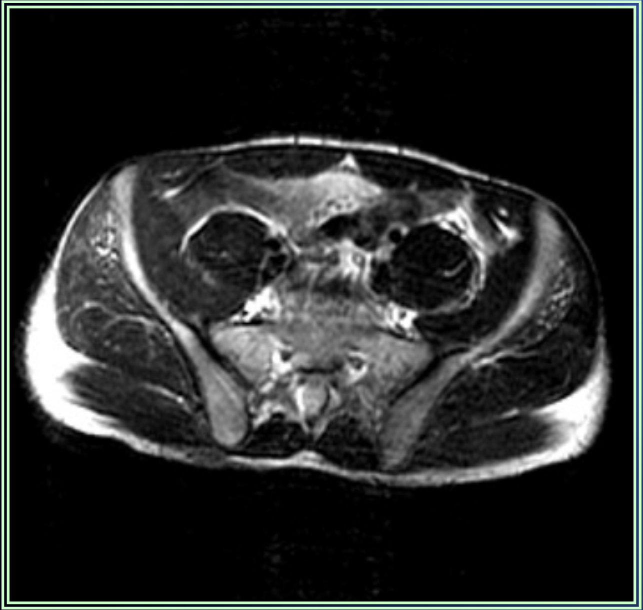
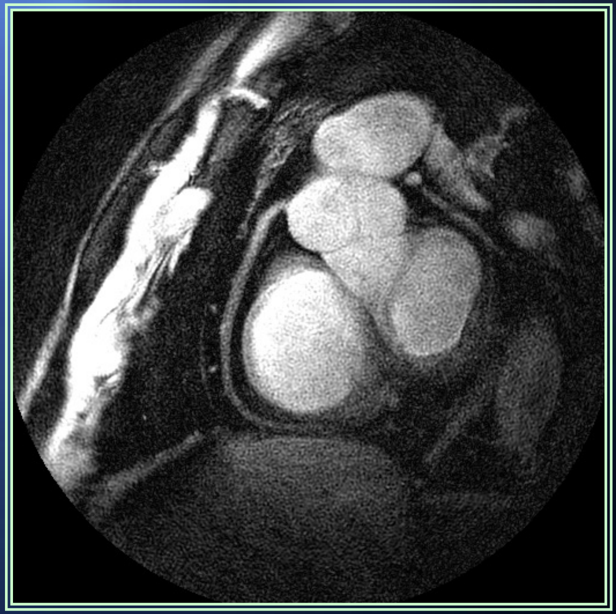
FROM SIGNALS TO IMAGES: ENCODING SPATIAL INFORMATION IN NMR



Abdomen

Spine



Heart / Coronary

Birth of MRI

Lauterbur and the first magnetic resonance images (from *Nature 1973*)

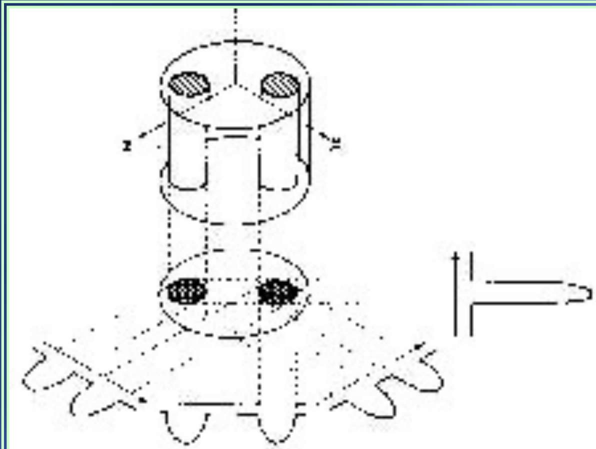


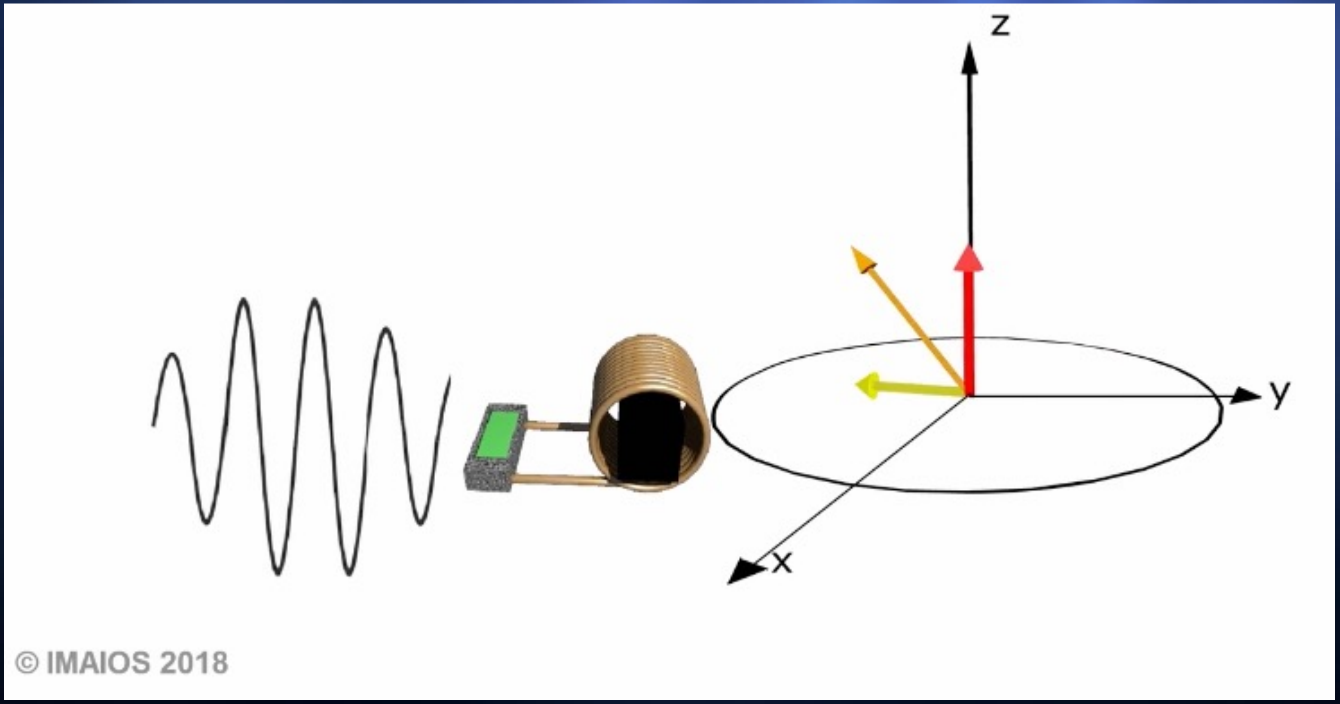
Fig. 1 Relationship between a three-dimensional object, its two-dimensional projection along the Y-axis, and four one-dimensional projections at 45° intervals in the XZ-plane. The arrows indicate the gradient directions.

Fig. 2 Proton nuclear magnetic resonance zeugmatogram of the object described in the text, using four relative orientations of object and gradients as diagrammed in Fig. 1.

In 1978, Mansfield presented his first image through the abdomen.

Concepts in MRI Spatial Localisation

- ✓ In the NMR experiments a signal is collected from the entire sample
  - no means of differentiating signals from different parts of the sample



Concepts in MRI Spatial Localisation

For imaging

- ✓ a means of encoding parts of the signal according to where they originate from in space is necessary
- ✓ then using these tags to ‘deconstruct’ the acquired signal and map it to spatial locations

