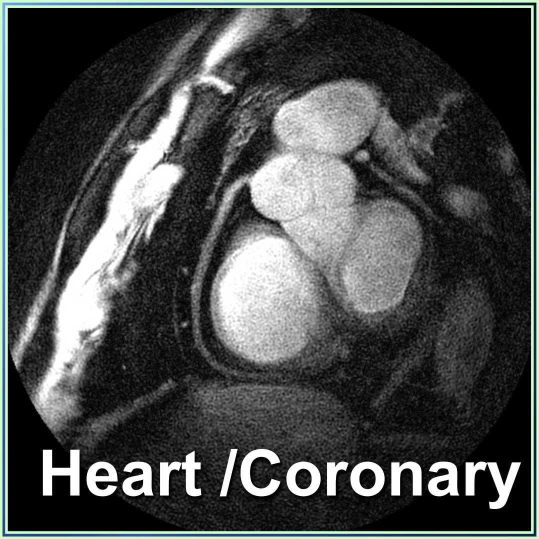
Magnetic Resonance Imaging
MRI


Abdomen


Spine


Heart /Coronary

Key resources:

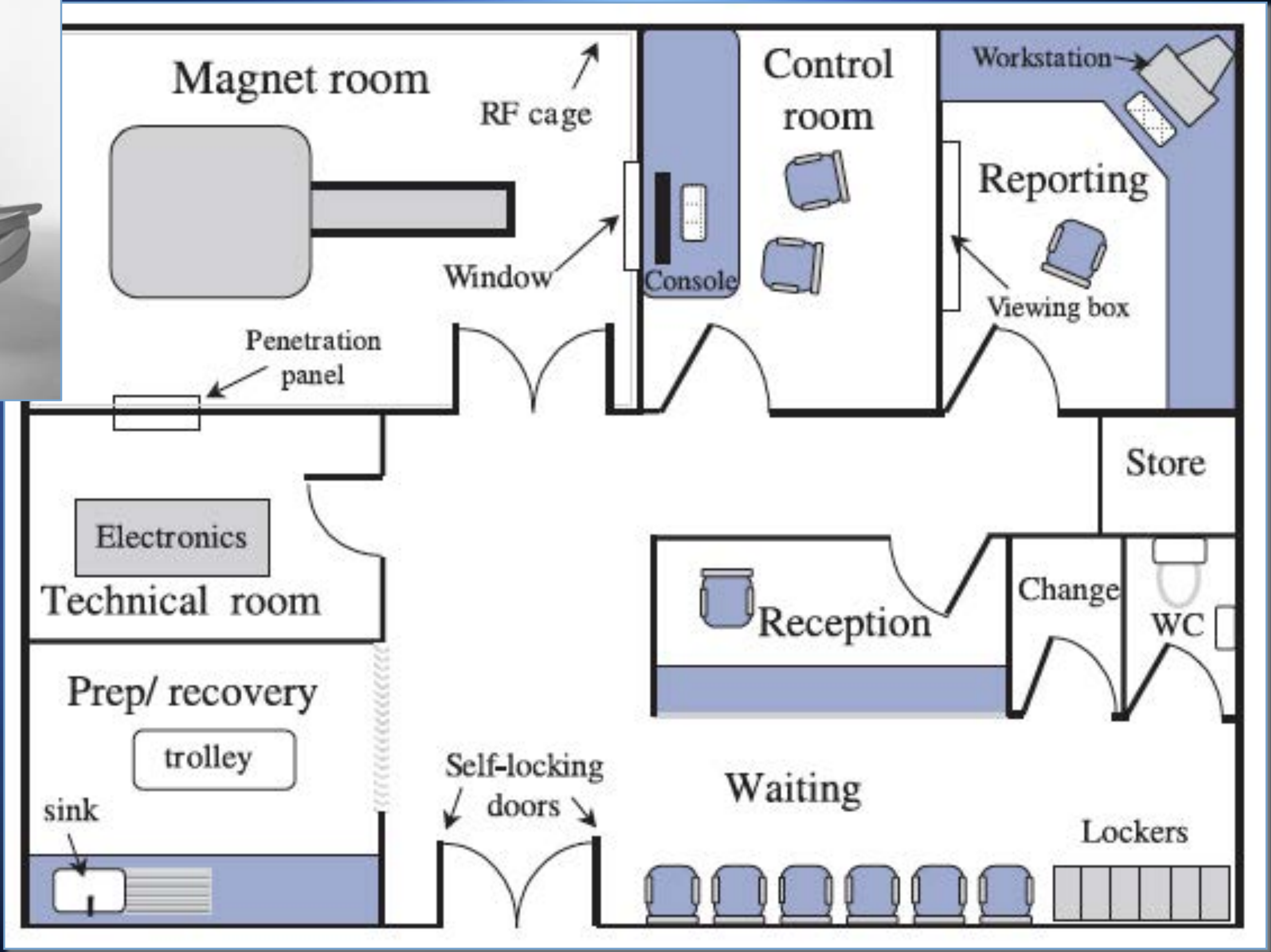
- ✓ "Introduction to Medical Physics" S. Keevil et al CRC Press (2022)
  - Chapter 7
- ✓ Some papers and link in Moodle
- ✓ the following web site:

http://www.imaios.com/en/e-Courses/e-MRI

Aims

- ✓ Understanding the principles of nuclear magnetic resonance (NMR)
- ✓ The knowledge of the technology of the NMR imager
- ✓ To learn how to optimize a MRI exam

Welcome to the MR unit



https://www.youtube.com/watch?v=8m_RUVrxhew

Historical Introduction

- ✓ 1930s and 1940s: The discovery of nuclear magnetic resonance (NMR) confirms predictions of the quantum theory
  - Nobel Prizes: Isidor Rabi, Felix Bloch, Edward Purcell
- ✓ 1971: Raymond Damadian discover differences between NMR signals obtained from normal tissue and from tumours
- ✓ 1973: Imaging methods by Paul Lauterbur and further developed by Peter Mansfield

MRI in medicine

- ✓ **High spatial resolution imaging of both structure and function**
- ✓ **Growing role in planning and guiding treatment and in the evaluation of patients’ response to therapy**
- ✓ **A tool for understanding of basic physiology and neuroscience**

Much of this flexibility can be exploited by programming existing scanners to obtain images in new ways, rather than requiring expensive hardware upgrades

NMR

- Nuclear
  - a phenomenon involving atomic nuclei
    - not all nuclei can be studied using NMR
  - almost all medical MRI involves only the nucleus of the hydrogen atom
- Magnetic
- Resonance

NMR

- ✓ Nuclear
- ✓ Magnetic
    - NMR involves the magnetic properties of nuclei
    - it occurs when nuclei are placed in a magnetic field
- ✓ Resonance

NMR

- ✓ Nuclear
- ✓ Magnetic
- ✓ Resonance
  - NMR occurs when energy is applied to the nuclei at a specific resonance frequency
    - ❖ nuclei absorb and then re-emit this energy
    - ❖ analysis of the resulting signal can reveal various properties of the material under study

NMR

- ✓ NMR is fundamentally a quantum mechanical phenomenon
- ✓ For most purposes in medical MRI, it is sufficient to adopt a ‘semi-classical’ approach
  - in some situations we will have to refer back to the quantum mechanical model
  - in others we will have to accept that our semi-classical approach is an approximation that doesn’t bear too much scrutiny!

Resonance

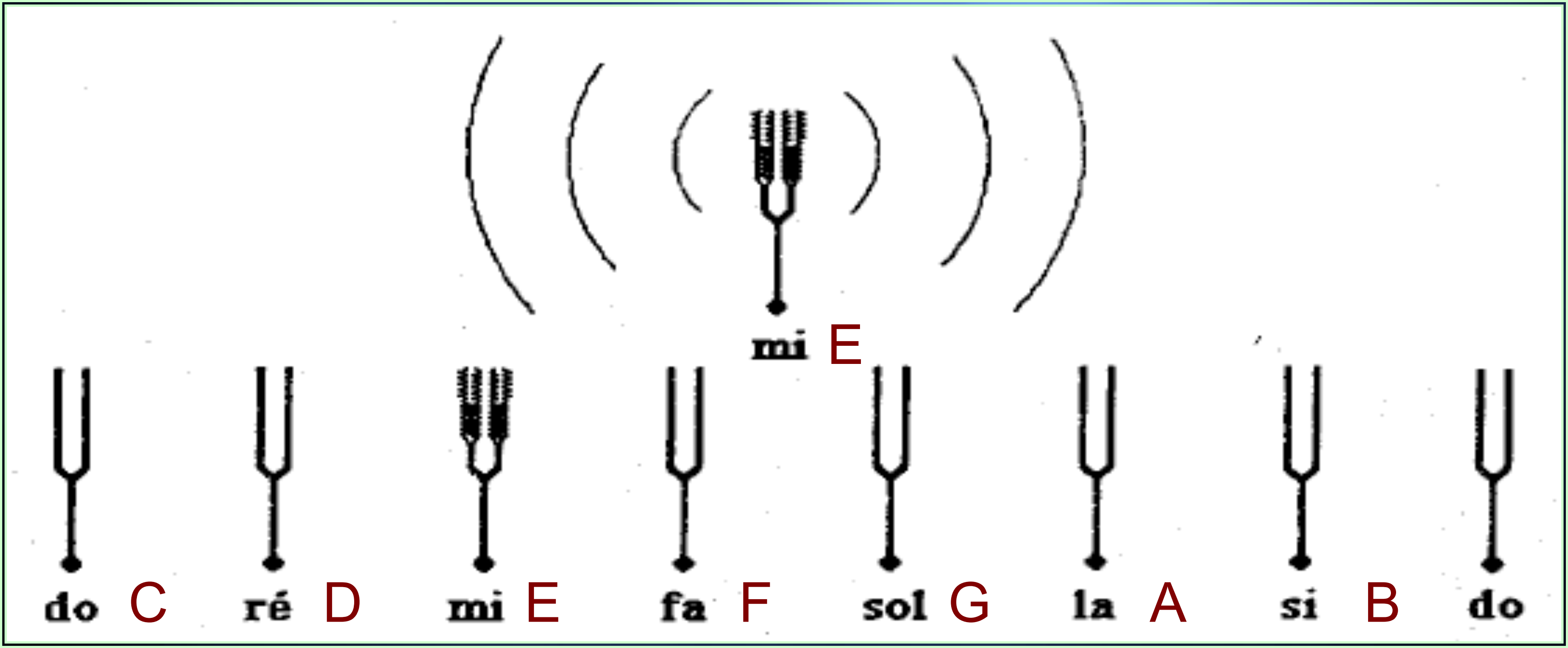
- ✓ Resonance occurs when a system can efficiently transfer energy between two different storage modes
  - such as kinetic energy and potential energy in the case of a pendulum

Swinging a child in a playground swing is an easy job because you are helped by its natural frequency.

