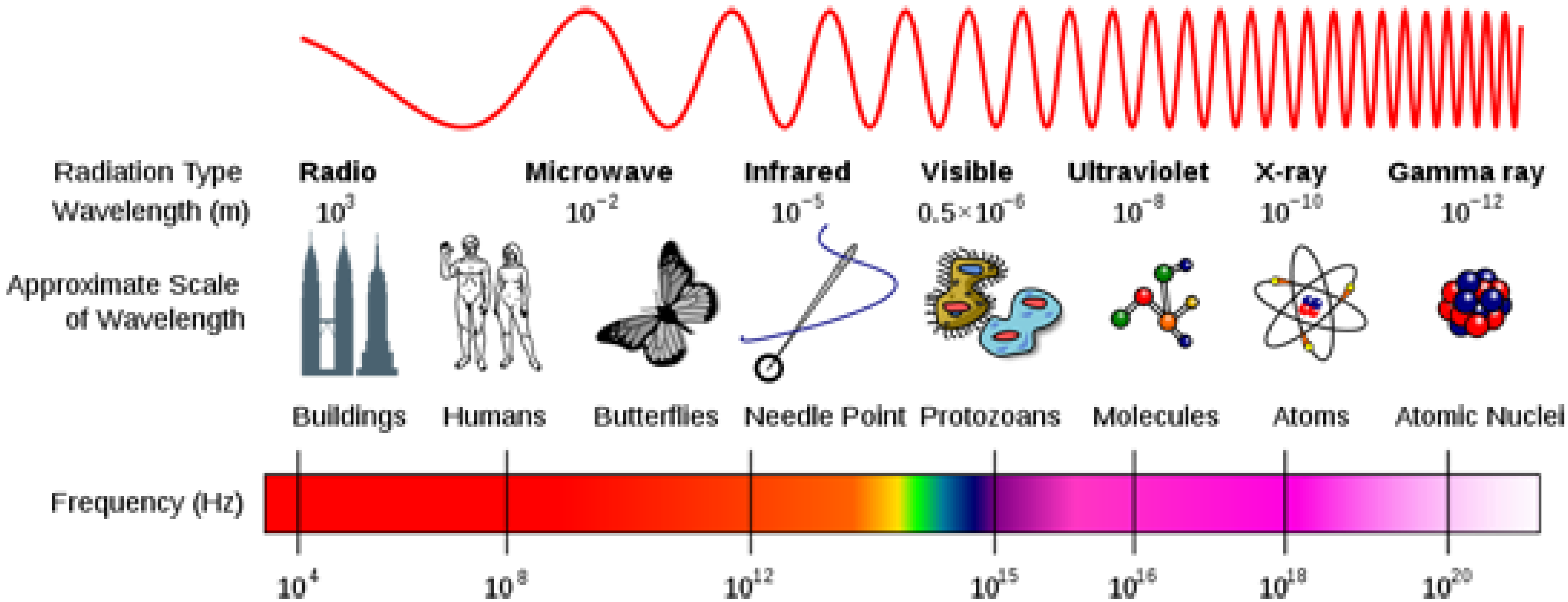
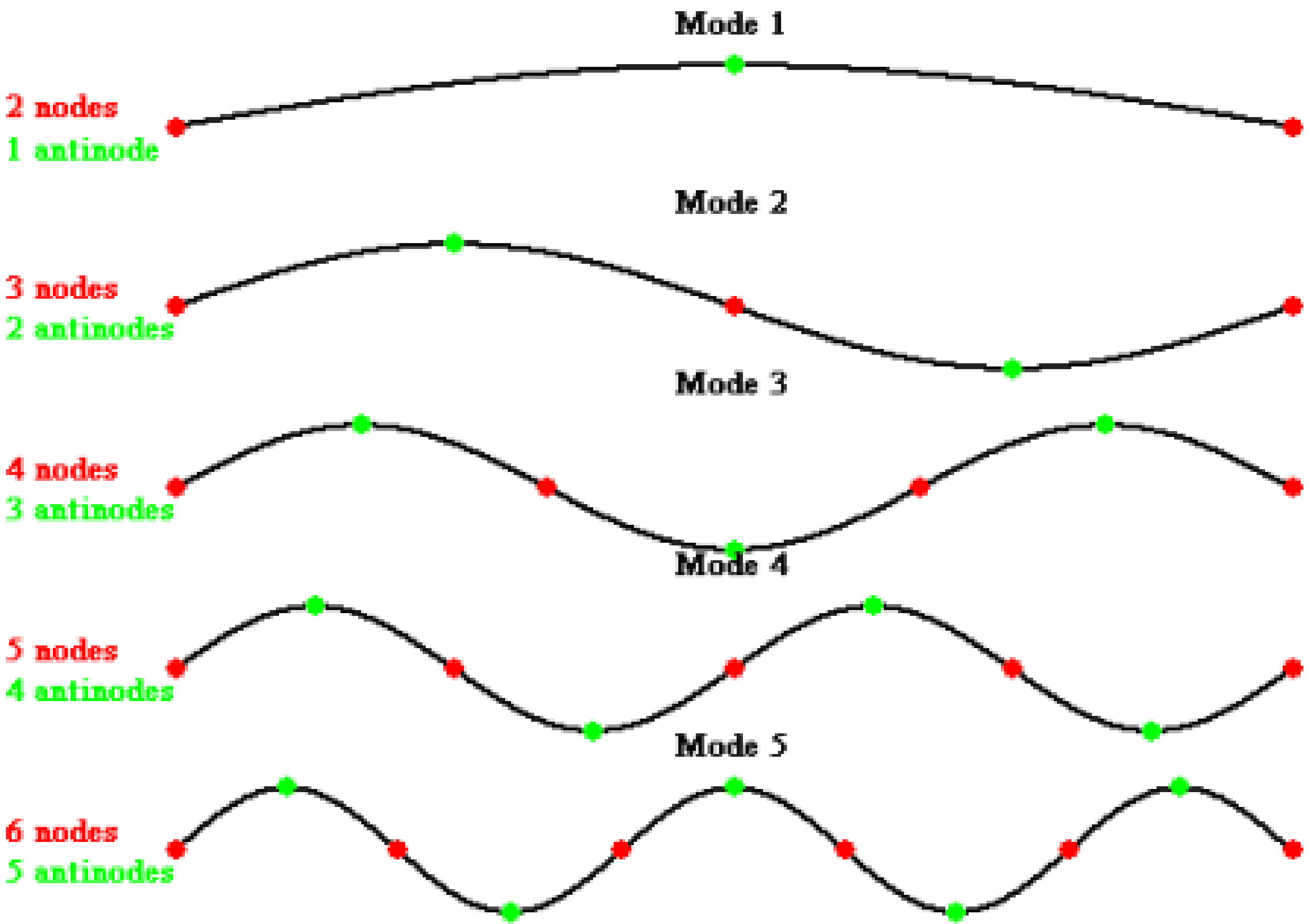