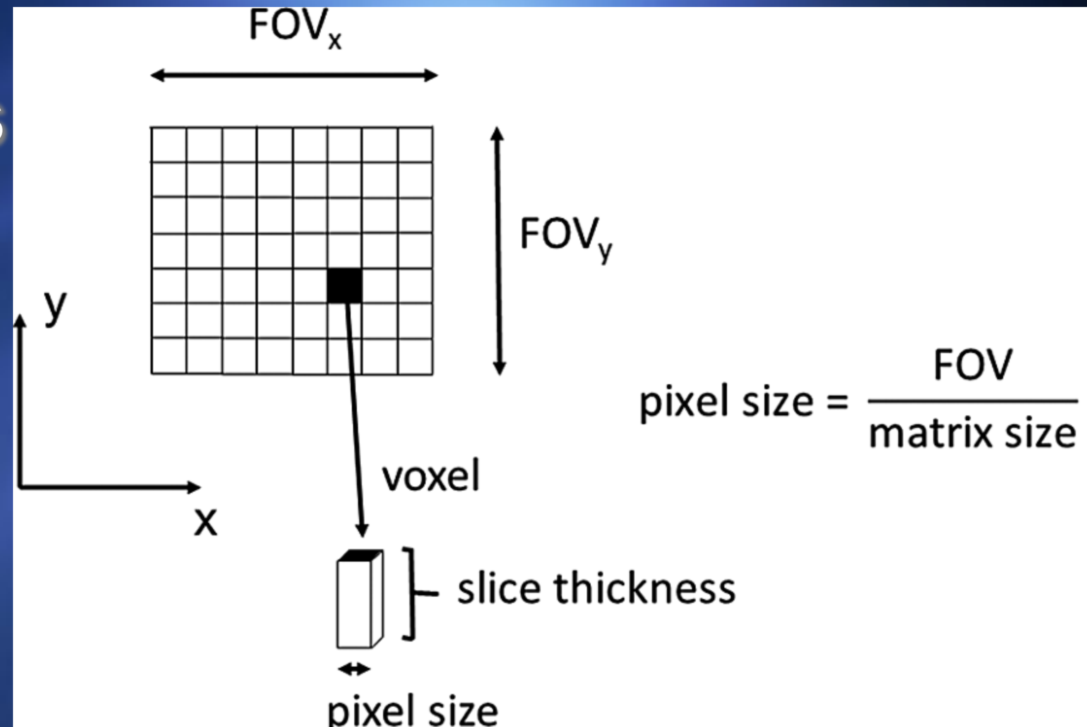
Concepts in MRI Spatial Localisation

- Signal originating from a 3D volume
  - the portion of the patient's body at the centre of the magnet and the sensitive volume of the receive RF coils
- Signals can be encoded in 3D and 3D image generated
  - more usual to first restrict the region from which signal is acquired to slices and then encode data in 2D
    - Slice selection

Concepts in MRI Spatial Localisation

In this example

- ✓ transaxial slices
- ✓ 2D spatial encoding
  - in the xy-plane



this is a convenient simplification and significantly underplays the capability of MRI in this respect!

NMR Signal localization

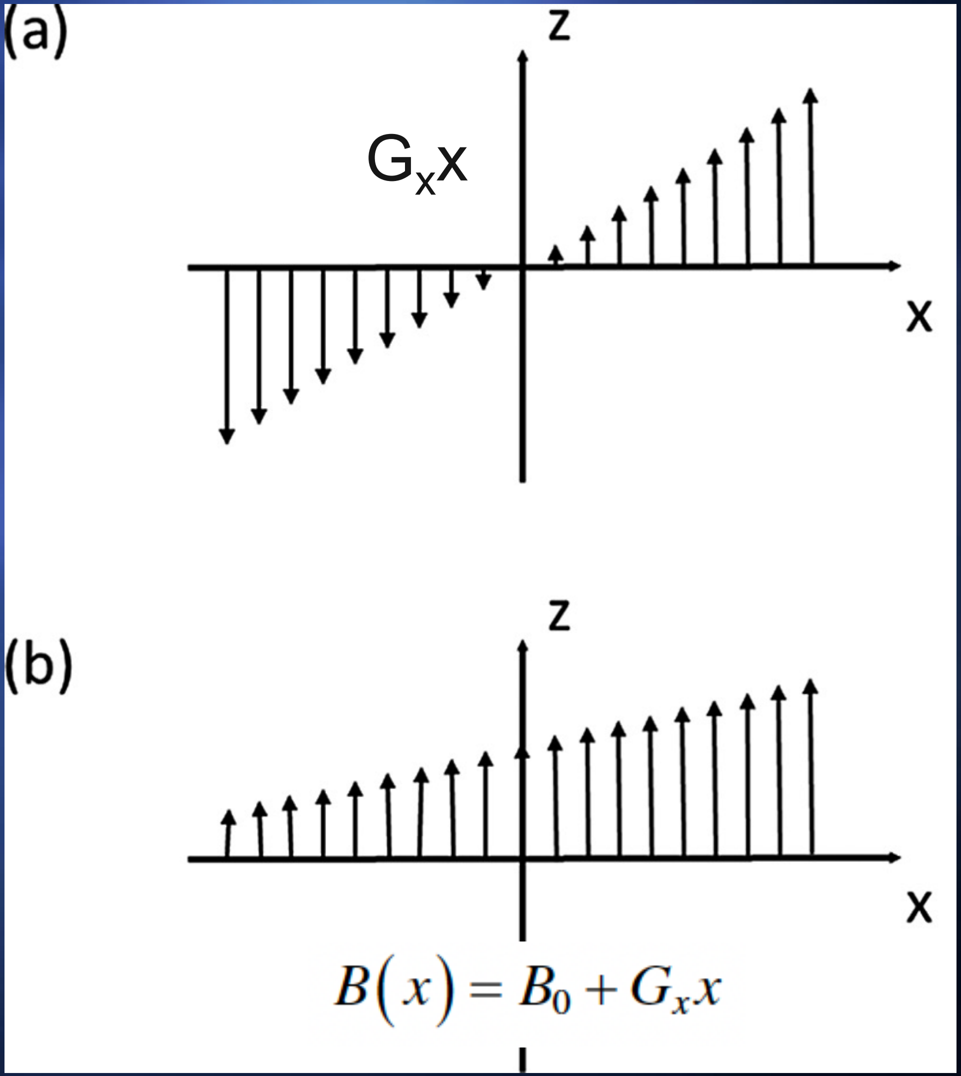
- ✓ In MRI the signal localization is based on magnetic fields gradients
  - G_x, G_y, G_z
- ✓ They produce a linear variation in magnetic field intensity in a direction in space
- ✓ This variation in magnetic field intensity is added to the B_0
  - B_0 is far more powerful

$$B(x) = B_0 + G_x x$$
$$B(y) = B_0 + G_y y$$
$$B(z) = B_0 + G_z z$$

NMR Signal localization

a) Magnetic field gradient with the field oriented along the z-axis but varying in strength along the x-axis
   - an 'x-gradient'

b) spatial variation in field strength along the x-axis due to B_0 and the gradient field



http://www.imaios.com/en/e-Courses/e-MRI/MRI-instrumentation-and-MRI-safety/gradients

NMR Signal localization

- ✓ In MRI the signal localization is based on magnetic fields gradients
- ✓ Gradient pulse applied for a short period of time
  - typically lasting 1–2 ms
- ✓ Gradient strength refers to how steeply the field strength varies with position
  - expressed in mTm^{-1}
  - Typical values of the order of 10 mTm^{-1}
  - B_0 usually 1.5 or 3 T

NMR Signal localization

In MRI the signal localization is based on magnetic fields gradients

- ✓ Generally speaking:
  - B_i vector
  - $\delta B_i/\delta x_j$ second order tensor
- ✓ In NMR the direction of the field remains the z-axis
  - $B_0 = B_z k$
  - $\delta B_z/\delta x$, $\delta B_z/\delta y$, $\delta B_z/\delta z$ fundamental gradients for MRI
    - Written as dB/dx, dB/dy, dB/dz or G_x, G_y, G_z

NMR Signal localization

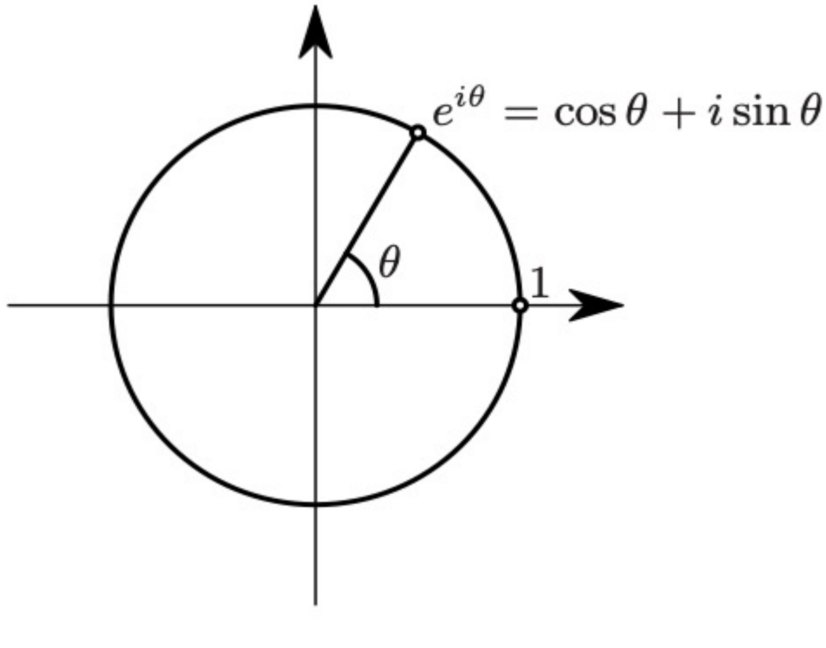
- 1D system
  - In position x_1 , N_1 protons, $T_2^*(x_1)$
  - in position x_2 N_2 protons $T_2^*(x_2)$
    - T2* may be different
- if B is constant
  - Larmor frequency is constant

$$S(t) = N_1 e^{-i\omega_0 t} e^{-t/T2^*(x_1)} + N_2 e^{-i\omega_0 t} e^{-t/T2^*(x_2)}$$