- ➢ But can you swing it at another frequency?

Resonance

Resonance occurs when a system can easily transfer energy between two or more different storage



https://www.youtube.com/watch?v=8xE_nT3QySo

Quantum mechanics

- ✓ **many physical variables that appear to be continuous on the larger scale of everyday life are actually quantised when very small objects are considered**
  - **atoms and nuclei**
- ✓ **quantised variables include energy, electric charge and <u>angular momentum</u>**

Quantum mechanics

- Angular momentum of atomic nucleus

$$\left|\mathbf{J}\right| = \hbar\sqrt{I(I+1)}$$

- $\hbar = h/2\pi$
  - h Planck's constant 6.63 × 10^{-34} J s
- *I* spin quantum number
  - a property of the nucleus
  - *I* can only take integer or half integer values (1/2, 1, 3/2, etc.)
    - which in turn restricts the angular momentum to certain values too

Quantum mechanics

- ✓ The value of *I* depends on the number of protons and neutrons
- ✓ Protons and neutrons have spin *I* = ½
  - protons pair off with each other in such a way that the spin of each pair of protons cancels out
- ✓ If there is an odd number of protons, the unpaired proton contributes *I* = 1/2
- ✓ Neutrons pair off in a similar way

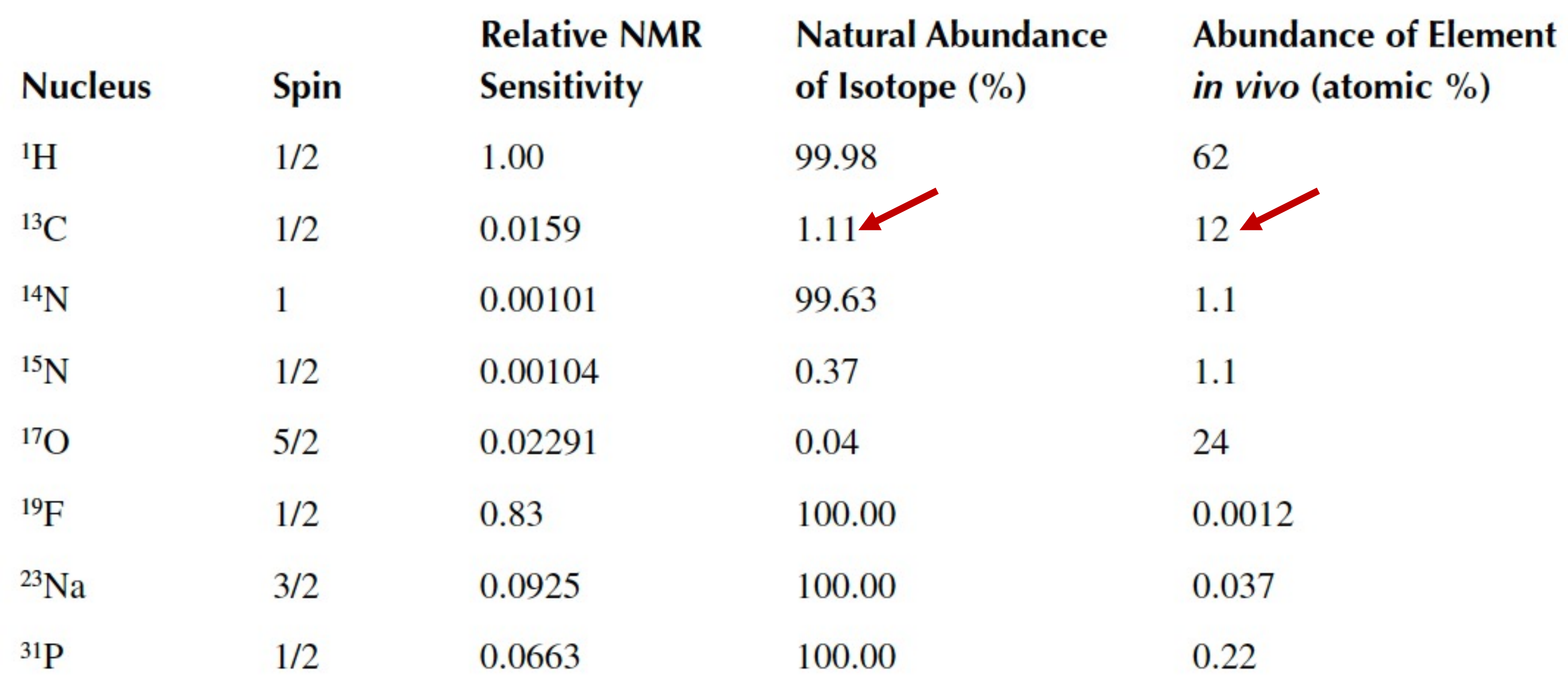
nuclei containing an even number of protons and an even number of neutrons have an overall spin I = 0

Magnetic Nuclei

A nucleus has magnetic properties if it has an odd number of protons and/or neutrons

- $^{1}H_{1}$ (proton), $^{2}H_{1}$ (Deuterium; odd-odd)
- $^{7}Li_{3}$ (Litium; even-odd)
- $^{13}C_{6}$ (Carbonium; even-odd)
- $^{14}N_{7}$ (Nitrogen; odd-odd)
- $^{19}F_{9}$ (Fluorine)
- $^{23}Na_{11}$ (Sodium)
- $^{31}P_{15}$ (Phosphor)
- $^{35}Cl_{17}$ (Chlorine)
- $^{63}Cu_{29}$ (Copper)
- $^{127}I_{153}$ (Iodine)

Nuclei with Nonzero Spin of Potential Biomedical Interest



Nucleus	Spin	Relative NMR Sensitivity	Natural Abundance of Isotope (%)	Abundance of Element *in vivo* (atomic %)
^{1}H	1/2	1.00	99.98	62
^{13}C	1/2	0.0159	1.11	12
^{14}N	1	0.00101	99.63	1.1
^{15}N	1/2	0.00104	0.37	1.1
^{17}O	5/2	0.02291	0.04	24
^{19}F	1/2	0.83	100.00	0.0012
^{23}Na	3/2	0.0925	100.00	0.037
^{31}P	1/2	0.0663	100.00	0.22

The Relative NMR Sensitivity indicates the amount of signal that would be obtained from numbers of each nucleus, relative to that of the ^{1}H nucleus

Magnetic Nuclei

- ✓ **More complex aspects of nuclear structure mean that some nuclei have higher spin quantum numbers**
  - **specifically “orbital angular momentum’**

- ✓ **The hydrogen nucleus, and most of the other nuclei that are of interest in biomedical applications of NMR, have I = 1/2**

^{1}H images

- ✓ ^{1}H nucleus combines the highest NMR sensitivity with natural abundance close to 100% and very high abundance in the body
- ✓ ^{1}H nuclei are predominantly found in water molecules and in lipids

other NMR-visible nuclei

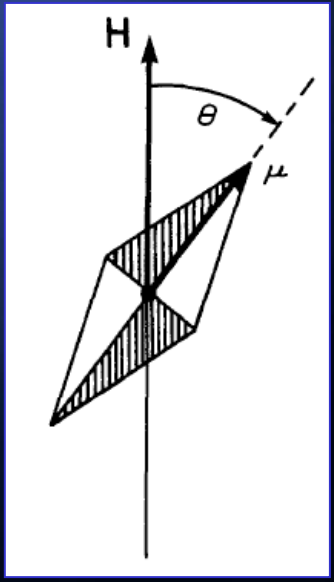
specific applications to investigate other NMR-visible nuclei

- ✓ spectroscopic studies of energy metabolism using ^{31}P
- ✓ molecular imaging using ^{19}F-containing compounds
  - introduced into the body artificially

Basic Concepts
Hydrogen Protons

- ✓ Magnetic Resonance Imaging (MRI) is based on ^{1}H protons
- ✓ Very qualitative introduction
  - a solid ball spinning on an axis, with the rate at which it is spinning determined by the quantum number I
  - a positively charged spinning proton acts like a tiny magnet

a crude representation of a quantum mechanical object that is at once a wave and a particle !!!

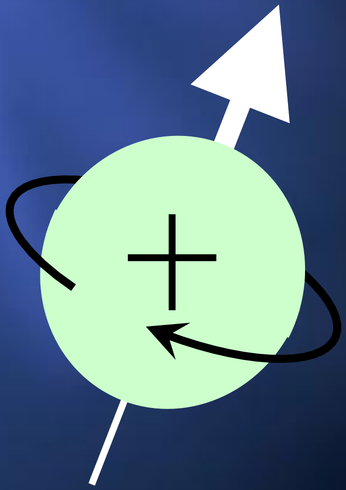


The physics of proton NMR

$$\mu = \gamma J$$

$$|\mathbf{J}| = \hbar\sqrt{I(I+1)}$$

- ✓ μ magnetic moment
- ✓ γ gyromagnetic ratio of ^{1}H
  - $\gamma = 42.57$ MHz T^{-1}
  - $\gamma = 2.68 \times 10^8$ rad s^{-1} T^{-1}
- ✓ J angular momentum