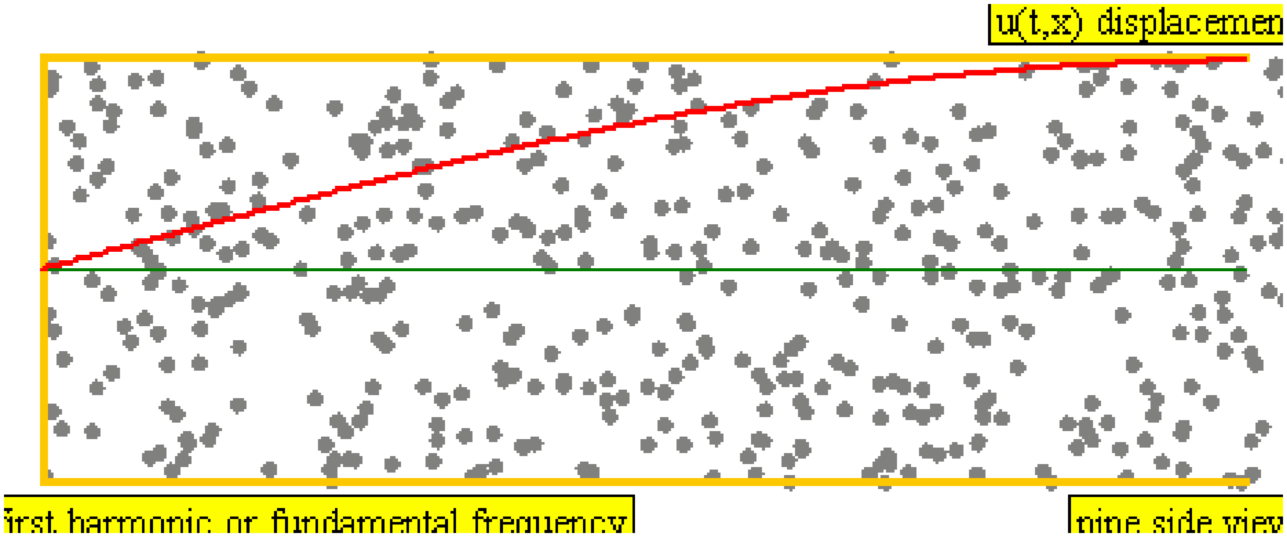