Phase notation

- ✓ **Complex exponential function**
  - Euler's formula, i imaginary unit

$$e^{i\omega t} = cos(\omega t) + i\, sin(\omega t)$$



Euler's definition of $e^{i\theta}$

NMR Signal localization

1D system

- ✓ If $B(x)=B_0 + x\, G_x$
  - $B(x_1)=B_0+x_1\, G_x$; $B(x_2)=B_0+x_2\, G_x$
  - $\omega_1=\gamma B(x_1)=\gamma[B_0+ x_1 G_x]$; $\omega_2=\gamma B(x_2)=\gamma[B_0+x_2 G_x]$

$$S(t) = N_1 e^{-i\omega_1 t}\, e^{-t/T2^*(x_1)} + N_2 e^{-i\omega_2 t}\, e^{-t/T2^*(x_2)}$$

Signal localization

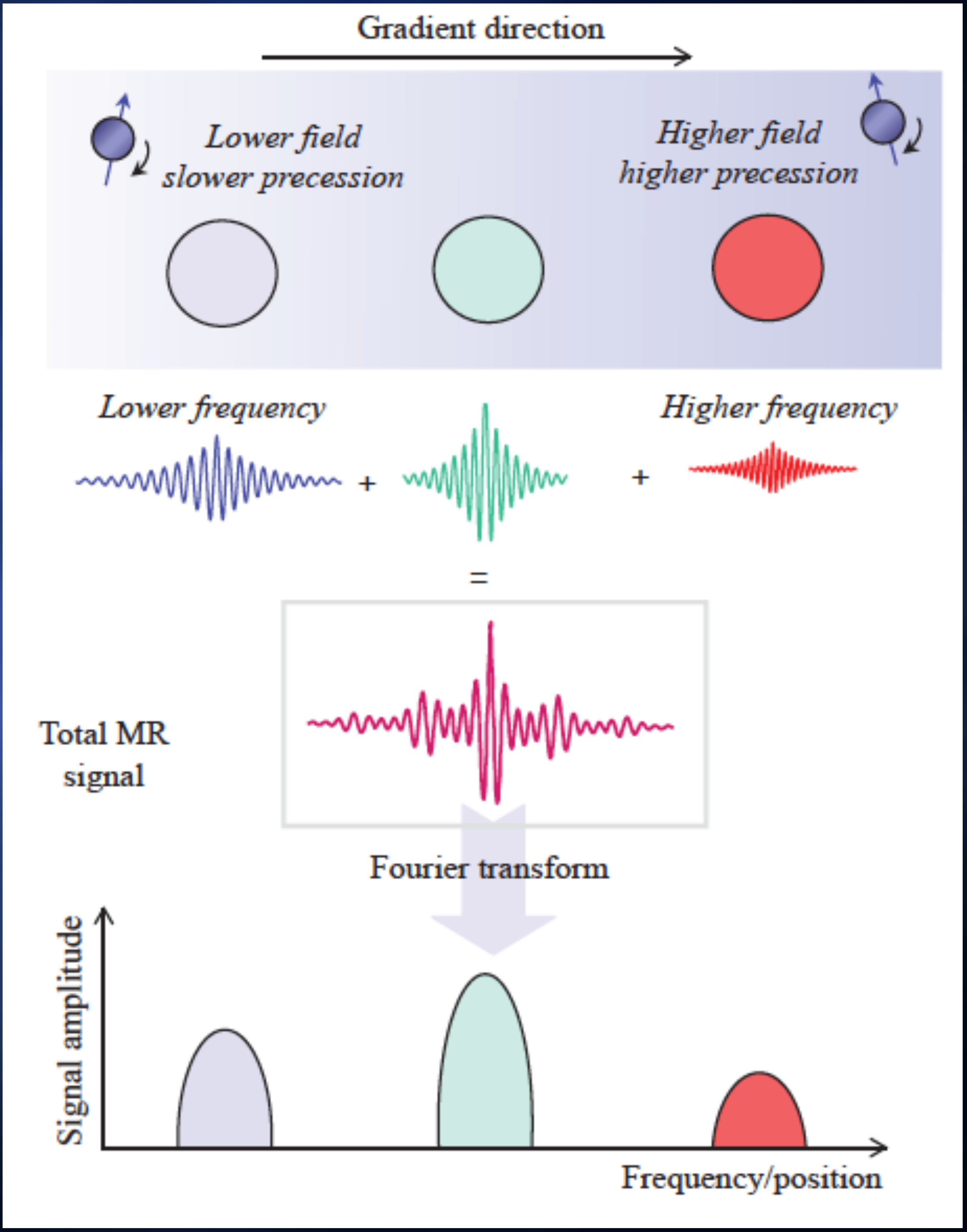
$$x_1=[\omega_1-\omega_0]/(\gamma G_x)$$

$$x_2 =[\omega_2-\omega_0]/(\gamma G_x)$$

$$\omega_0 = \gamma B_0$$

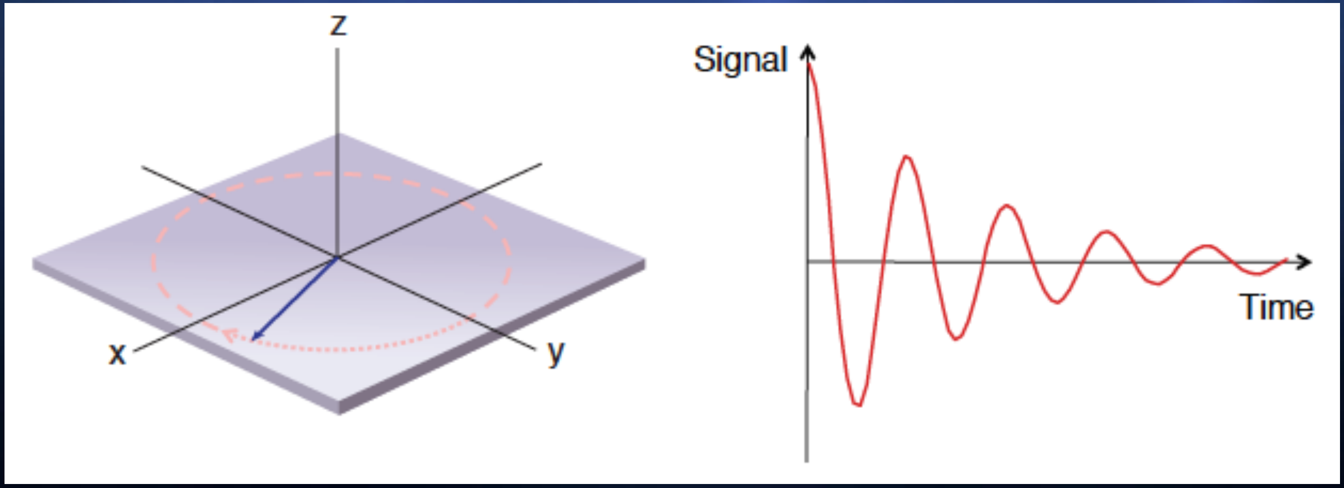
NMR signal and Fourier Transform

"MRI From Picture to Proton" D.W. McRobbie et al.
Ch 8 Spaced Out: Spatial Encoding

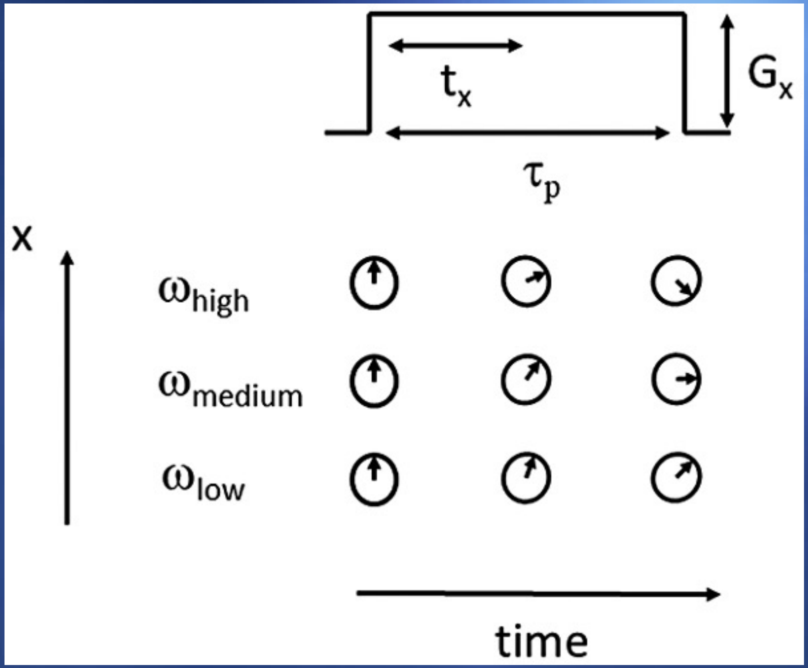


Larmor frequency in NMR

- ✓ The frequency of the RF pulse
  - applied to nutate M into the transverse plane
- ✓ The frequency at which M_{xy} precesses around the z-axis
- ✓ the frequency at which the FID or echo signal oscillates