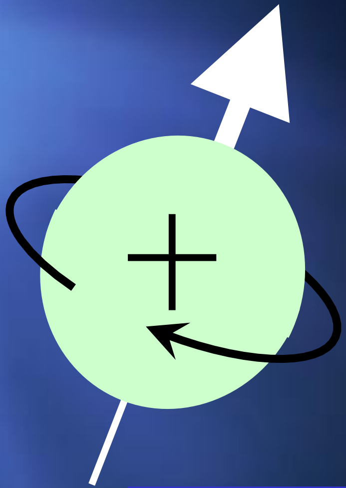


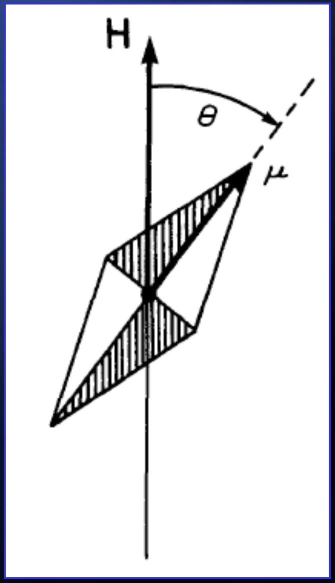
Angular momentum of an atomic nucleus is due to internal structures of p and n (quarks)

https://www.imaios.com/en/e-mri/nmr/nuclear-spin

Effect of the Static Magnetic Field

- When a magnetic field B_0 is applied to a sample containing nuclear dipoles the dipoles align themselves in specific orientations
  - with respect to the applied field
- The orientations depend on m_I
  - m_I the magnetic quantum number
  - m_I can take a range of values in integer steps from $-I$ to I
    - I spin quantum number



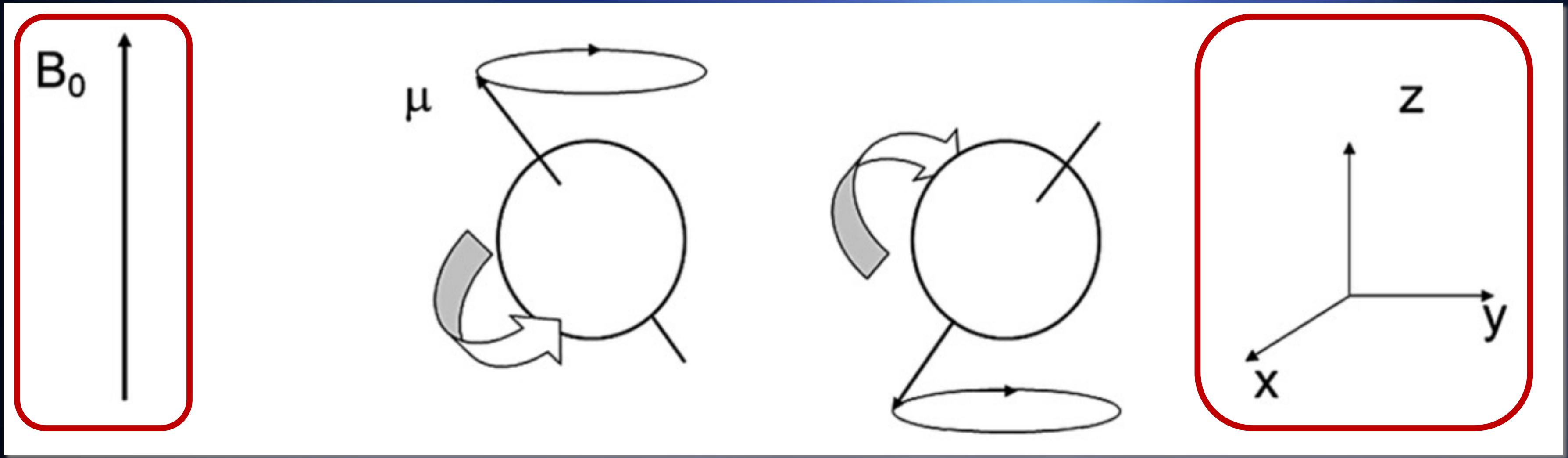


The physics of proton NMR

- ✓ ^1H spin quantum number: $I = 1/2$
- ✓ ^1H magnetic quantum number: $m_I = \pm 1/2$
- ✓ μ can adopt 2 possible orientations
- ✓ μ_z is the component lying along the direction of the static magnetic field B_0

$$\mu_z = \gamma\hbar m_I = \pm\frac{1}{2}\gamma\hbar$$

Spin-up and spin-down



μ_z *the component lying along the direction of the static magnetic field* B_0

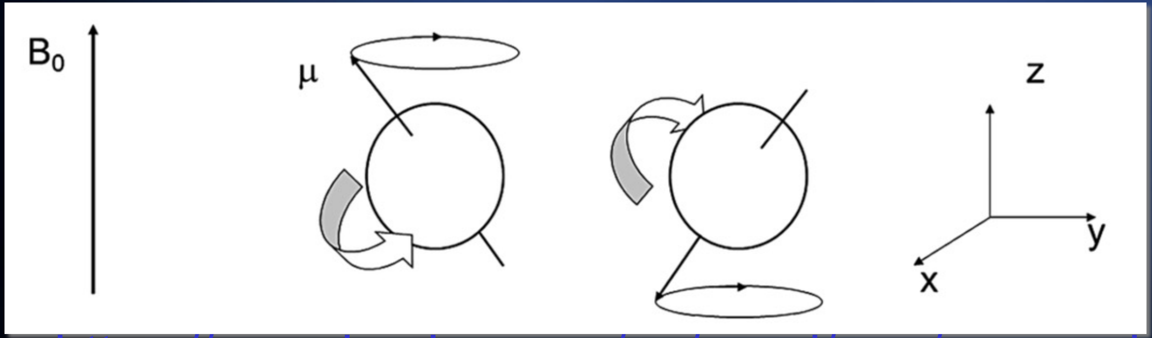
- ✓ 2 orientations corresponding to $m_I = \pm 1/2$
- ✓ μ lies at an angle of 54.7° to B_0

the xy-plane is known as the transverse plane

Larmor frequency

According to classical electromagnetism

- ✓ a magnetic moment lying at an angle to a magnetic field experiences a torque
  - turning force
- ✓ This causes the magnetic moment to rotate, or precess, around the direction of the B_0 at the Larmor frequency ω_0

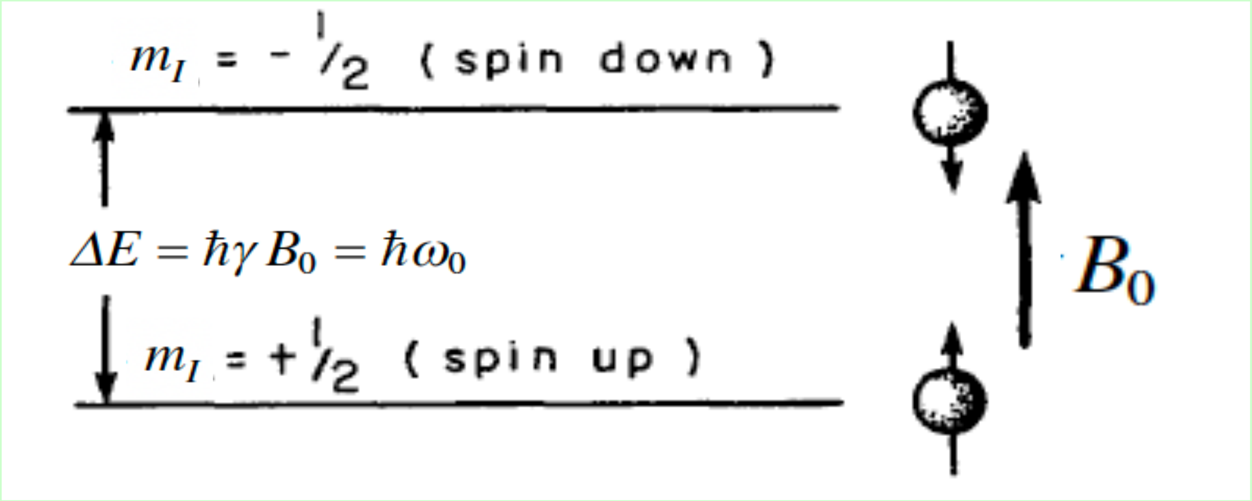


$$\omega_0 = \gamma B_0$$

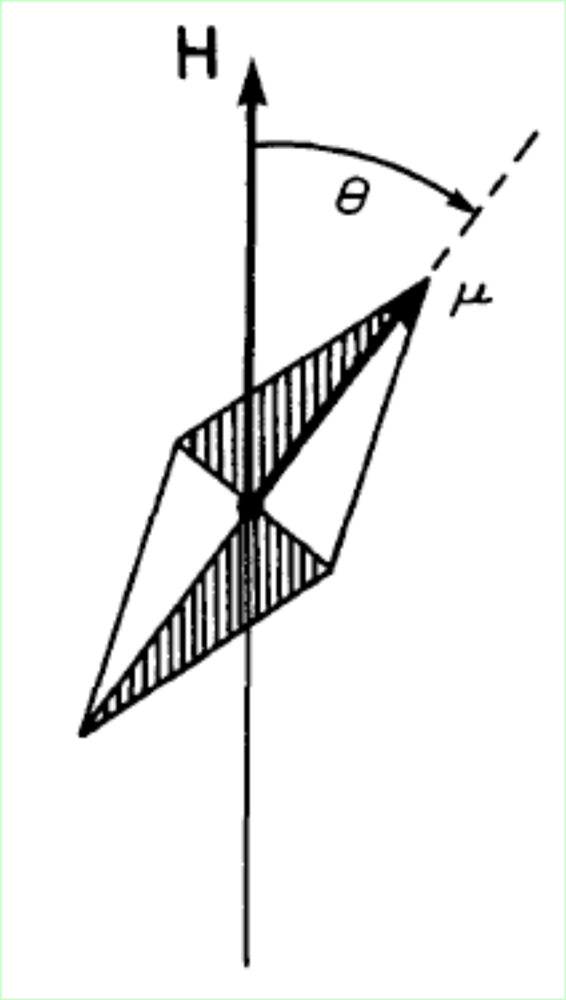
https://www.imaios.com/en/e-mri/nmr/precession-and-larmor-frequen

The energy of a magnetic moment in a magnetic field

- ✓ Energy of the nuclei $E = \pm \frac{1}{2}\hbar\gamma B_0$
  - I=1/2
  - Larmor frequency $\omega_0 = \gamma B_0$
- ✓ Energy difference between the energy of nuclei in the 2 orientations