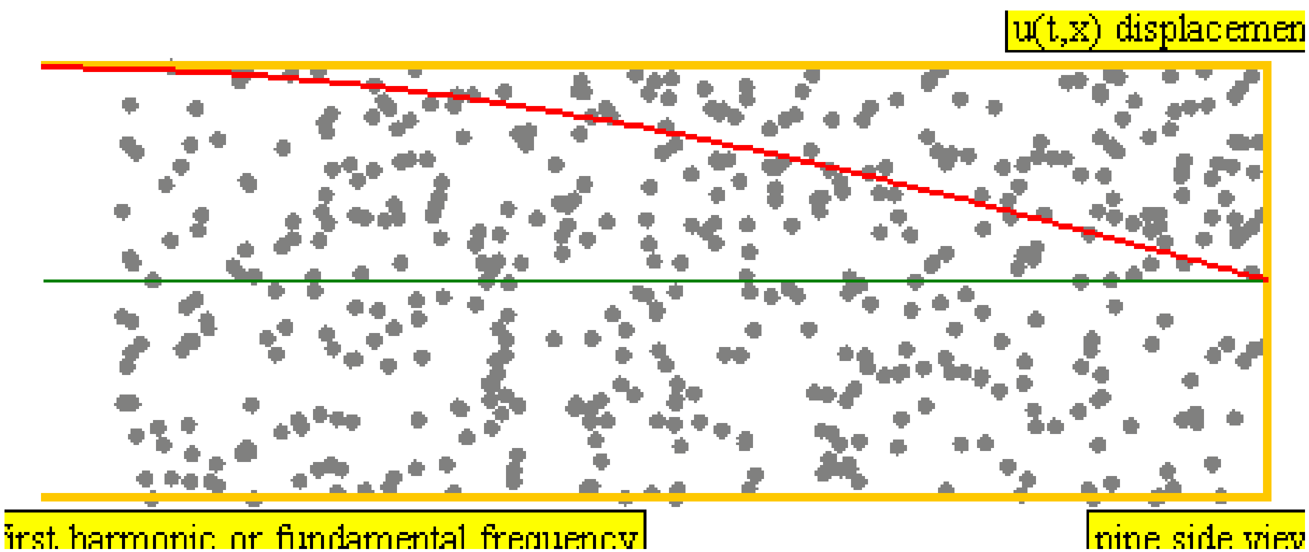
Radiation Type
Wavelength (m)
Radio
10^{3}
Microwave
10^{-2}
Infrared
10^{-5}
Visible
0.5×10^{-6}
Ultraviolet
10^{-8}
X-ray
10^{-10}
Gamma ray
10^{-12}
Approximate Scale of Wavelength
Buildings
Humans
Butterflies
Needle Point
Protozoans
Molecules
Atoms
Atomic Nuclei
Frequency (Hz)
10^{4}
10^{8}
10^{12}
10^{15}
10^{16}
10^{18}
10^{20}


