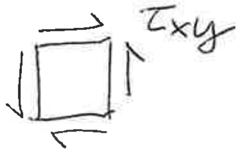
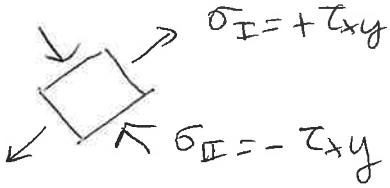
DIMOSTRAZIONE DEL FATTO CHE $G = \dfrac{E}{2(1+\nu)}$

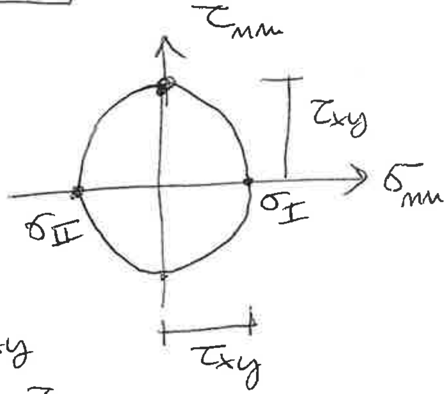
SI BASA SUL CONFRONTO DEL POTENZIALE ELASTICO φ PER DUE DIVERSE RAPPRESENTAZIONI DELLO STATO TENSIONALE DI TAGLIO PURO.

① 

$$\varphi_{①} = \frac{1}{2}\left(2\,\tau_{xy}\,\varepsilon_{xy}\right), \qquad \varepsilon_{xy} = \frac{\tau}{2G}$$

$$\varphi_{①} = \frac{1}{2}\,\frac{\tau_{xy}^2}{G}$$

② 



$$\varphi_{②} = \frac{1}{2}\left(\sigma_{xx}\,\varepsilon_{xx} + \sigma_{yy}\,\varepsilon_{yy}\right)$$

con $\sigma_{xx} = \sigma_I = \tau_{xy}$

$\sigma_{yy} = -\sigma_{II} = -\tau_{xy}$

Dal legame costitutivo:

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\,\sigma_{yy}\right] = \frac{1+\nu}{E}\,\tau_{xy}$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\,\sigma_{xx}\right] = -\frac{1+\nu}{E}\,\tau_{xy}$$

$$= \frac{1}{2}\left(\tau_{xy}\,\frac{1+\nu}{E}\,\tau_{xy} + (-\tau_{xy})\left(-\frac{1+\nu}{E}\,\tau_{xy}\right)\right)$$

$$= \frac{1+\nu}{E}\,\tau_{xy}^2$$

Ma $\varphi_{①} = \varphi_{②} \Rightarrow \dfrac{1}{2}\,\dfrac{\cancel{\tau_{xy}^2}}{G} = \dfrac{1+\nu}{E}\,\cancel{\tau_{xy}^2} \Rightarrow G = \dfrac{E}{2(1+\nu)}$