The physics of the compass needle



- ✓ The torque in the magnetic needle = μ x H
  - *Vector product*
- ✓ potential energy

$$-\mu \cdot H = -\mu H \cos\theta$$

  - *Scalar product*
  - Minimum: θ=0, cos θ=1
  - Maximum: θ=180, cos θ=-1

Effect of the Static Magnetic Field

Nature prefers the lowest energy state

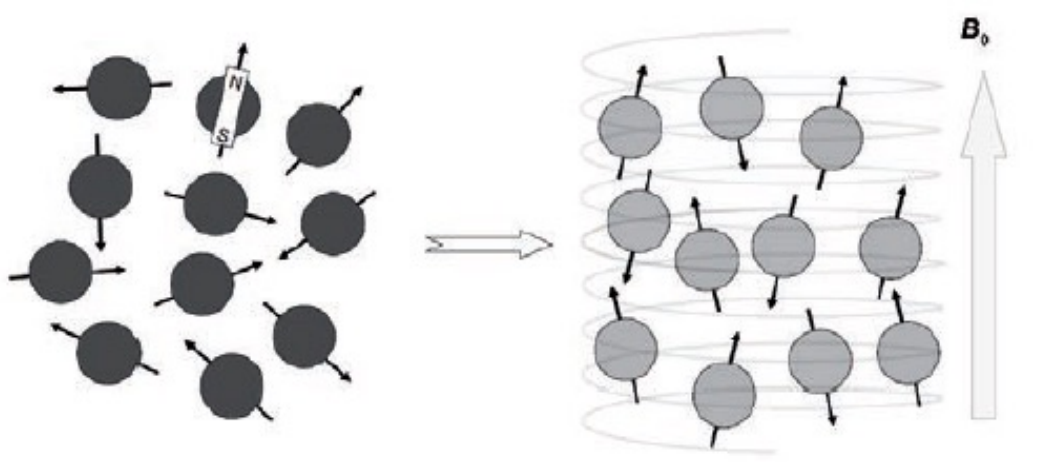
- ✓ more nuclei in the lower energy level than the higher one
- ✓ the difference between the populations of the 2 energy levels

$$\frac{N\uparrow - N\downarrow}{N} \approx \frac{\hbar\gamma B_0}{2kT}$$

- k is Boltzmann’s constant
  - ❖ 1.38 × 10−23 J K^{-1}

Effect of the Static Magnetic Field

- ✓ the difference in population amounts to only a few nuclei per million
  - A few ppm
- ✓ the small excess of nuclei oriented along the B_0 direction gives rise to the NMR signal
  - $B_0 \sim 1\text{-}3$ T

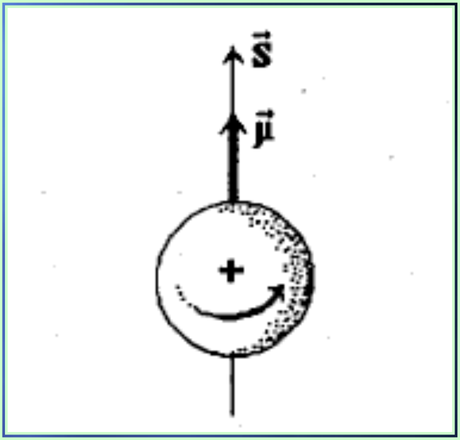


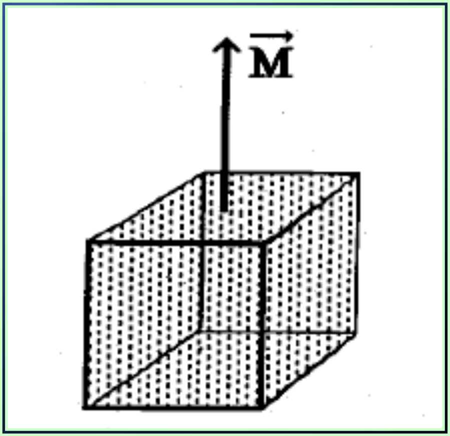
$$\frac{N\uparrow - N\downarrow}{N} \approx \frac{\hbar\gamma B_0}{2kT}$$

https://www.imaios.com/en/e-mri/nmr/net-magnetization

The Bulk Magnetization

$$\vec{M} = \sum \vec{\mu}$$

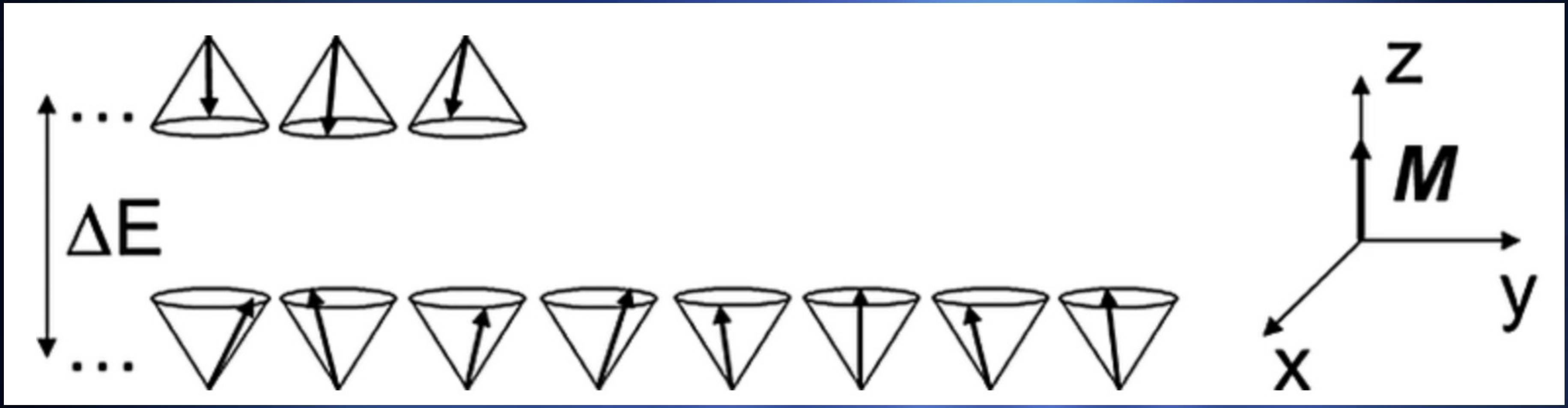




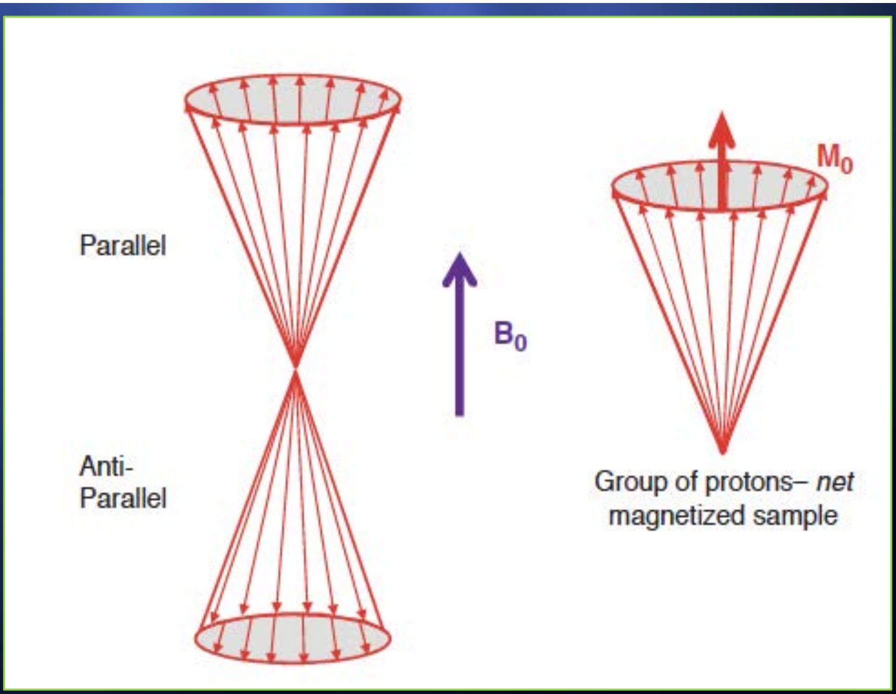
- ✓ M is the net magnetization (magnetic moment) of a sample placed in a magnetic field
  - typically >> 10^{18} nuclei
- ✓ M lies along the B_0 direction

http://www.imaios.com/en/e-Courses/e-MRI/NMR/Net-magnetization

The Bulk Magnetization



$$\frac{N\uparrow - N\downarrow}{N} \approx \frac{\hbar\gamma B_0}{2kT}$$



Effect of the Radiofrequency Field

The resonance of NMR

Additional magnetic fields, which vary in time are used to manipulate the direction and size of M

- M bulk magnetisation vector
- to collect a signal from the spin system

✓ classical electromagnetism is used

✓ quantum-mechanical description is briefly considered

Classical description of a spin in a magnetic field

R.L. Dixon and K.E. Ekstrand

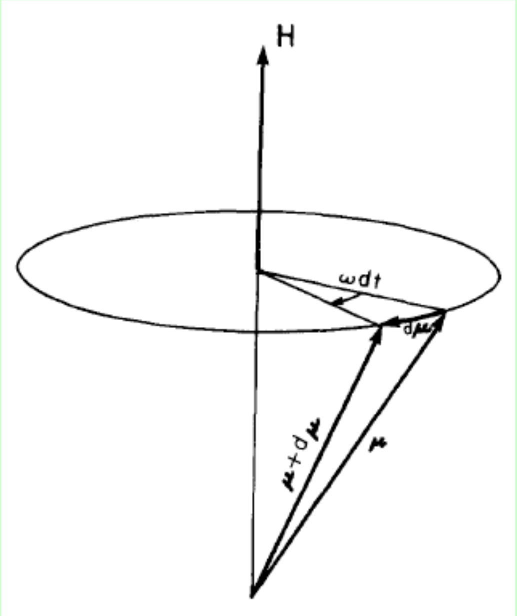
Medical Physics vol 9 pp 807-818 1982

When a force is exerted on a spinning object (S), it tends to move at right angles to the force

$$\frac{d\vec{S}}{dt} = \vec{\mu} \times \vec{H}$$

$$\vec{\mu} = \gamma \vec{S}$$

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma \vec{H}$$



- **the magnitude of μ remains constant**
  - **since $d\mu$ is perpendicular to both μ e H (or B)**
- **The motion is a precession of μ about H (or B)**
- **Angular (Larmor) frequency $\omega = \gamma H = \gamma B$**

http://www.imaios.com/en/e-Courses/e-MRI/NMR/Precession-and-Larmor-frequency

Classical description of a spin in a magnetic field

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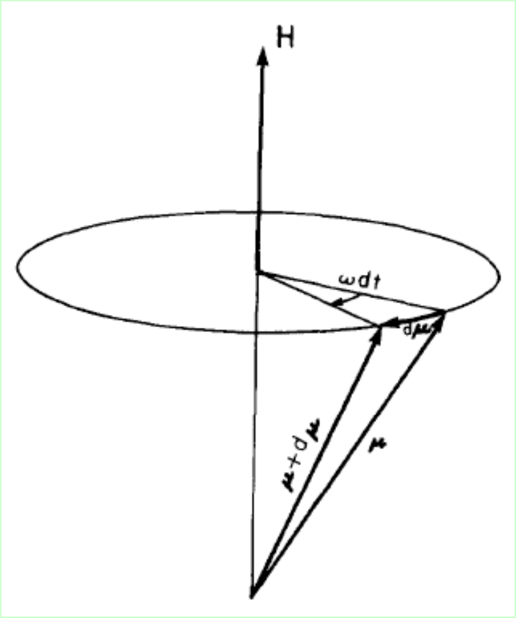