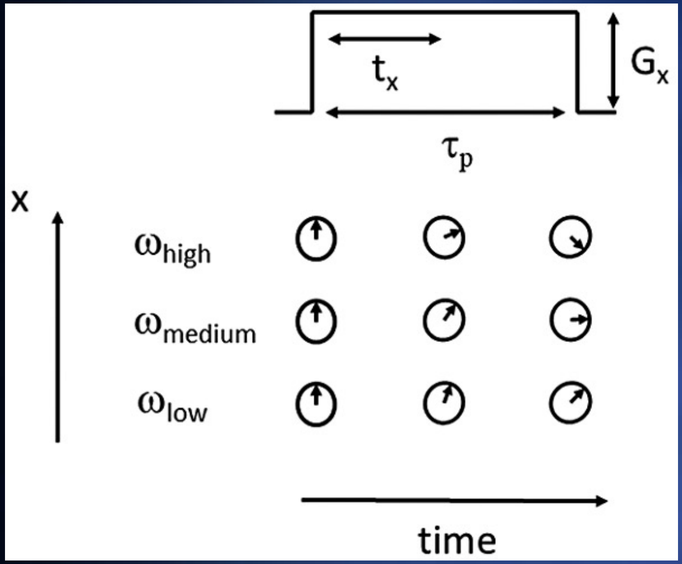


Gradient, frequency and phase



Variation in frequency and phase of transverse magnetisation in the presence of a magnetic field gradient

Gradient pulse



$$\omega(x) = \gamma B(x) = \gamma(B_0 + G_x x)$$

Frequency variation is

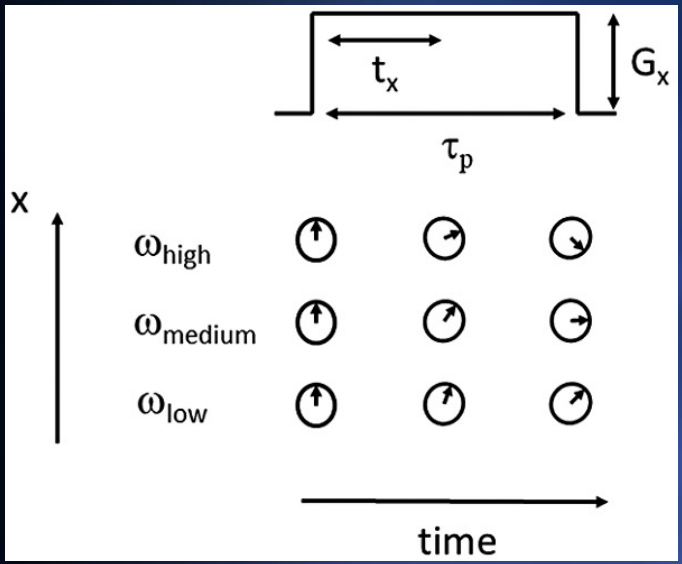
✓ constant

- magnetisation at a given location precesses at the same frequency throughout the gradient pulse

✓ transient

- at the end of the gradient pulse magnetisation goes back to precessing at the Larmor frequency determined by B_0
  - regardless of position

Gradient pulse



difference in phase develops during the gradient pulse

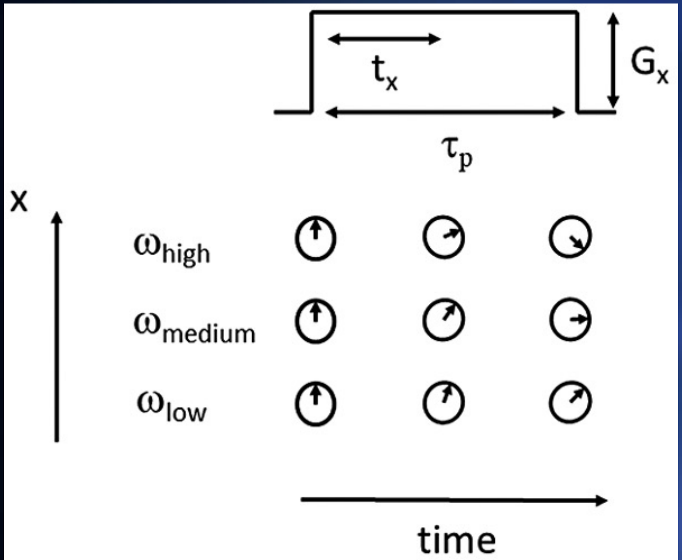
- ✓ at any point in time (t_x) there is a variation in phase along the x-axis

$$\varphi(x) = \omega(x)t_x = \gamma B(x)t_x = \gamma(B_0 + G_x x)t_x$$

The phase variation

- ✓ increases with time during the gradient pulse
- ✓ is persistent

Gradient pulse

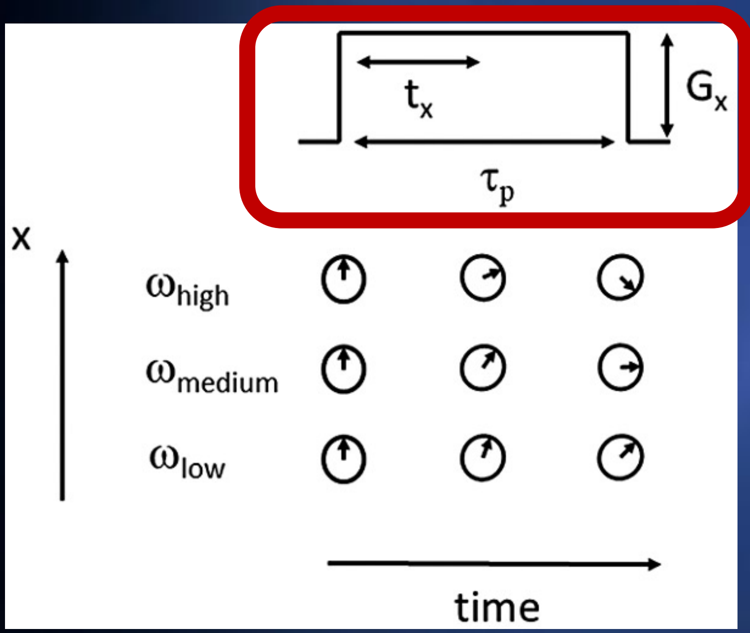


$$\varphi(x) = \omega(x)t_x = \gamma B(x)t_x = \gamma(B_0 + G_x x)t_x$$

The phase variation:

- ✓ increases with time during the gradient pulse
- ✓ is persistent
  - the phase distribution built up at the end of the gradient pulse remains until the transverse magnetisation decays away
    - ❖ or another gradient is applied

Gradient pulse



- ✓ **The phase variation imposed by a gradient depends on the product of the gradient strength (G_x) and its duration τ_p**
- ✓ **$G_x\tau_p$ is referred to as the gradient 'area'**
  - **See gradients depicted in pulse sequence diagrams**

$$\varphi(x) = \omega(x)\,t_x = \gamma B(x)\,t_x = \gamma\left(B_0 + G_x x\right)t_x$$

