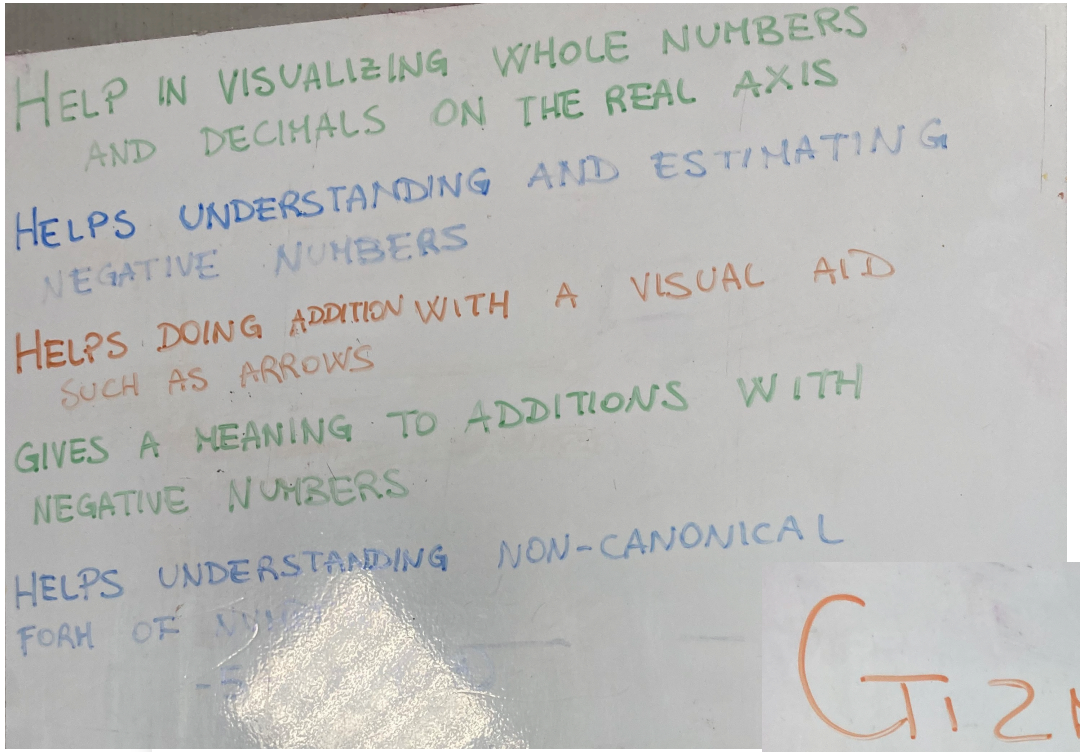
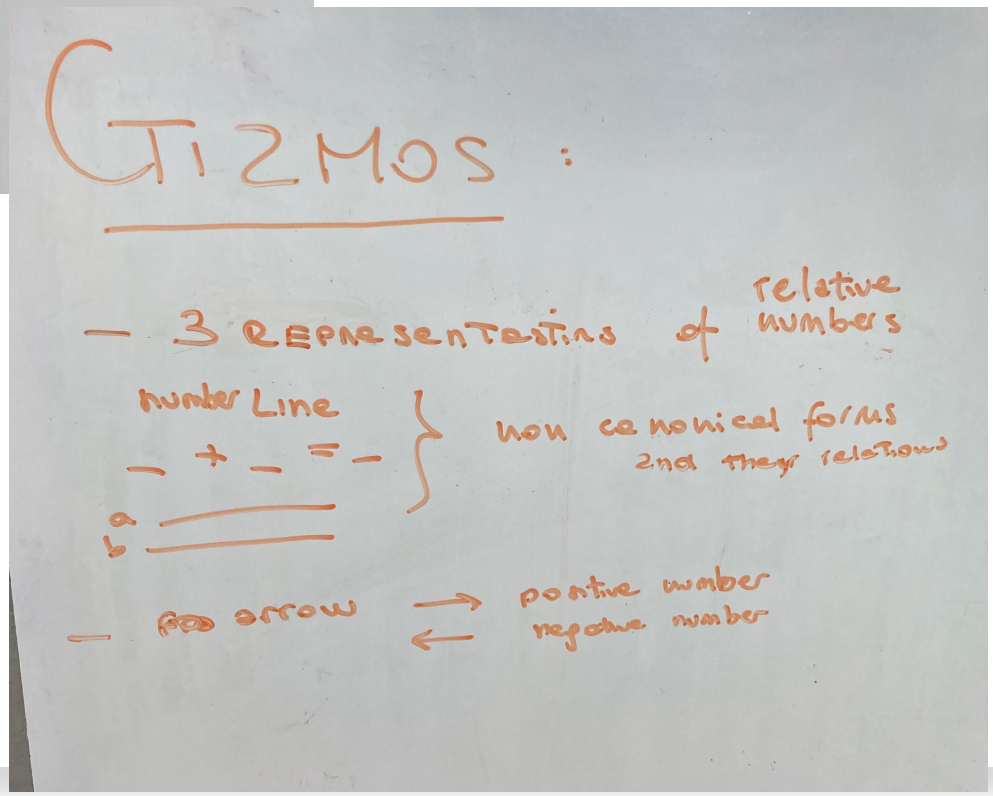
Relative Numbers Lab