Why do certain buildings fall in earthquakes?
Using *analogies* to understand *resonant frequency**
*Natural frequency of vibration determined by the physical parameters of the vibrating object.
IRIS
Classroom demo at end of this animation
NSF

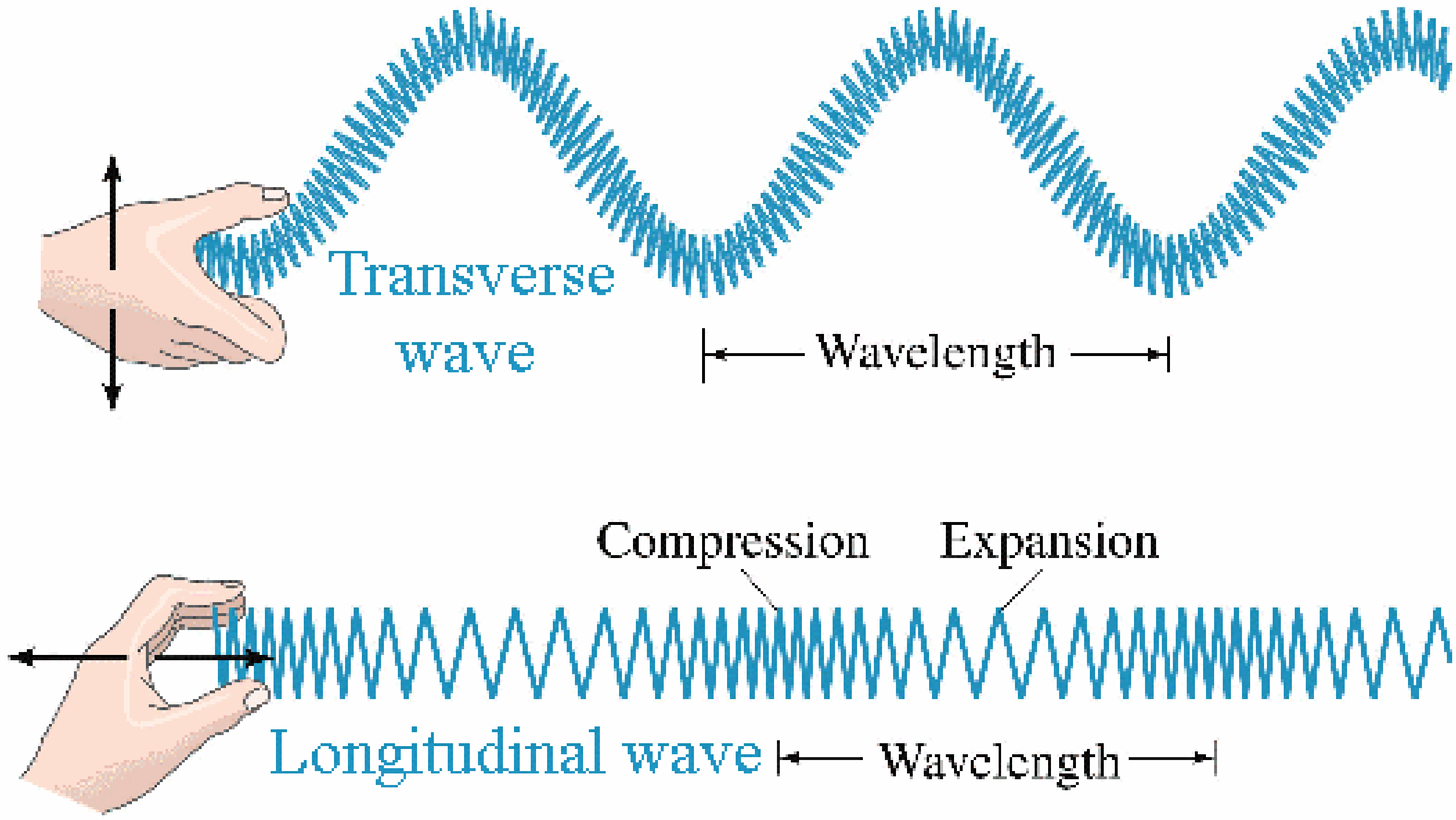

Mode 1
2 nodes
1 antinode
Mode 2
3 nodes
2 antinodes
Mode 3
4 nodes
3 antinodes
Mode 4
5 nodes
4 antinodes
Mode 5
6 nodes
5 antinodes

isvr


u(t,x) displacemen
irst harmonic or fundamental frequency
pipe side view

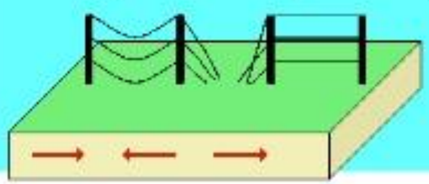

u(t,x) displacemen
irst harmonic or fundamental frequency
pipe side view

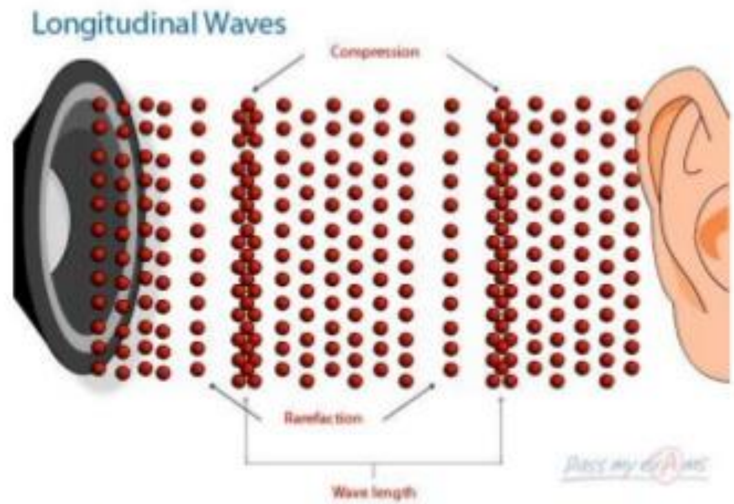
http://scienceprimer.com/embed/waveType.min.html