Towards spatial localization

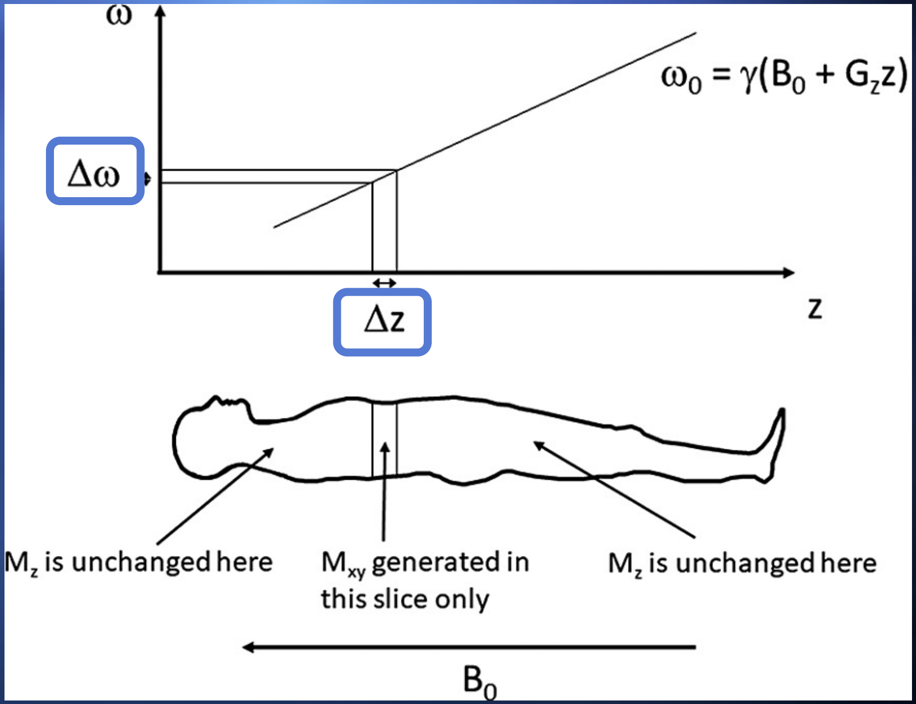
3 stages

- ✓ **Selecting a transaxial slice**
- ✓ **Encoding 2D spatial information into the signal obtained from that slice**

- https://www.imaios.com/en/e-Courses/e-MRI/Signal-spatial-encoding/Frequency-encoding
- https://www.imaios.com/en/e-Courses/e-MRI/Signal-spatial-encoding/Phase-encoding

Slice Selection

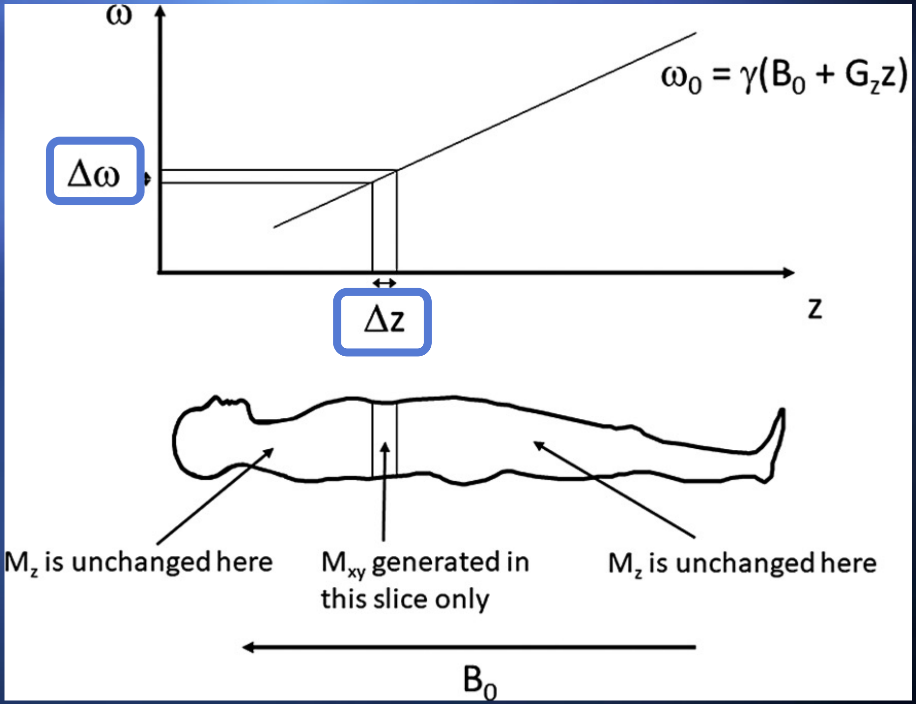
- While the gradient G_z is switched on, a 90º RF pulse is applied
  - not at a single frequency but containing a range of frequencies $\Delta\omega$
- M_{xy} is generated within a slice in which the frequency content of the RF pulse matches the spatially varying ω_L



Slice Selection

a 90^0 RF pulse containing a range of frequencies $\Delta\omega$

$$\Delta z = \frac{\Delta\omega}{\gamma G_z}$$



thickness of the slice is controlled by

- ✓ the frequency content of the RF pulse
- ✓ the gradient strength

www.imaios.com/en/e-Courses/e-MRI/Signal-spatial-encoding/Slice-select

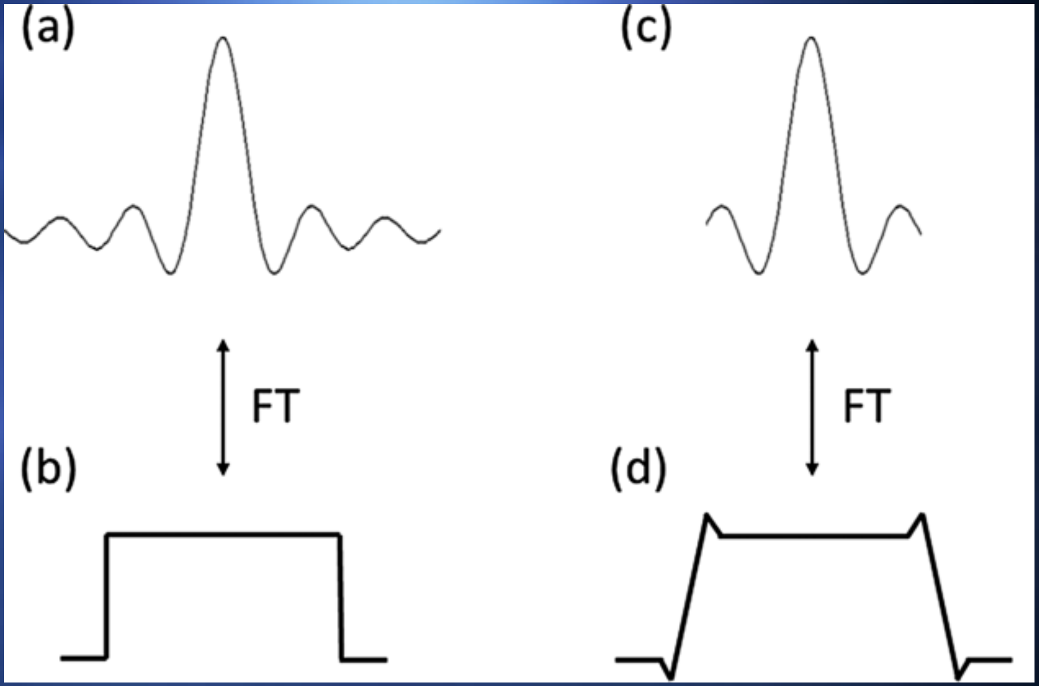
What shape of RF pulse?

- ✓ assuming we want the selected slice to have a sharp ‘top hat’ shaped profile
- ✓ the pulse shape in the time domain needs to be the Fourier transform of a ‘top hat’ function
  - sinc function

$$\mathrm{sin}c(x) = \frac{\sin(x)}{x}$$

What shape of RF pulse?

- sinc function is infinitely long so inevitably to be truncated
  - for practical purposes
- degradation of the profile of the excited slice
- reduction in spatial resolution in the through-slice direction
  - generation of artefacts



Slice Selection and phase

- ✓ elements of that magnetisation at different positions along the z-axis begin to precess at different frequencies
  - i.e. at different positions within the thickness of the slice
- ✓ get out of phase with each other
- ✓ leading to a loss of signal
  - undesirable !!!!!