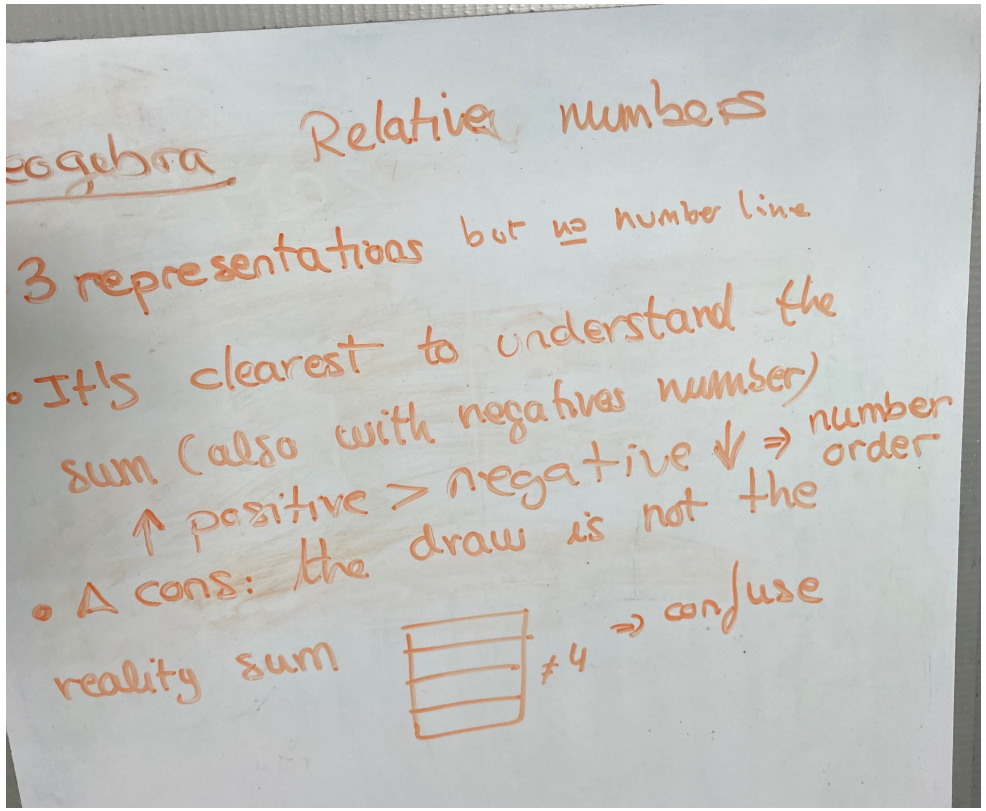
19/10/2023 – Technology in Mathematics Education