A group of isolated classical spin precesses ad infinitum

B and H fields

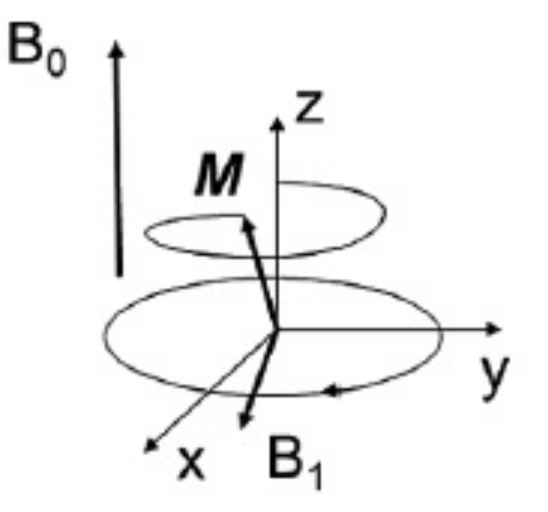
- ✓ Both B and H are called magnetic field
- ✓ B is measured in Tesla (T) or Gauss (G)
  - 1T=10000G
- ✓ In vacuum H is proportional to B
- ✓ In medium

$$\mathbf{H} \equiv \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$

  - M magnetization vector field
  - μ_0 permeability of free space

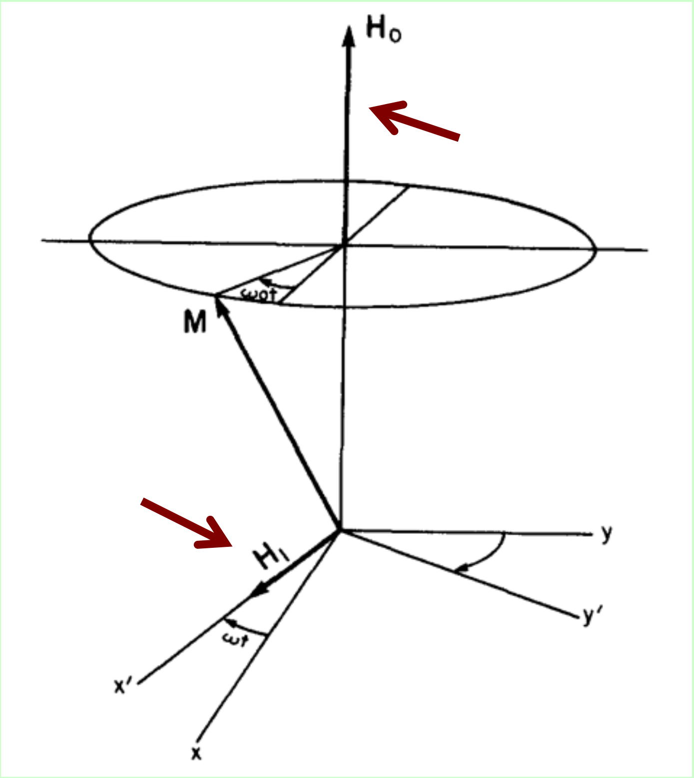
Effect of the Radiofrequency Field

- ✓ B_1 is orthogonal to B_0
  - it lies in the transverse plane
- ✓ It is much weaker than B_0
  - of the order of μT
- ✓ It is applied for a short period of time
  - typically hundreds of μs



Motion of M in an RF field

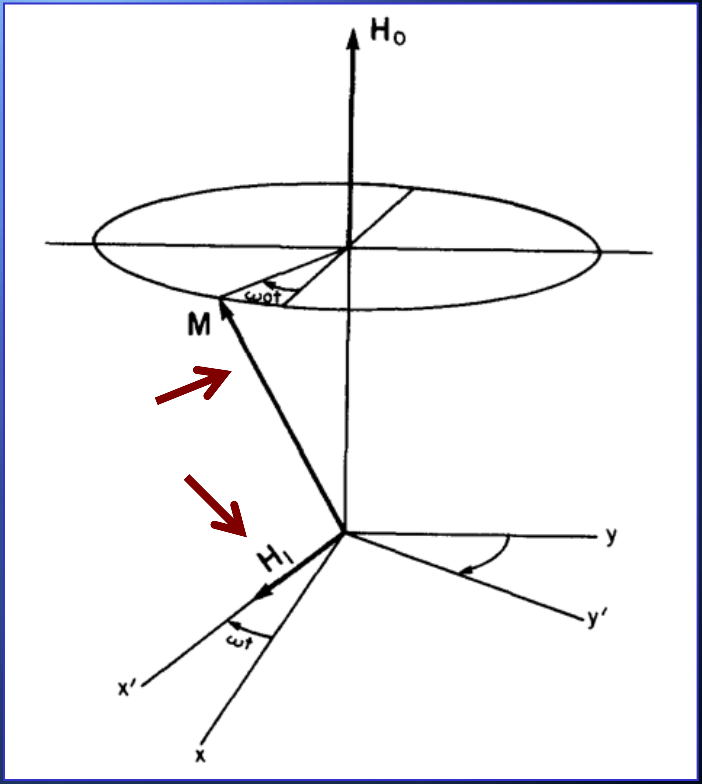
- magnetic moments experience a torque in the presence of B_0 which causes them to precess
- In the same way, there is precession of M around the direction of B_1
  - causing the magnetisation vector M to tip (or nutate) away from the z-axis



Motion of M in an RF field

- In order for nutation of M to happen, the B_1 field must be kept 'in step' with the precessing nuclei

- the B_1 field must itself be rotating in the transverse plane at the Larmor frequency ω_0



Larmor frequency in MRI

- ✓ Larmor frequency lies in the same frequency range as the RF of the electromagnetic spectrum
  - 10–100 MHz
  - B_1 is often termed the RF magnetic field
- ✓ B_1 can be generated by using 2 RF coils aligned along the x- and y-axes
  - generating sinusoidal magnetic fields 90° out of phase and so add up to produce a circularly polarised magnetic field

Motion of M in an RF field

This is the resonance part of NMR

- ✓ nutation only occurs if B_1 is applied at the Larmor frequency
- ✓ this fact allows to choose which nuclear species to interrogate
  - B_0 generates bulk magnetisation from all nuclei that have nonzero spin
    - ^{1}H, ^{13}C, ^{31}P
  - The choice of the frequency of B_1 allows to select which of these to study

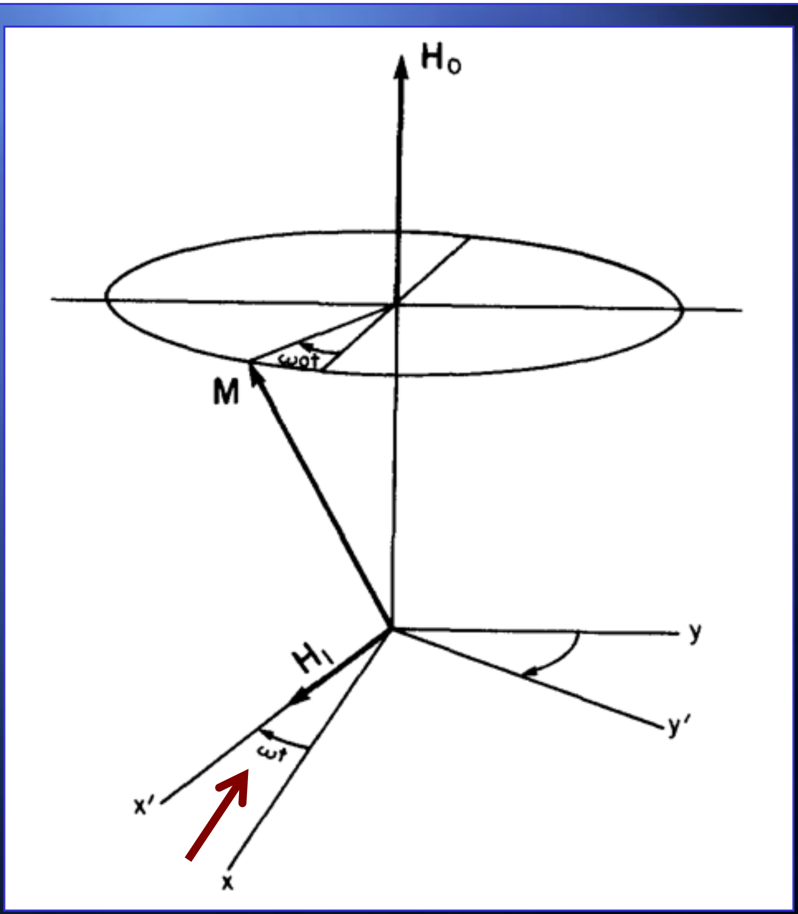
Motion of M in an RF field

- ✓ if $\omega=\omega_0$ B_1 stays in constant position relative to M
  - ω frequency of the RF pulse
- ✓ Thus the small torque of B_1 on M over many precessions has significant effect