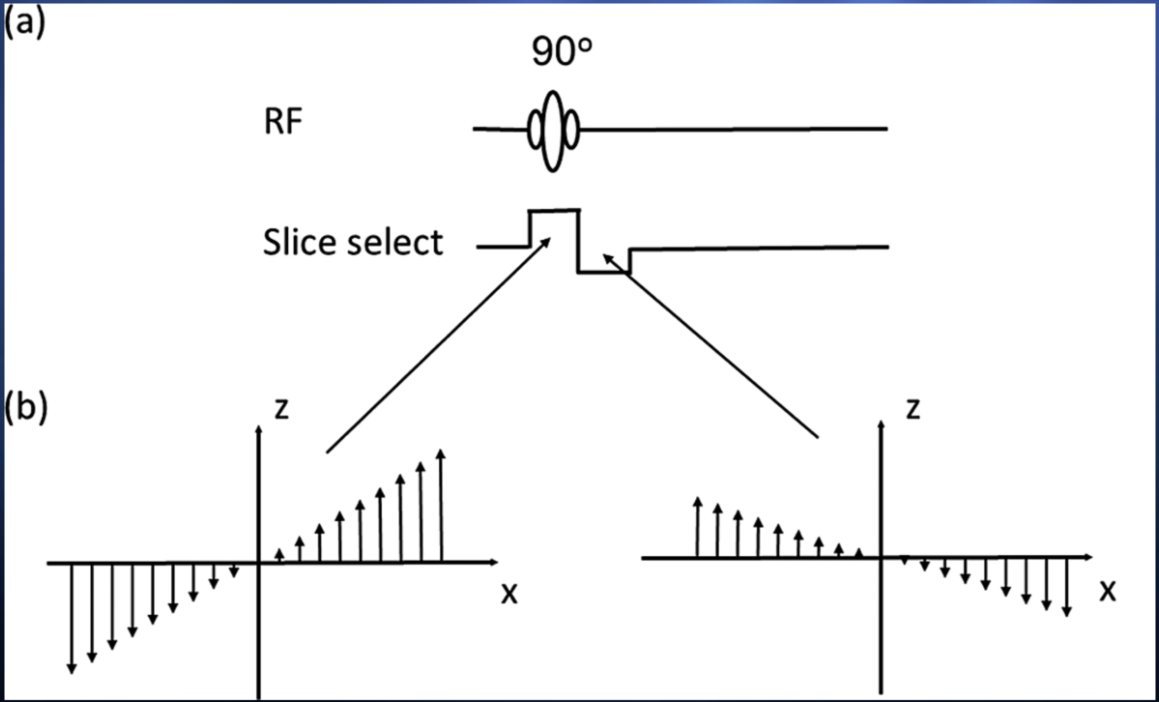
A correction is needed

Slice Selection

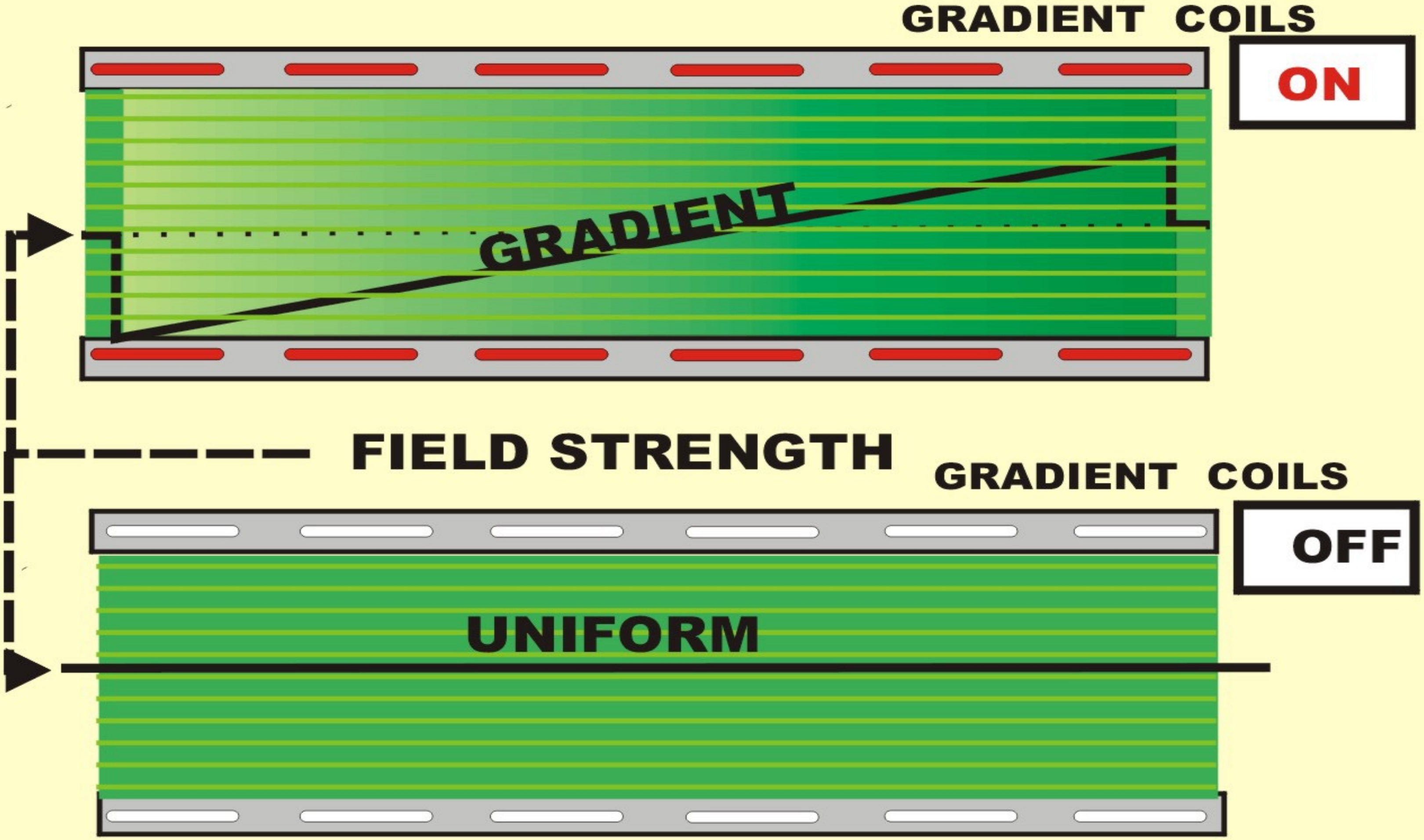
- phase imparted by a gradient persists until we apply another gradient
- To reverse this dephasing we can apply another gradient in the opposite direction to the slice selection gradient and with half the ‘area’
  - assuming that nutation occurs all at once in the middle of the gradient pulse
    - there is always some residual dephasing in the through-slice direction

Slice Selection

- ✓ phase imparted by a gradient persists
- ✓ To reverse this dephasing another gradient is applied in the opposite direction with half the ‘area’



A MAGNETIC FIELD GRADIENT

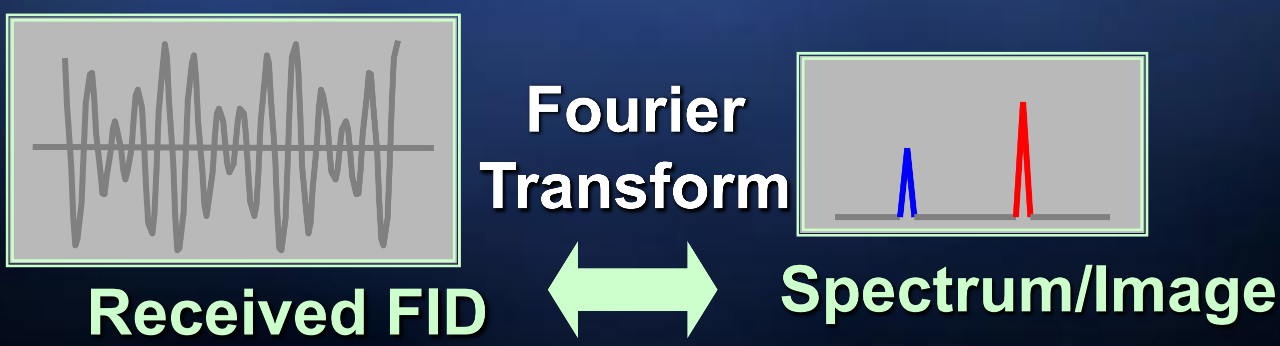


Sprawls

Frequency Encoding

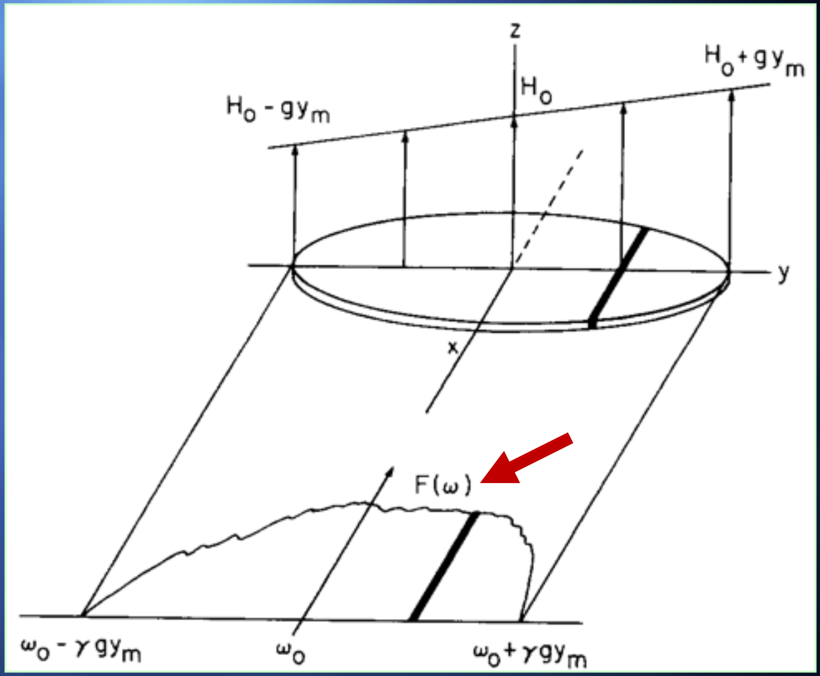
Application of a G_x while the echo signal is being acquired