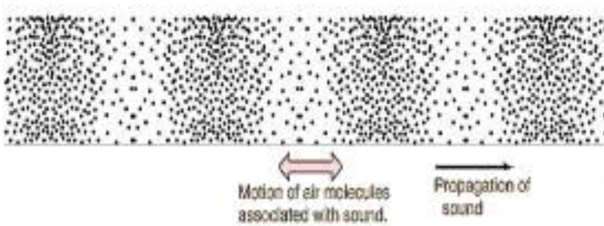

Transverse
wave
Wavelength
Compression
Expansion
Longitudinal wave
Wavelength

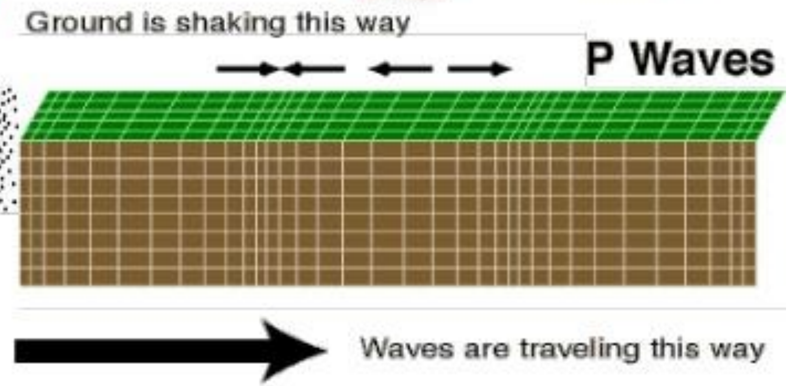
Longitudinal waves

- **Examples?**
  - Sound waves
  - P waves (earthquake)