$\omega=\omega_0$ is

the resonance condition

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma(\vec{H}_0 + \vec{H}_1(t))$$



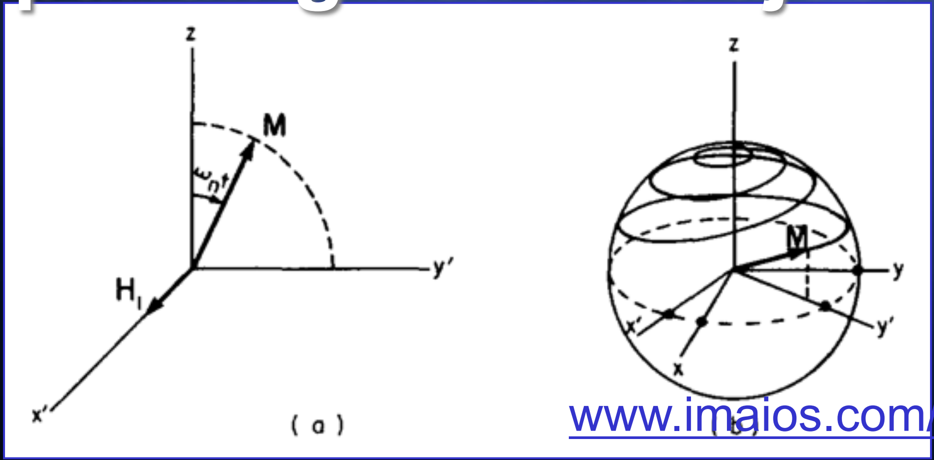
http://www.imaios.com/en/e-Courses/e-MRI/NMR/Excitation

Motion of M in an RF field

Once B_1 is applied:

- ✓ M tips away from the z-axis because of precession around B_1
- ✓ it will also precess around B_0

The overall motion is quite complicated: the magnetisation vector following an expanding helical trajectory



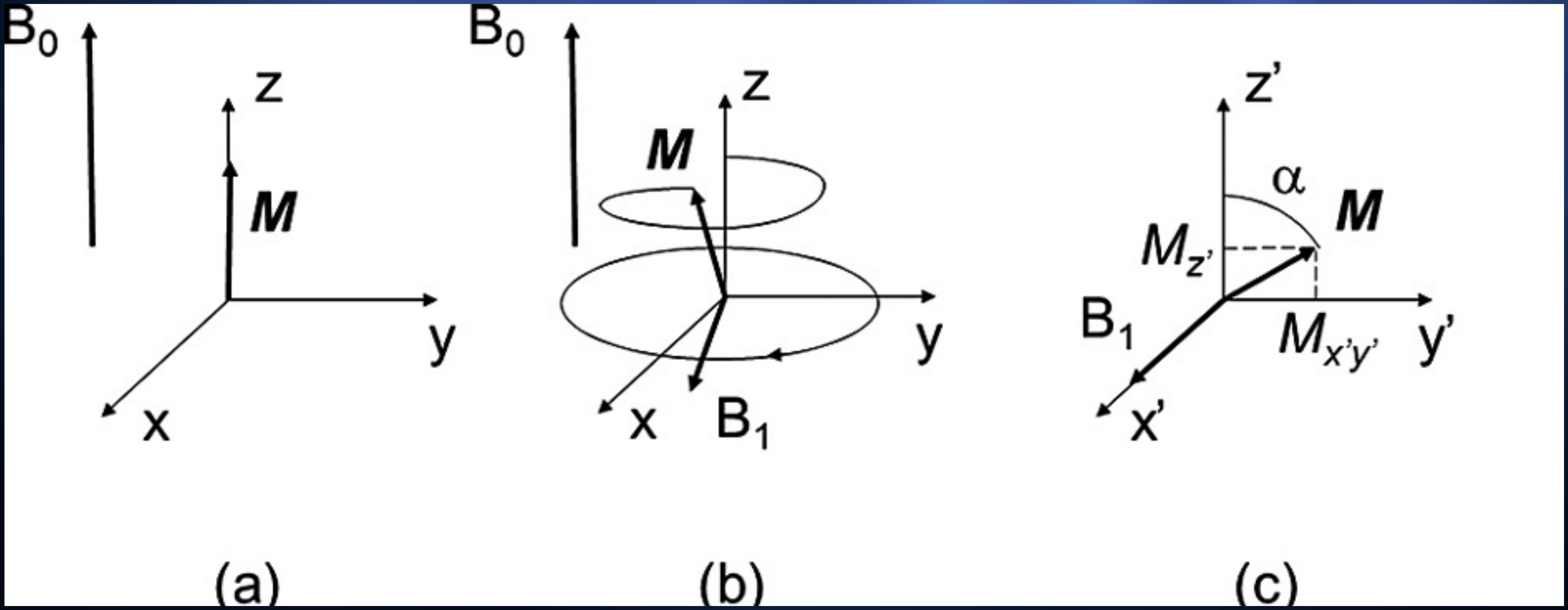
www.imaios.com/en/e-Courses/e-MRI/NMR/Excitation

Motion of M in an RF field

a) Static position

b) the magnetisation vector M following an expanding helical trajectory

- laboratory frame of reference

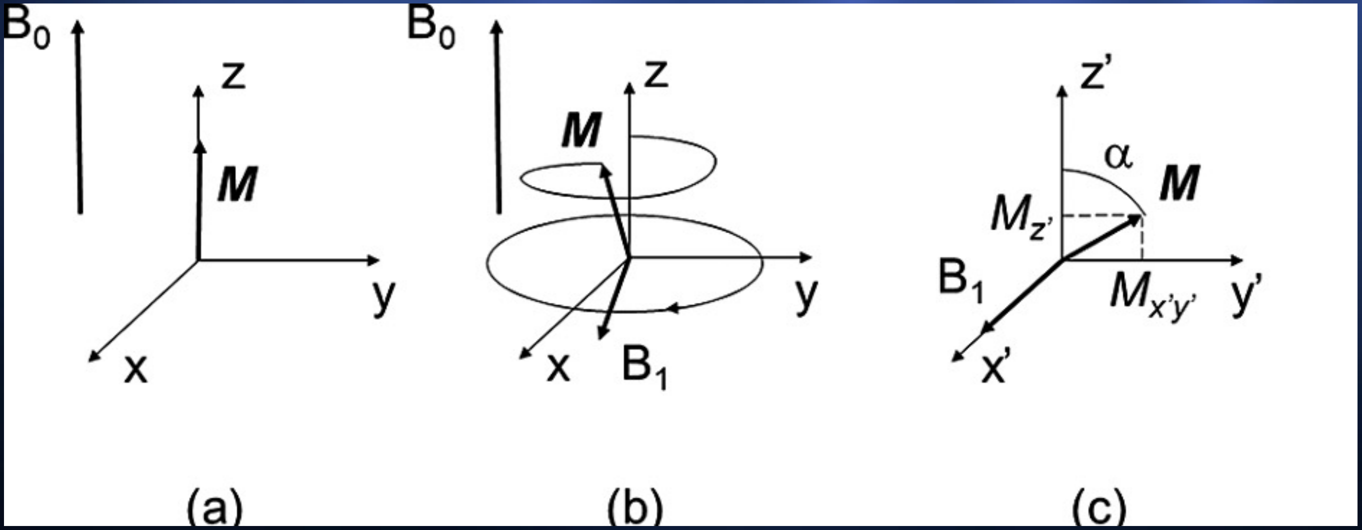


(a) (b) (c)

The rotating frame

c) rotating frame of reference moving at the Larmor frequency (ω_0)

- the motion of M is simplified to nutation from the z′-axis towards the x′y′ plane
- x′y′z′ the rotating frame



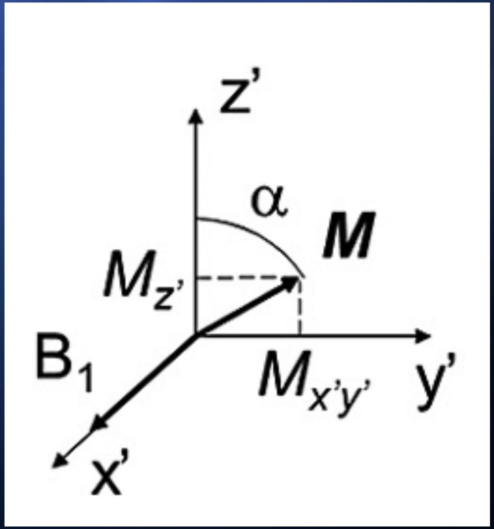
(a) (b) (c)

The flip angle α

In the rotating frame of reference

- the motion of M is simplified to nutation from the z′-axis towards the x′y′ plane
- nutation continues for as long as the B_1 field is present (t_{RF})
- The nutation angle α
  - Or flip angle