- ✓ The precession frequency of M_{xy} will depend on position along the x-axis
- ✓ The acquired echo signal will also contain a range of frequencies
    - the strength of the signal at each frequency corresponding to the amount of signal coming from different locations along x



Frequency Encoding

- Application of G_y while the signal is acquired
- The precession frequency of M_{xy} will depend on position
- The signal will also contain a range of frequencies
- The strength of the signal at each frequency corresponding to the amount of signal coming from different locations



$F(\omega)$ = FT $S(t)$

$S(t)$ the signal collected by the coil

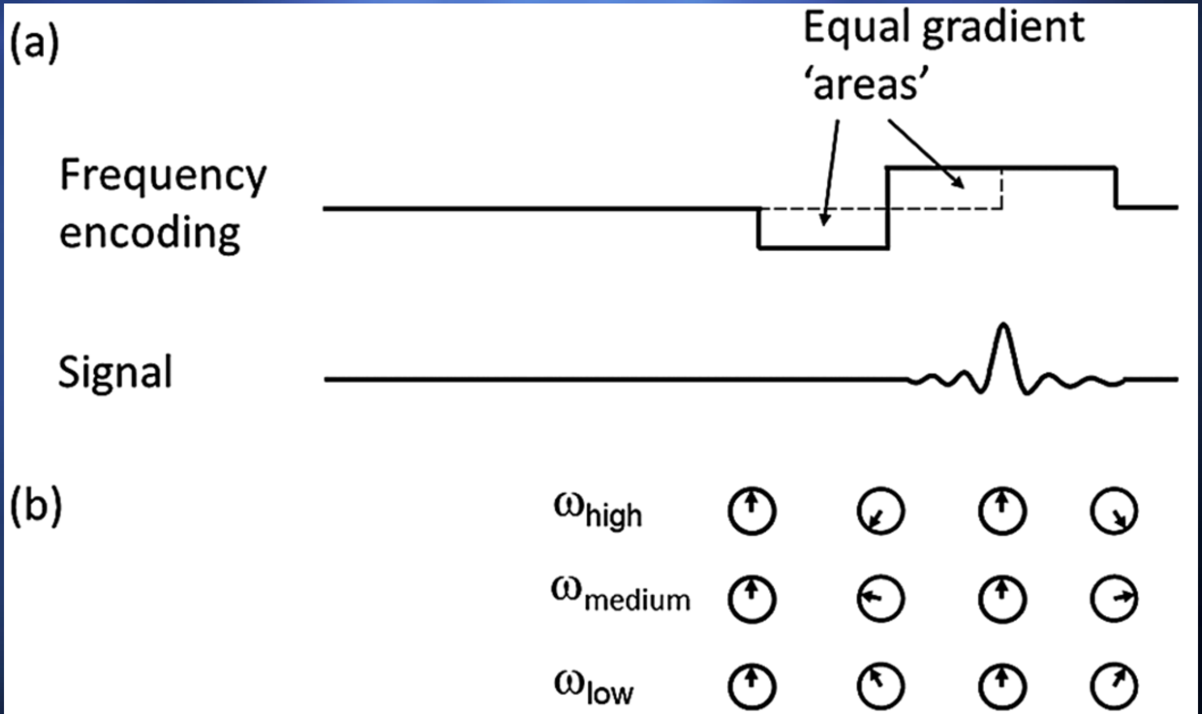
Frequency Encoding and phase

- ✓ The acquired echo signal will also contain a range of frequencies
- ✓ Differences in frequency due to magnetic field gradients result in accumulation of differences in phase
- ✓ The frequency differences that we are using for spatial encoding will result in dephasing of magnetisation at different locations along the x-axis
  - destroying the echo signal !

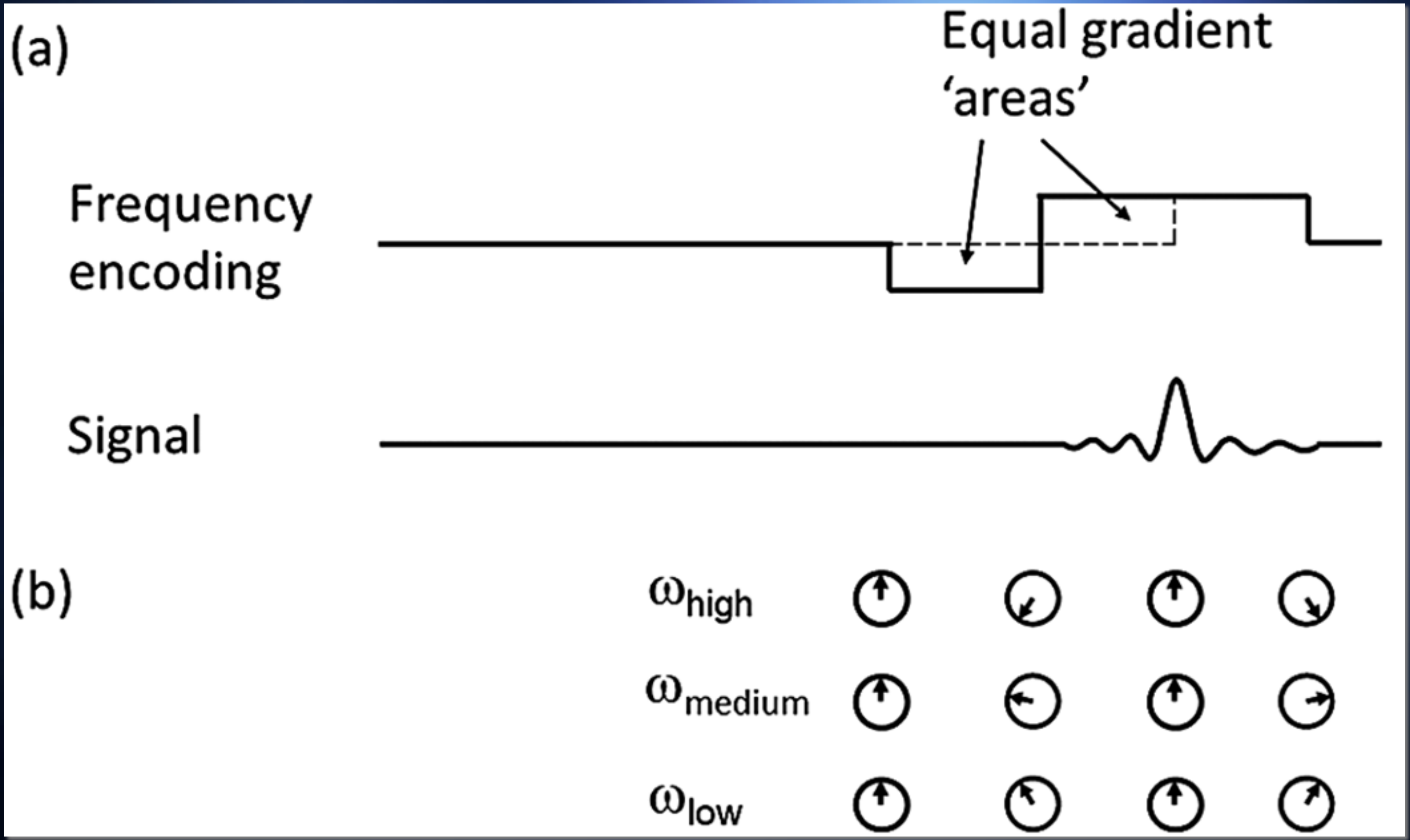
Frequency Encoding

- ✓ Before frequency encoding, a gradient is applied in the opposite direction to the frequency encoding gradient and of half the 'area'

The dephasing caused by this gradient is reversed during the first half of the frequency encoding gradient pulse