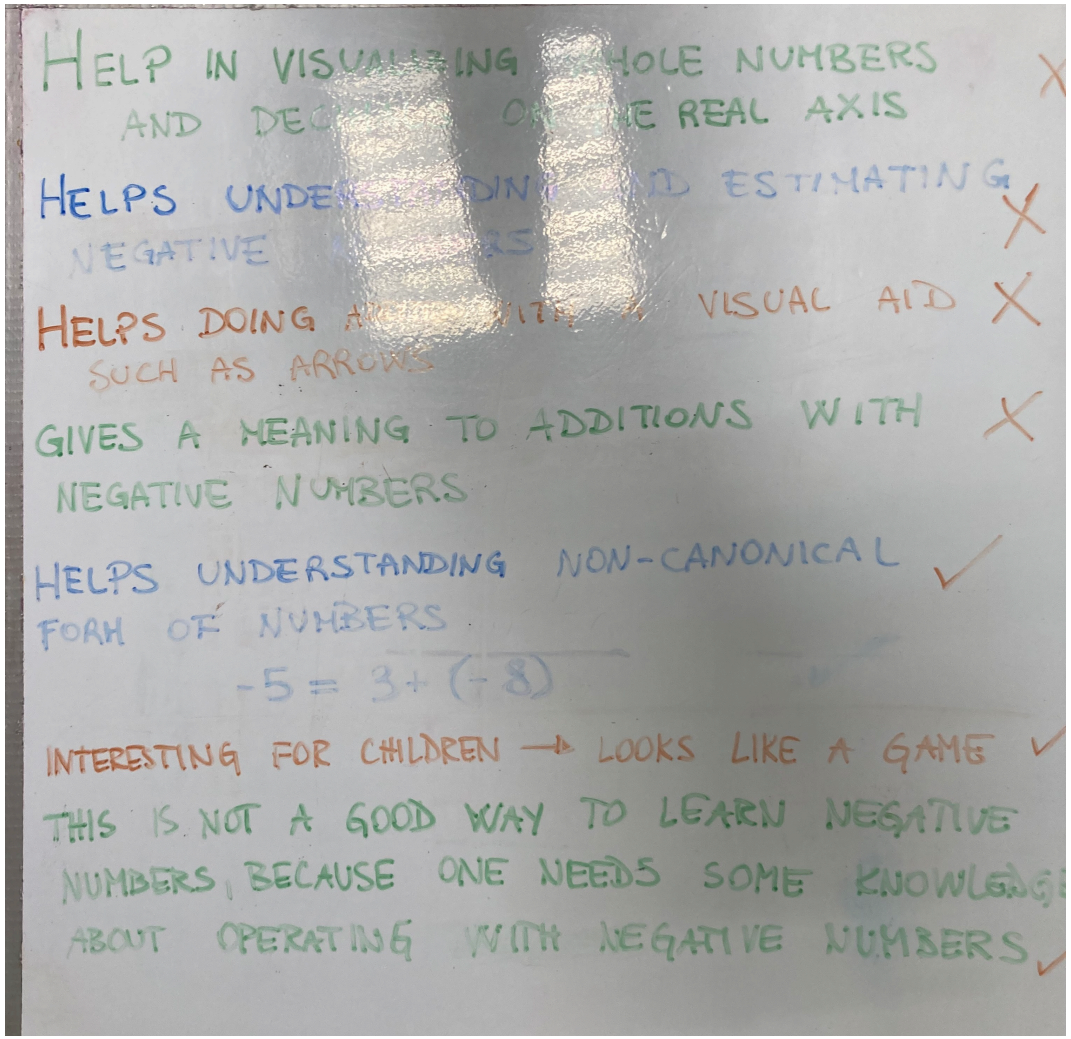
Gizmo's exploration





Geogebra's exploration





Desmos’s exploration

https://teacher.desmos.com/activitybuilder/custom/5f1aef204f0d9708e294e462?lang=it&collections=5f8a43db06b0d9a8bd84c3cf%2C5f8a446e06b0d9a8bd84c3dd

https://digipad.app/p/532145/ba03c68102e07

- COLORS to distinguish between positive AND NEGATIVE NUMBERS

- How the multiplication affects the product between relative numbers and the order relation.