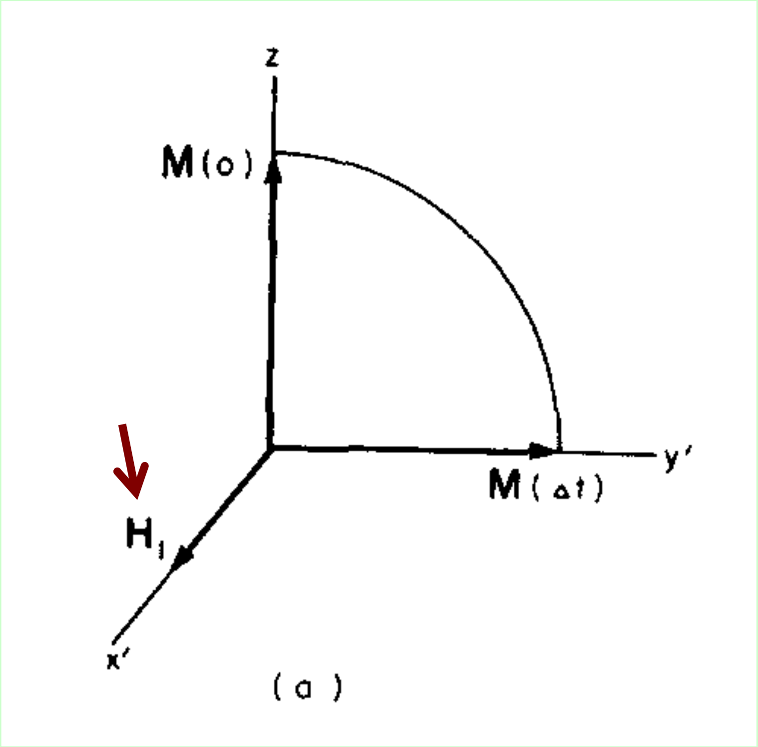
$$\alpha = \gamma B_1 t_{RF}$$



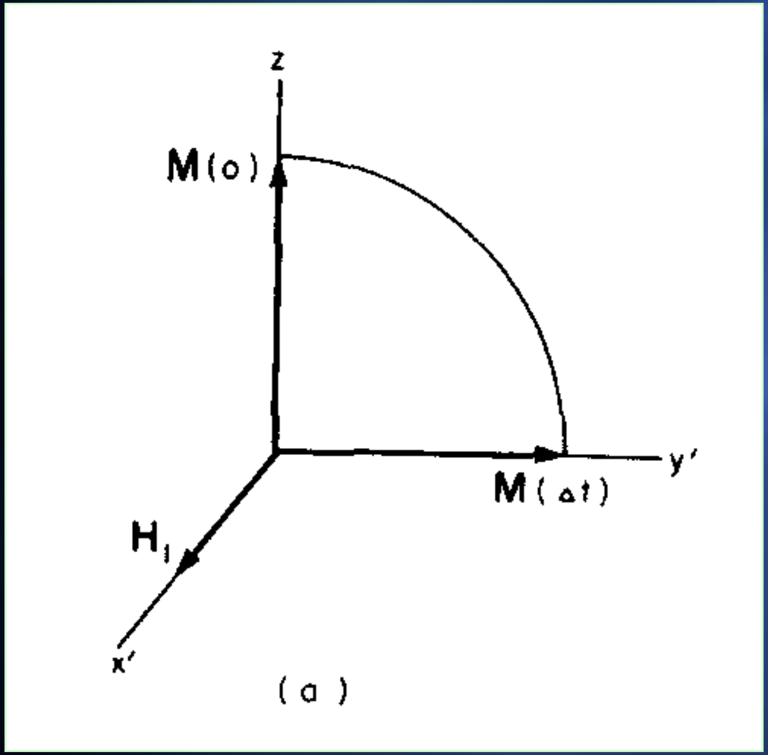
The 90º RF pulse in the rotating system

$$\boxed{\omega_1 t_{RF} = \gamma B_1\ t_{RF} = \frac{\pi}{2}}$$

- RF pulse at resonance:
  - M rotates about x’ in the zy’ plane
  - Angular velocity $\omega_1 = \gamma B_1$
- t_{RF} defines the flip angle
  - 90º pulse: $\omega_1 t_{RF} = \pi/2$
- M at the end of the pulse lies along the y’ axis
  - in the laboratory frame M precesses in the xy plane at frequency ω_0



The 90° RF pulse: how long is t_{RF} ?



$H_1 = 10^{-3}$ T ; protons

t_{RF} ?

$\nu' = 42.5$ kHz

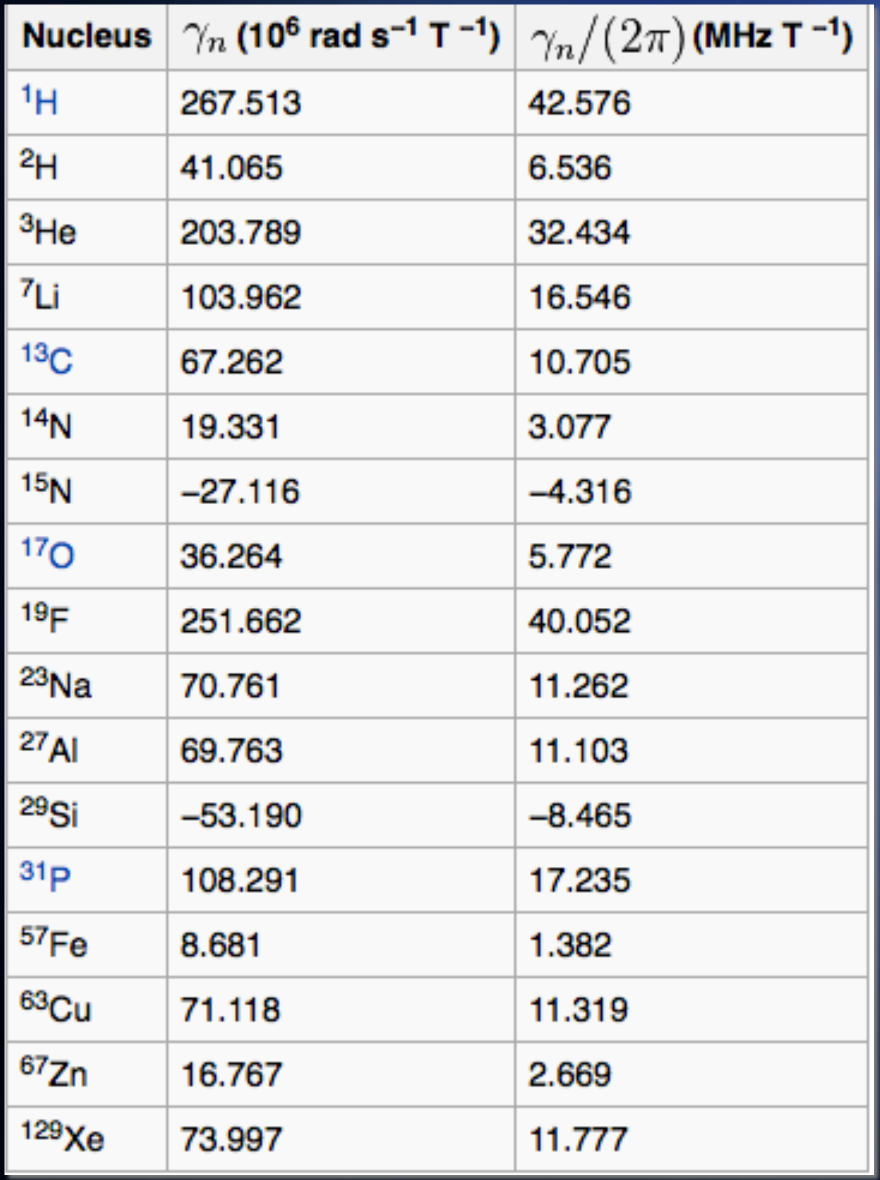
$t_{RF} = 1/(4\nu') \sim 6$ μsec

How long a 180° pulse ?

$\nu = 1/T$

T period of the motion

Larmor frequency



Nucleus	γ_n (10^6 rad s^{-1} T^{-1})	$\gamma_n/(2\pi)$ (MHz T^{-1})
^1H	267.513	42.576
^2H	41.065	6.536
^3He	203.789	32.434
^7Li	103.962	16.546
^{13}C	67.262	10.705
^{14}N	19.331	3.077
^{15}N	−27.116	−4.316
^{17}O	36.264	5.772
^{19}F	251.662	40.052
^{23}Na	70.761	11.262
^{27}Al	69.763	11.103
^{29}Si	−53.190	−8.465
^{31}P	108.291	17.235
^{57}Fe	8.681	1.382
^{63}Cu	71.118	11.319
^{67}Zn	16.767	2.669
^{129}Xe	73.997	11.777

- ✓ Angular Larmor frequency (rad/s)

$$\omega=\gamma H$$

- ✓ Larmor frequency (MHz)

$$\nu=\gamma H/2\pi$$

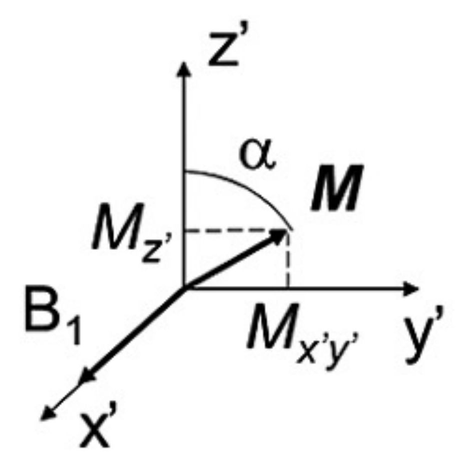
Motion of M in an RF field

The net effect of the RF pulse is to reduce the longitudinal magnetisation (M_z) while generating transverse magnetisation (M_{xy})

$$M_{z'} = |\mathbf{M}|\cos\alpha = |\mathbf{M}|\cos\left(\gamma B_1 t_{RF}\right)$$

$$M_{y'} = |\mathbf{M}|\sin\alpha = |\mathbf{M}|\sin\left(\gamma B_1 t_{RF}\right)$$

$$M_{x'} = 0$$



assuming B_1 along the x′-axis in the rotating frame, so that the transverse magnetisation lies along the y′-axis

https://www.imaios.com/en/e-mri/nmr/excitation

Quantum mechanics ?

- ✓ an electromagnetic field of frequency ω_0 carries energy $E = \hbar\,\omega_0$
- ✓ this is the energy gap between up and down energy levels
- ✓ μ in the lower energy level are able to absorb energy from the RF field
  - if the RF frequency matches the Larmor frequency
- ✓ The effect is to reduce the difference between the populations of the two energy levels, and hence the size of the longitudinal magnetisation

Quantum mechanics ?