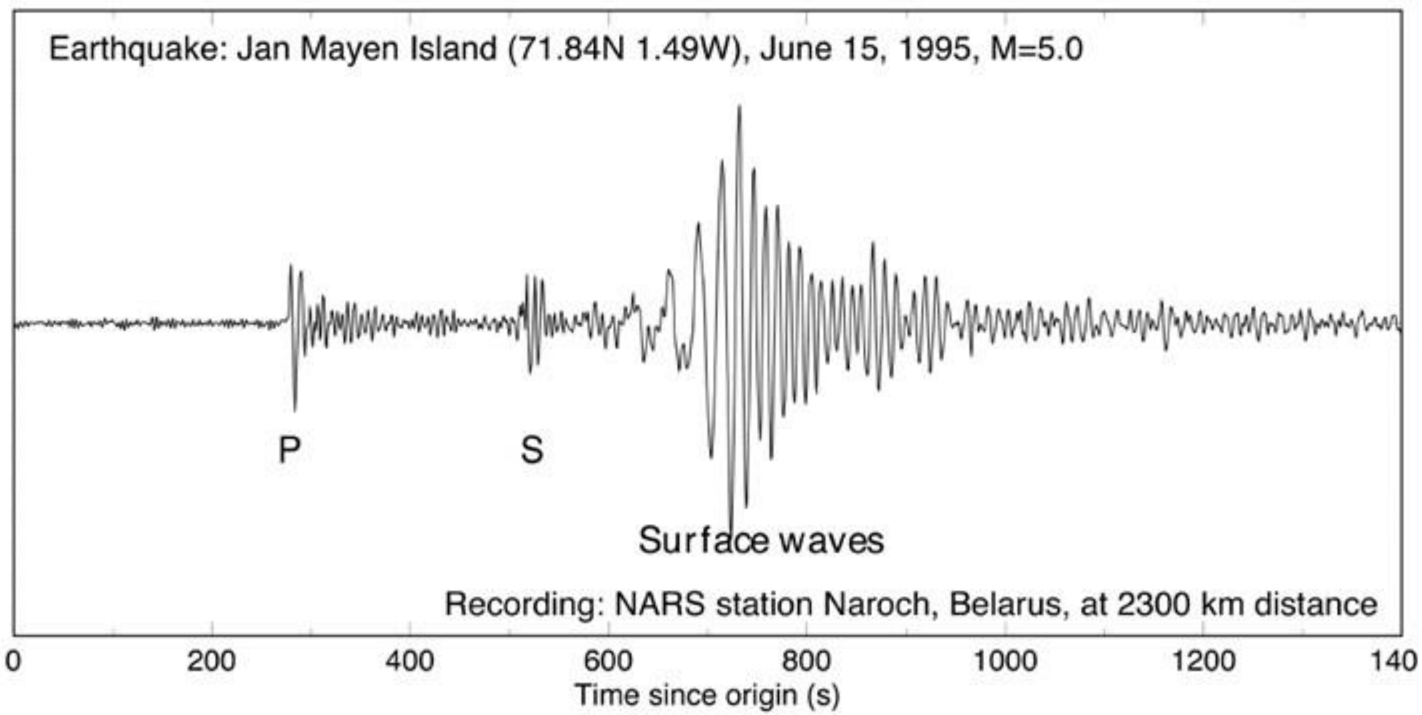




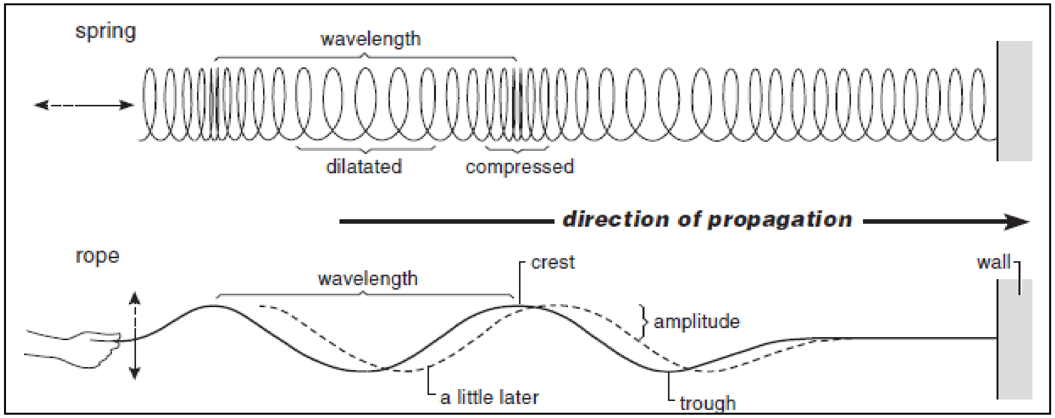




Seismograms

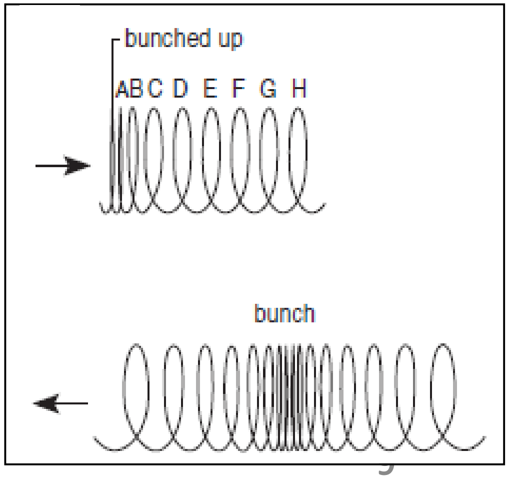


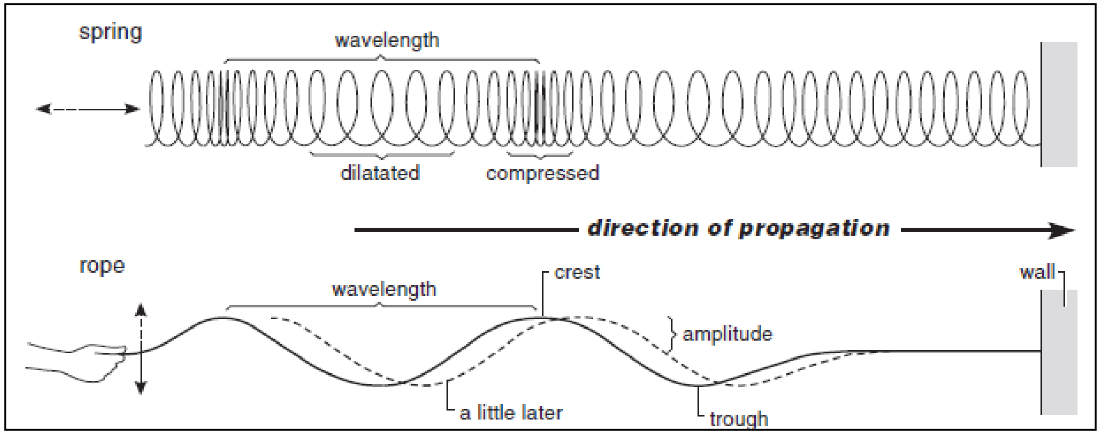
• An example of a wave is a water wave, but so also are waves produced by shaking a rope attached at its further end, or pushing in and out a long spring. If the end is moved rhythmically, a series of disturbances travels along the rope or spring: figure shows them before the disturbances have reached the further end.



• Consider the spring. When it is held stretched but stationary, the turns will be equally spaced because the tension of the spring upon each turn is the same from either side.

