Systems Dynamics

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Lecture 5

Model Identification from Data

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System Identification

Modelling
Identification
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Disciplines providing tools to **estimate** variables and unknown parameters and to **design models** of natural and artificial systems using **experimental data**.

Why do we need models?

*Constructing models for a slice of reality and studying their properties is really what science is about. The **models** – "the hypotheses", "the laws of nature", "the paradigms" – can be of a more or less formal character, but they all have the **fundamental property** that they try **to link observations to some pattern**.*

L. Ljung, T. Glad, "Modeling of Dynamic Systems", Prentice Hall, 1994

System Identification: an Introduction

“Transparent Box” vs. “Black Box” Modeling Approach

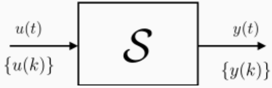
"Transparent Box" Modeling Approach

So far, approach undertaken to devise dynamical models:

Inputs ("causes")

$u(t) \in \mathbb{R}^m$

$\{u(k) \in \mathbb{R}^m\}$



Outputs ("effects")

$y(t) \in \mathbb{R}^p$

$\{y(k) \in \mathbb{R}^p\}$

Definition of the "system" entity to be analysed $\Longrightarrow$ Physical laws, a priori knowledge, heuristic considerations, statistical evidence, etc. $\Longrightarrow$ *Mathematical models*: algebraic and/or differential and/or difference equations

A Different (Data-Based) Approach

Experimental data, sensor measurements, historical data, etc.

"Black-box" modeling approach

Input, output and disturbance variables are characterized in terms of numerical sequences. These are the data to be used to determine the dynamical model.

“Black-Box” Modeling Approach (Identification)

Given: $\{y(k)\}$, $\{u(k)\}$

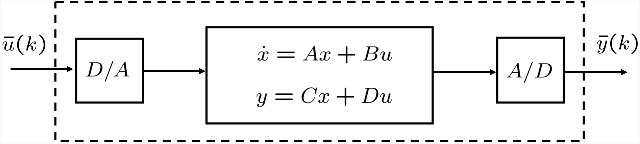
Discrete-time model:

- continuous-time system and data obtained by sampling
- discrete-time system and data inherently discrete-time

Finite-difference equations

Continuous-Time System and Data Obtained by Sampling

Linear, time-invariant case:



$u(t) = \bar{u}(k)$

$t_k \leq t < t_{k+1}$

Recall the ***step-invariant transform***

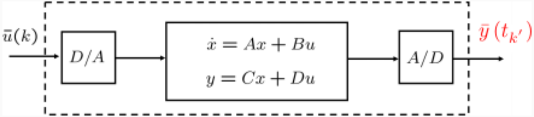
$\bar{y}(k) = y\left(t_k\right)$

$$\begin{cases} \bar{x}\left[(k+1)\right] = \bar{A}\bar{x}\left(k\right) + \bar{B}\bar{u}\left(k\right) \\ \bar{y}\left(k\right) = \bar{C}\bar{x}\left(k\right) + \bar{D}\bar{u}\left(k\right) \end{cases}$$

Letting $t_{k+1} - t_k = \Delta$

$$\bar{A} = e^{A\Delta} \qquad \bar{B} = \int_0^{\Delta} e^{Ar} B\, dr \qquad \bar{C} = C \qquad \bar{D} = D$$

Continuous-Time System and Data Obtained by Sampling (cont.)



$u(t) = \bar{u}(k)$

$t_k \leq t < t_{k+1}$

What if the output is sampled at $t_{k'} \neq t_k$ with $t_k \leq t_{k'} < t_{k+1}$?

$\bar{y}(k) = y(t_{k'})$

- Let's recall the expression

$$y(t) = C\, e^{A\,(t-t_0)}\, x\,(t_0) + C \int_{t_0}^{t} e^{A(t-\tau)} Bu(\tau)\, d\tau + Du(t)$$

for the movement of the output of a continuous-time LTI system (from "*Fundamentals of Automatic Control*").

Continuous-Time System and Data Obtained by Sampling (cont.)

- Let's consider t_k as initial time instant (i.e $t_0 = t_k$), the instant $t_{k'}$ as final time instant and let's assume $t_{k'} - t_k = \alpha$. Recall also the stair-wise behavior of the input signal: $u(t) = \bar{u}(k)\,,\ t_k \le t < t_{k+1}$.

$$y\,(t_{k'}) = C\,e^{A\alpha}\,x\,(t_k) + C\left(\int_{t_k}^{t_{k'}} e^{A(t_{k'}-\tau)}B\,d\tau\right)u(t_k) + Du(t_k)$$

- Substitute $r = t_{k'} - \tau$ into the integral term and rewrite the expression

$$y\,(t_{k'}) = C\,e^{A\alpha}\,x\,(t_k) + C\left(\int_0^{\alpha} e^{Ar}\,dr\right)Bu(t_k) + Du(t_k)$$

- Let's compare with the discrete-time output expression

$$\bar{y}(k) = \bar{C}\bar{x}(k) + \bar{D}\bar{u}(k)$$

Continuous-Time System and Data Obtained by Sampling (cont.)