- ✓ The effect of the RF pulse is to reduce the difference between the populations of the two energy levels, and hence the size of the longitudinal magnetisation
- ✓ Less obviously, the excited nuclei are made to precess in phase with each other, so that net magnetisation M_{xy} is generated in the transverse plane

Quantum-mechanical description of a spin in a magnetic field

- ✓ Hemiltonian equal to potential energy
- ✓ Stationary solution
  - if H_0 is time-independent, along z
  - stationary states are eigenstates of angular momentum S_z
- ✓ E_m energy levels are

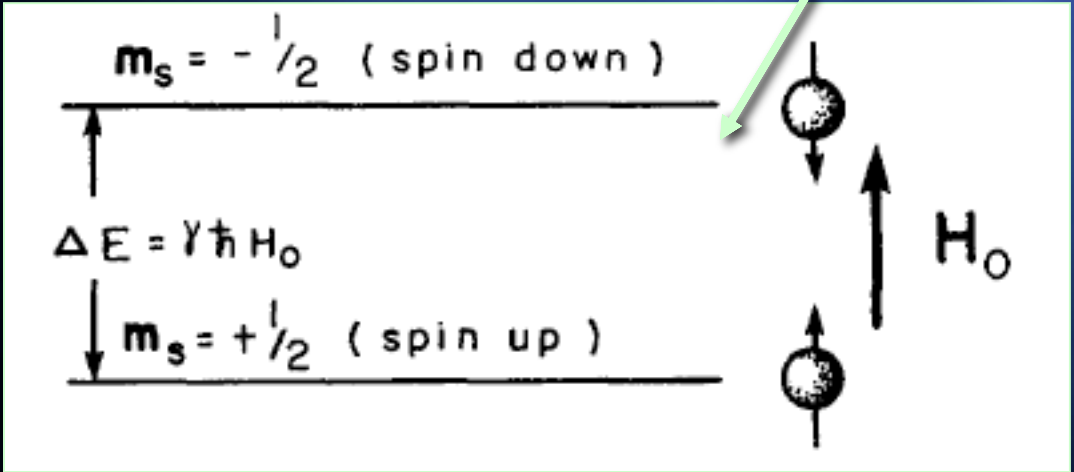
$$\mathscr{H} = -\mu \cdot H$$

$$\mathscr{H} = -\gamma H_0 S_z$$

$$E_{m_s} = -\gamma \hbar H_0 m_s$$

$$-s \le m_s \le s$$

$$\Delta E = \hbar \gamma H_0 = \hbar \omega_0$$



Larmor frequency

http://www.imaios.com/en/e-Courses/e-MRI/NMR/nuclear-spin

Quantum-mechanical description of a spin in a magnetic field

- ✓ The transition from lower to upper state is made by energy absorption
- ✓ Radio frequency energy

$$E=(h\omega_0)/2\pi=h\nu_0$$

  - proton $\nu_0(\text{MHz})=42.58\ H_0(\text{T})$
    - ❖ $\nu_0=\omega_0/2\pi$
- ✓ The magnetic dipole transitions are influenced only by the magnetic component H' of the RF field, and <u>only by x and y components</u> $\boxed{\langle m_s' | \vec{\mu}\cdot H' | m_s \rangle}$
  - Since transition matrix element is no zero only for μ_x and μ_y

The physics of proton NMR
Dixon & Ekstrans
Medical physics 1982

Quantum-mechanical description of a spin in a magnetic field

- ✓ RF irradiation at ν_0 can cause de-excitation
  - Stimulated emission of a photon
- ✓ ω_0 obtained from the Hamiltonian is the same as classical Larmor frequency but the state of the quantum-mechanical spin time-independent →
  - No precession !

$$\langle \mu_x \rangle = \langle \mu_y \rangle = 0$$

$$\langle \mu_z \rangle = \pm \frac{1}{2} \gamma \hbar$$

- ✓ A wave function which is an equal mixture of spin-up and down states the expectation values are →
  - A precession of <μ> in the xy plane !

$$\langle \mu_x \rangle = \frac{1}{2} \gamma \hbar \cos(\omega_0 t)$$

$$\langle \mu_y \rangle = -\frac{1}{2} \gamma \hbar \sin(\omega_0 t)$$

$$\langle \mu_z \rangle = 0$$

http://www.imaios.com/en/e-Courses/e-MRI/NMR/Net-magnetization

Quantum-mechanical description of a spin in a magnetic field

- ✓ The expectation value of the magnetic moment of an ensemble of spins
  - [𝒳,μ] is the commutator of the Hemiltonian

$$\frac{d}{dt}\langle\mu\rangle = \frac{i}{\hbar}\langle[\mathscr{H},\mu]\rangle$$

- ✓ It may be shown
  - Commutation properties of angular momentum operators

$$\frac{d}{dt}\langle\mu\rangle = \langle\mu\rangle \times \gamma H$$

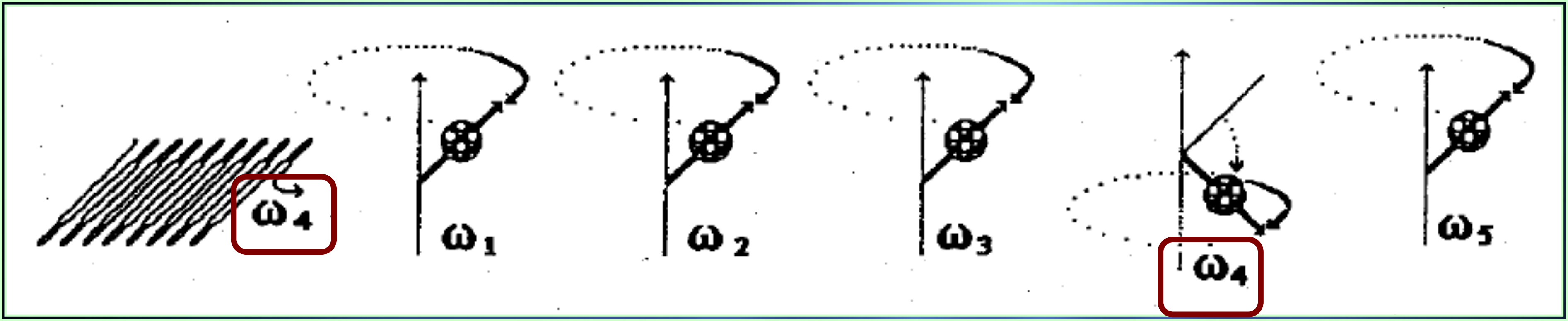
- ✓ For an ensemble of N noninteracting spin with an associated magnetic moment
  - M the observable quantity

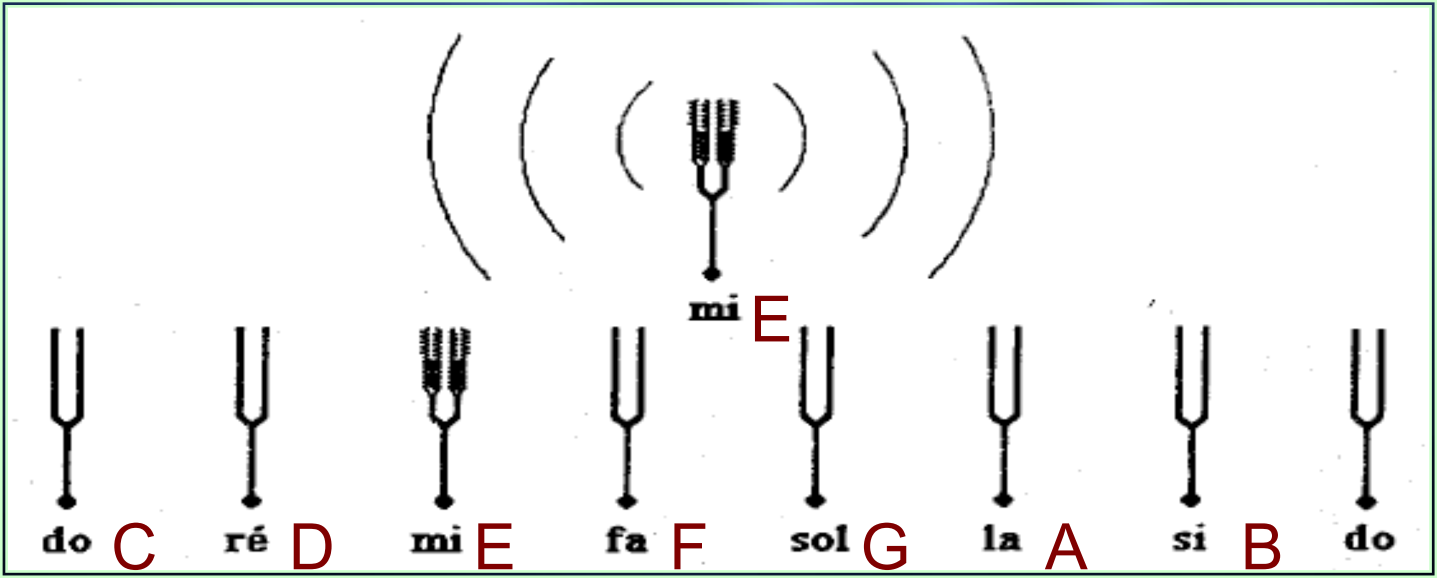
$$M = N\langle\mu\rangle$$

- ✓ M follows the classical equation!
  - Individual spins undergo complicated behavior

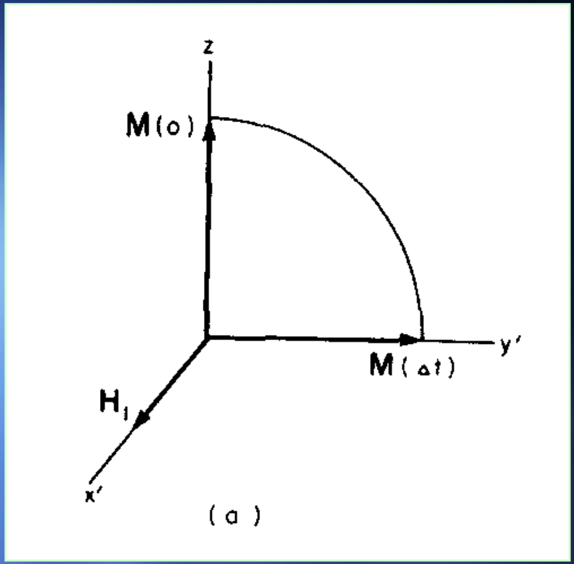
$$\frac{d}{dt}M = M \times \gamma H$$

Effect of the Radiofrequency Field





homework



A hydrogen sample is at equilibrium in a 1.5 Tesla magnetic field.

A constant B_1 field of 2.34×10^{-4} T is applied along the +x'-axis for 25 μs

- ✓ What is the direction of the net magnetization vector after the B_1 field is turned off?

MRI: Basic Physics & a Brief History
a video

https://www.youtube.com/watch?v=djAxjtN_7VE