• A rhythmic pushing in and pulling out of the end of the spring produces a regular series of compressions and dilatations that move along, forming waves. Though the waves travel along, the spring does not, each turn only oscillating about its stationary position. Similarly, water is not moved along by water waves, which is why you cannot propel a ball across a pond by generating waves behind it by throwing in stones

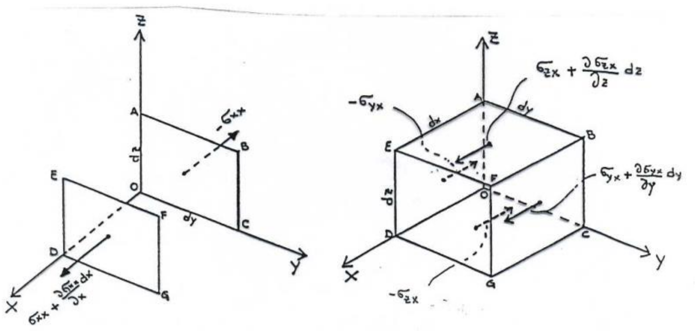




- Wavelength, λ, is the repeat length, conveniently measured between successive crests or compressions.
- The amplitude, a, is the maximum displacement from the stationary position.
- The waves travel along at some speed called, in seismology, the seismic velocity, v (a velocity should specify the direction of propagation as well as its speed, but this is often neglected in seismology).
- The number of crests or compressions that pass any fixed point on the rope, spring, and so on, in one second is the frequency, f, measured in Hz (Hertz oscillations, or cycles, per second).
- In one second, f wave crests, each λ metres apart, will pass a point; and by the time the last one has passed, the first one will have travelled a distance of f times λ metres. As velocity is the distance travelled in one second:

$$v = f \cdot \lambda \qquad [Eq\ 1]$$

Considerando l'equilibrio di un parallelepipedo per traslazioni abbiamo visto che la risultante delle forze elastiche agenti lungo l'asse x (e analogamente lungo y e z) è:



$$\left(\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \sigma_{yx}}{\partial y}+\frac{\partial \sigma_{zx}}{\partial z}\right)dxdydz$$

Usando la legge di Newton: **F=ma** otteniamo:

$$\left(\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \sigma_{yx}}{\partial y}+\frac{\partial \sigma_{zx}}{\partial z}\right)dxdydz=\rho dxdydz\frac{\partial^2 u}{\partial t^2}$$

Con ρ la densità del parallelepipedo e u la componente lungo x dello spostamento. Assumiamo che le altre forze (gravità) non varino apprezzabilmente attraverso il parallelepipedo. L'equazione si semplifica esprimendo gli sforzi in termini di deformazioni e le deformazioni in termini di spostamento:

$$\sigma_{ij}=\lambda\theta\delta_{ij}+2\mu e_{ij}$$ Legge di Hooke

$$e_{ii}=\frac{\partial u_i}{\partial x_i}$$ Deformazioni normali

$$e_{ji}=\frac{1}{2}\left(\frac{\partial u_j}{\partial x_i}+\frac{\partial u_i}{\partial x_j}\right)$$ Deformazioni di taglio

$$\rho\frac{\partial^2 u}{\partial t^2}=\frac{\partial}{\partial x}\left(\lambda\Delta+2\mu e_{xx}\right)+\frac{\partial}{\partial y}\left(2\mu e_{yx}\right)+\frac{\partial}{\partial z}\left(2\mu e_{zx}\right)$$

$$\rho\frac{\partial^2 u}{\partial t^2}=\frac{\partial}{\partial x}\left(\lambda\Delta+2\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right]$$

$$\rho\frac{\partial^2 u}{\partial t^2}=\frac{\partial}{\partial x}\left(\lambda\Delta+2\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right]$$

Assumendo uno spazio omogeneo, cioè μ e λ costanti nello spazio:

$$\rho\frac{\partial^2 u}{\partial t^2}=\lambda\frac{\partial\Delta}{\partial x}+2\mu\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial^2 u}{\partial y^2}+\mu\frac{\partial^2 v}{\partial x\partial y}+\mu\frac{\partial^2 w}{\partial x\partial z}+\mu\frac{\partial^2 u}{\partial z^2}=$$

$$=\lambda\frac{\partial\Delta}{\partial x}+\mu\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)=$$

$$=(\lambda+\mu)\frac{\partial\Delta}{\partial x}+\mu\nabla^2 u$$

Analogamente le componenti delle forze lungo y e z porteranno alle equazioni per v e w:

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + \mu)\frac{\partial \Delta}{\partial x} + \mu \nabla^2 u$$

$$\rho \frac{\partial^2 v}{\partial t^2} = (\lambda + \mu)\frac{\partial \Delta}{\partial y} + \mu \nabla^2 v$$

$$\rho \frac{\partial^2 w}{\partial t^2} = (\lambda + \mu)\frac{\partial \Delta}{\partial z} + \mu \nabla^2 w$$

Usando la notazione tensoriale o vettoriale →

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (\lambda + \mu)\frac{\partial \Delta}{\partial x_i} + \mu \nabla^2 u_i$$

Queste equazioni valgono per un disturbo generale che si propaga in un mezzo omogeneo, isotropo perfettamente elastico assumendo deformazioni infinitesime e assenza di forze di corpo (gravità ovvero forze che descrivono la sorgente sismica!)

