega') = \Omega \in \mathcal{F}$

SE $A' \in \mathcal{G}' \Rightarrow \overline{A'} \in \mathcal{G}'$?

$$f^{-1}(\overline{A'}) = \overline{\underbrace{f^{-1}(A')}_{\in \mathcal{F}}} \in \mathcal{F}$$

$$(A'_n) \in \mathcal{G}' \Rightarrow \bigcup A'_n \in \mathcal{G}'$$

$$\underbrace{f^{-1}\left(\bigcup A'_n\right)}_{\shortparallel} \in \mathcal{F} \; ?$$

$$\bigcup_n \underbrace{f^{-1}(A'_n)}_{\in \mathcal{F}} \in \mathcal{F}$$

$\mathcal{G}'$ È UNA σ-ALGEBRA $\overset{\text{MINIMALITÀ}}{\supseteq} \sigma(\mathcal{E}')$

$\mathcal{E}' \subset \mathcal{G}'$

$$\underbrace{\sigma(\mathcal{E}')}_{=\mathcal{F}'} \subset \mathcal{G}'$$