- If $t_{k'} \neq t_k$ then

$$\bar{C} = C\, e^{A\alpha} \qquad \bar{D} = C \left(\int_0^{\alpha} e^{Ar}\, dr \right) B + D \qquad t_{k'} - t_k = \alpha\, (< \Delta)$$

- When $t_{k'} = t_k$ obviously

$$\bar{C} = C \qquad \bar{D} = D$$

- In both the cases the following expressions hold (remember the *step-invariant transform*)

$$\bar{A} = e^{A\Delta} \qquad \bar{B} = \int_0^{\Delta} e^{Ar} B\, dr \qquad \Delta = t_{k+1} - t_k\ \forall k$$

“Black Box” Modeling: an Example

As usual, let’s assume Δ as the sampling time.

Output sequence (unknown)

$$3\,y\left[(k+1)\Delta\right] - 2\,y\left[k\Delta\right] + 4\,y\left[(k-1)\Delta\right] =$$

$$2\,u\left[k\Delta\right] - u\left[(k-1)\Delta\right]$$

Input sequence (known)

Typical Notations

- With sampling-time Δ enhanced:

$$3y\left[(k+1)\Delta\right] - 2y\left[k\Delta\right] + 4y\left[(k-1)\Delta\right] = 2u\left[k\Delta\right] - u\left[(k-1)\Delta\right]$$

- Compact:

$$3y_{k+1} - 2y_k + 4y_{k-1} = 2u_k - u_{k-1}$$

General Expression

- Typical framework: linear finite-difference equations with constant coefficients.
- General expression takes on the form:

$$a_n y_{k+n} + a_{n-1} y_{k+n-1} + \cdots + a_0 y_k = b_m u_{k+m} + b_{m-1} u_{k+m-1} + \cdots + b_0 u_k$$

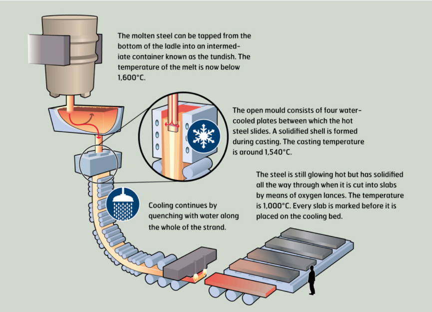
with given initial conditions

$$\{y_k : k = -n\,,\ -(n-1)\,,\ \ldots\,,\ 0\}$$

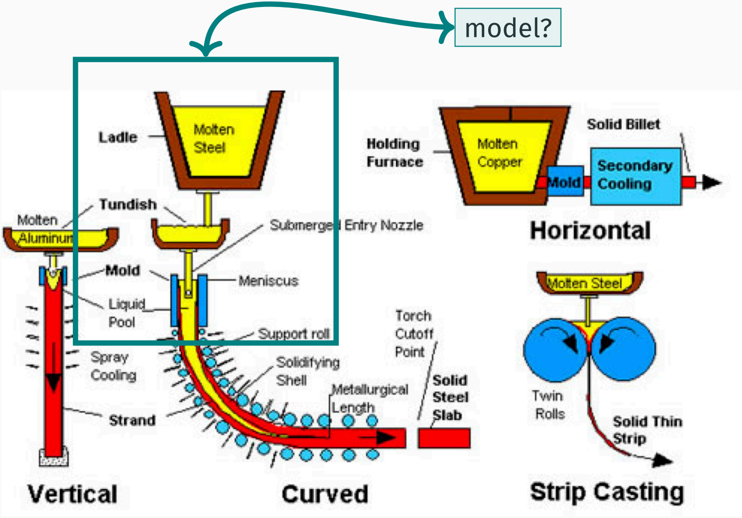
and known input sequence u_k .

An Example: a Real Application

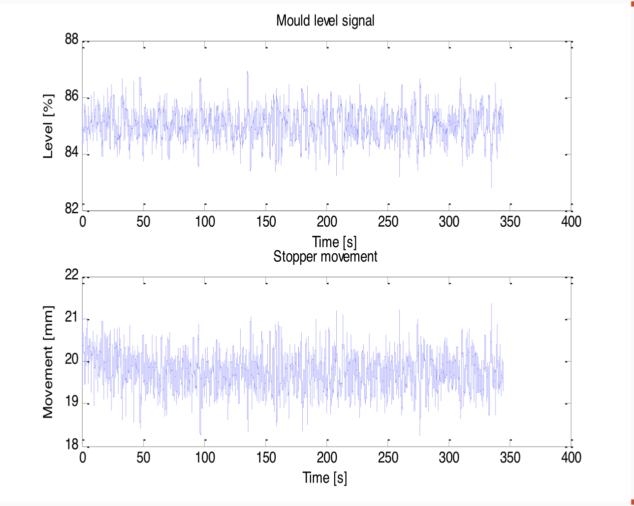
A Real Application: Steel Continuous Casting



A Real Application: Steel Continuous Casting (cont.)



A Real Application: Steel Continuous Casting (cont.)



"Black-box" modeling approach

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