- With one same sample of numbers we can have many different solutions. Ex. [1] [3] [-2] [-4]

↓ We can obtain

$[-4] \times [3] < [-2] \times [1]$

$[3] \times [1] < [-2] \times [-4]$

- Observe and Explain (not only computation) to see how it works.

- Autovalutation

HELPS PRACTICING WITH ALL OPERATIONS AND NEGATIVE NUMBERS

DISTINGUISHES NEGATIVE NUMBERS FROM POSITIVE ONES BY COLOR AND HELPS UNDERSTAND THAT THE PRODUCT OF TWO NEGATIVE NUMBERS GIVES A POSITIVE NUMBER.

HELPS UNDERSTANDING INEQUALITY SIGNS

OFFERS HINTS TO CONSTRUCT EXPRESSIONS WITH INTEGERS, NOT ONLY SOLVING

ASKS TO DESCRIBE PROCEDURES AND THE REASONING BEHIND SOME OF THE WORK

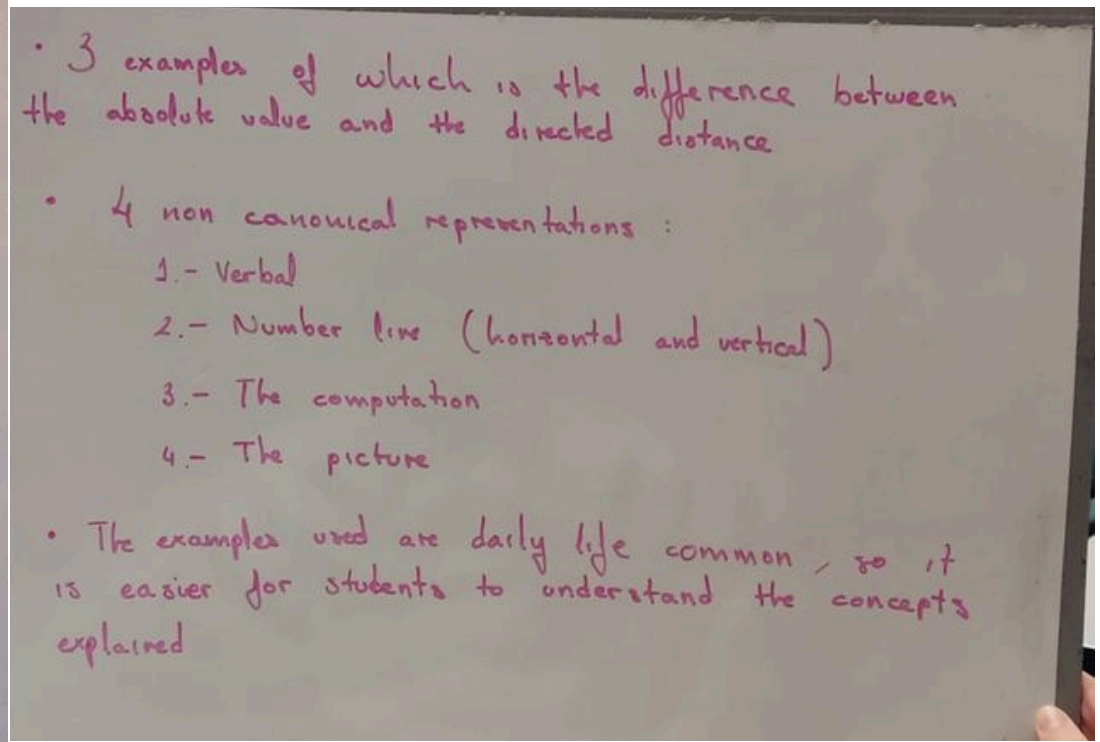
ASKS TO DESCRIBE AND COMMUNICATE ABOUT THE DIFFICULTIES THEY HAVE ENCOUNTERED

Phet’s exploration

https://phet.colorado.edu/en/simulations/number-line-distance

https://digipad.app/p/532145/ba03c68102e07

- UNDERSTANDING HOW THE ABSOLUTE VALUE WORKS AND HOW TO VISUALIZE IT AS A DISTANCE.
- STUDENTS ALREADY NEED TO KNOW WHAT A NEGATIVE NUMBER IS.
- HELPS PRACTICING WITH SUM OF NEGATIVE INTEGERS.
- SHOWS REALISTICS SITUATIONS IN WHICH WE CAN USE THE CONCEPT OF DISTANCE.
- TURNING VERTICA THE AXIS HELPS SWITCHING THINGS UP FROM USUAL AND GIVE A DIFFERENT INSIGHT ON THIS CONCEPTS
- THERE IS NOT A COLOR SCHEME



Detachment From the Minus Sign

Linchevski and Livneh (1999) attributed the DFMS error to a misunderstanding relating to a lack of 'structure sense'. They defined structure sense as being able to identify all the equivalent forms of an expression and "the ability to discriminate between the forms relevant to the task—generally one or two forms—and all the others" (p. 175). In the example of the operation $237+89-89+67-92+92$, this consists of considering $237+89-89+67$ or $237+67$ as equivalent forms relevant to the task, whereas $237+89-89+67-184$ should not be recognised as an equivalent form.

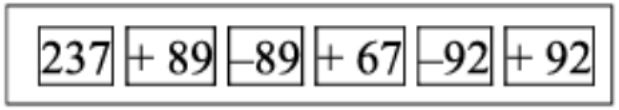
A lack of structure sense