Riscriviamo ora queste equazioni in un'altra forma che ci permetterà di capire le due forme di propagazione di un disturbo attraverso un corpo solido. Deriviamo parzialmente rispetto ad x, y, z rispettivamente le equazioni per u, v, w:

$$\rho \frac{d^2}{dt^2}\left(\frac{\partial u}{\partial x}\right) = (\lambda+\mu)\frac{\partial^2 \Delta}{\partial x^2} + \mu \nabla^2 \left(\frac{\partial u}{\partial x}\right)$$

$$\rho \frac{d^2}{dt^2}\left(\frac{\partial v}{\partial y}\right) = (\lambda+\mu)\frac{\partial^2 \Delta}{\partial y^2} + \mu \nabla^2 \left(\frac{\partial v}{\partial y}\right)$$

$$\rho \frac{d^2}{dt^2}\left(\frac{\partial w}{\partial z}\right) = (\lambda+\mu)\frac{\partial^2 \Delta}{\partial z^2} + \mu \nabla^2 \left(\frac{\partial w}{\partial z}\right)$$

Sommando le tre equazioni

$$(\lambda+\mu)\left(\frac{\partial^2 \Delta}{\partial x^2} + \frac{\partial^2 \Delta}{\partial y^2} + \frac{\partial^2 \Delta}{\partial z^2}\right) + \mu \nabla^2 \Delta = \rho \frac{d^2 \Delta}{dt^2}$$

$$(\lambda+\mu)\nabla^2 \Delta + \mu \nabla^2 \Delta = \rho \frac{d^2 \Delta}{dt^2}$$

$$(\lambda+2\mu)\nabla^2 \Delta = \rho \frac{d^2 \Delta}{dt^2}$$

$$\frac{(\lambda+2\mu)}{\rho}\nabla^2 \Delta = \frac{d^2 \Delta}{dt^2}$$

Questa è un'equazione d'onda che descrive la propagazione di una dilatazione cubica Δ. Questo tipo di onda viene detto ONDA P e si propaga con una velocità α data da:

$$\alpha = \sqrt{\frac{\lambda+2\mu}{\rho}}$$

Proviamo ora a derivare parzialmente rispetto ad y l'equazione per u e rispetto ad x quella per v:

$$\rho\frac{d^2}{dt^2}\left(\frac{\partial u}{\partial y}\right)=(\lambda+\mu)\frac{\partial^2\Delta}{\partial x\partial y}+\mu\nabla^2\left(\frac{\partial u}{\partial y}\right)$$

$$\rho\frac{d^2}{dt^2}\left(\frac{\partial v}{\partial x}\right)=(\lambda+\mu)\frac{\partial^2\Delta}{\partial x\partial y}+\mu\nabla^2\left(\frac{\partial v}{\partial x}\right)$$

Sottraendo le due equazioni

$$\rho\frac{d^2}{dt^2}\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)=\mu\nabla^2\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)$$

In modo analogo ritroviamo anche:

$$\rho\frac{d^2}{dt^2}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)=\mu\nabla^2\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)$$

$$\rho\frac{d^2}{dt^2}\left(\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\right)=\mu\nabla^2\left(\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\right)$$

Ricordando che $2\Theta_z=\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)$ rappresenta una rotazione nel piano x-y e che $2\underline{\Theta}=\left(\underline{\nabla}\times\underline{u}\right)_x$ cioè

La componente x del rotore degli spostamenti, possiamo riscrivere le ultime tre equazioni in modo elegante come:

$$\mu\nabla^2\left(\underline{\nabla}\times\underline{u}\right)=\rho\frac{d^2}{dt^2}\left(\underline{\nabla}\times\underline{u}\right)$$

$$\frac{\mu}{\rho}\nabla^2\left(\underline{\nabla}\times\underline{u}\right)=\frac{d^2}{dt^2}\left(\underline{\nabla}\times\underline{u}\right)$$

Questa è un'equazione d'onda che descrive la propagazione di un disturbo rotazionale. Questo tipo di onda non comporta variazioni di volume ed è detta ONDA S e si propaga con una velocità β data da:

$$\beta=\sqrt{\frac{\mu}{\rho}}$$