The lack of structure sense referred to by Linchevski and Livneh (1999) is not unrelated to the restrictive understanding of subtraction as 'taking away', which is often observed in students (Selter et al., 2012). In this view, numbers represent 'unsigned' concrete quantities on which operations are performed in order to find the answer. Selter et al. (2012) believe that this computational view is too one-sided and probably leads to restricted mathematical thinking.

A restrictive understanding of subtraction

Vlassis, J., Demonty, I. The role of algebraic thinking in dealing with negative numbers. *ZDM Mathematics Education* **54**, 1243–1255 (2022).

In relational thinking, detecting the structure of an expression means ‘seeing’ the expression holistically. In a numerical expression such as 237 + 89 − 89 + 67 − 92 + 92, for example, this would involve ‘seeing’ the expression as follows:



This holistic view requires the expression to be considered as a sum of signed components separated by an implicit plus sign.

This view is essential in algebra, for example when it is necessary to reduce polynomial expressions. It is not necessary in numerical operations, since it is always possible to proceed computationally, step by step, to find the answer. However, it is very useful for carrying out operations efficiently

The minus sign has to be treated as attached to the number that follows it

ARITHMETIC THINKING

OPERATIONAL VALUE – external to the number (semiotic aspect)

In mathematics everything related to signs, symbols and relations between symbols is called semiotic. Semiotic includes all signs that are visual and verbal.

ONTOLOGICAL VALUE – part of the number (semantic aspect)

ALGEBRAIC THINKING

In mathematics , this is strictly related to the mathematical objects' meaning.

Table 1 Exploring the three meanings of the minus sign will allow students to differentiate among them.

Problem	Meaning of the Minus Sign
1. $5 - 8 = \square$	Subtraction as a binary operation
2. $\square + 5 = -2$	A symbolic representation for a negative number
3. Which is larger, $- -4$ or -4?	The *opposite of*, a unary operation

CURRICULUM INSIGHT

Lisa L. Lamb, Jessica Pierson Bishop, Randolph A. Philipp, Bonnie P. Schappelle, Ian Whitacre, & Mindy Lewis. (2012). Developing Symbol Sense for the Minus Sign. *Mathematics Teaching in the Middle School*, *18*(1), 5–9.