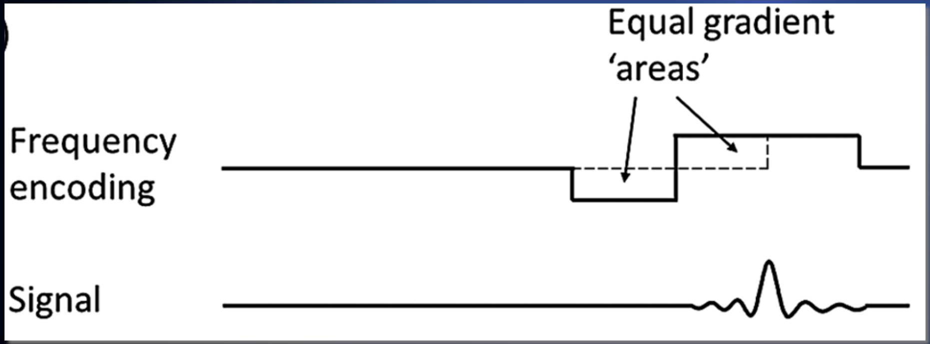


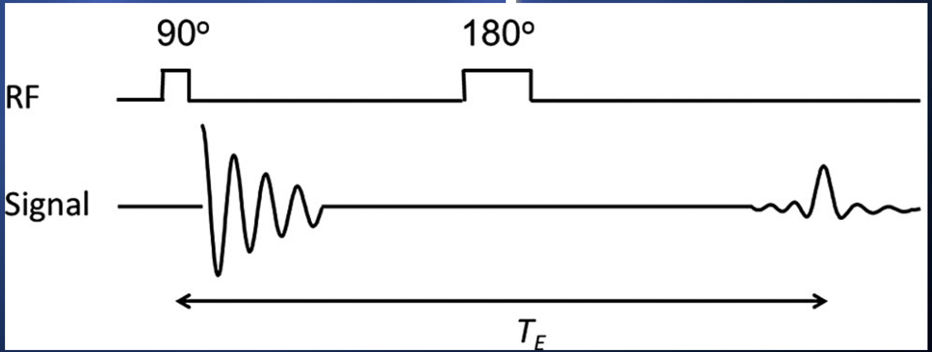
Gradient echo



Spin echo & gradient echo

- the timing of the gradient echo is determined by the amplitudes and durations of the gradients on the frequency encoding axis
  - it occurs at the point in time at which the net gradient 'area' is zero
- the timing of the spin echo is determined by the time interval between the 90^0 and 180^0 pulses





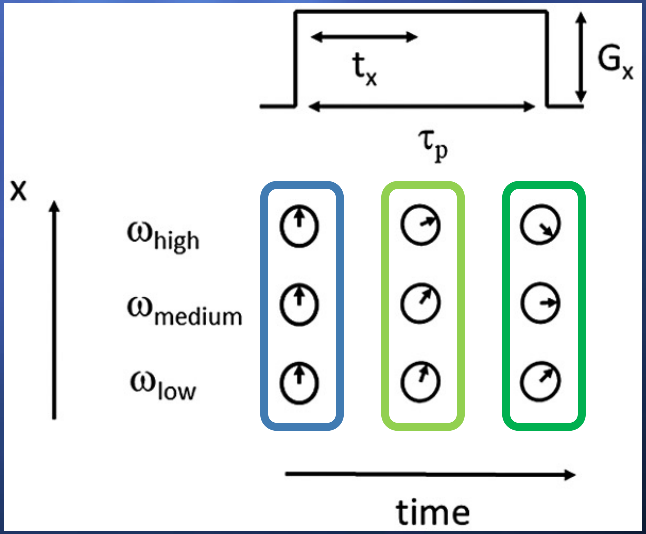
it is the task of the pulse sequence designer to ensure that the 2 echo conditions coincide in time

Frequency or Phase Encoding

consider how the phase of magnetisation develops in the presence of the frequency encoding gradient

- ✓ **Each of the phase distributions results from precession of M_{xy} in a constant gradient for a different period of time**

$$\varphi(x,t_x) = \gamma G_x x t_x$$



- **t_x is a timepoint during the acquisition of the echo**
  - **The signal timepoints are illustrative of digital samples collected**
    - **256 or 512 data points**

Phase Encoding

- ✓ A gradient on the y-axis of strength G_y and duration t_y is applied between excitation of M_{xy} and collection of the echo

$$\varphi(y, t_y) = \gamma G_y y t_y$$

- ✓ a phase distribution is imposed onto M_{xy}
  - using a gradient of the appropriate 'area'
    - product of strength and duration
- ✓ this phase distribution persists once the gradient is switched off

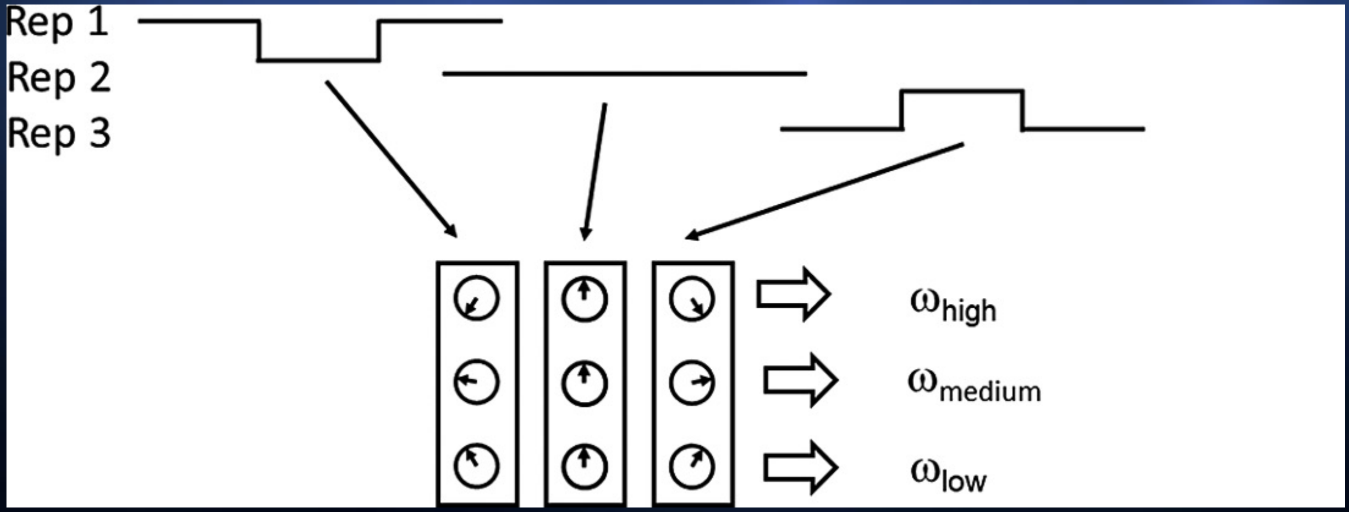
Phase Encoding

- ✓ The entire experiment is repeated many times applying the whole pulse sequence each time with a different G_y amplitude
  - 90º pulse, 180º echo pulse and echo acquisition with G_x
- ✓ A different variation in phase along the y-axis would be ‘locked up’ in each resulting echo
- ✓ If we then lined up the signals next to each other, the evolution of phase across the set of signals would be exactly the same as the evolution of phase between samples of the frequency-encoded echo

Phase Encoding

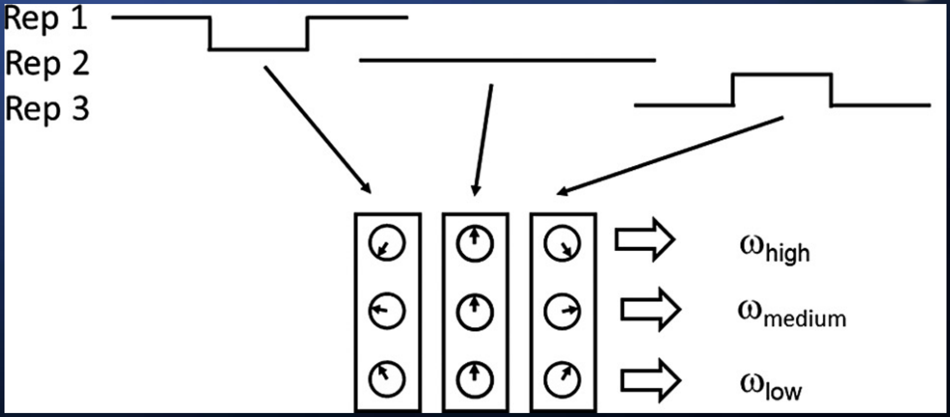
In this example, we perform an NMR experiment 3 times

- ✓ **During each repetition, G_y is applied**
- ✓ **G_y has a different amplitude each time**
- ✓ **We collect a spin-echo signal each time**
- ✓ **Locked up inside each signal is a different phase distribution along the y-axis**



Phase Encoding

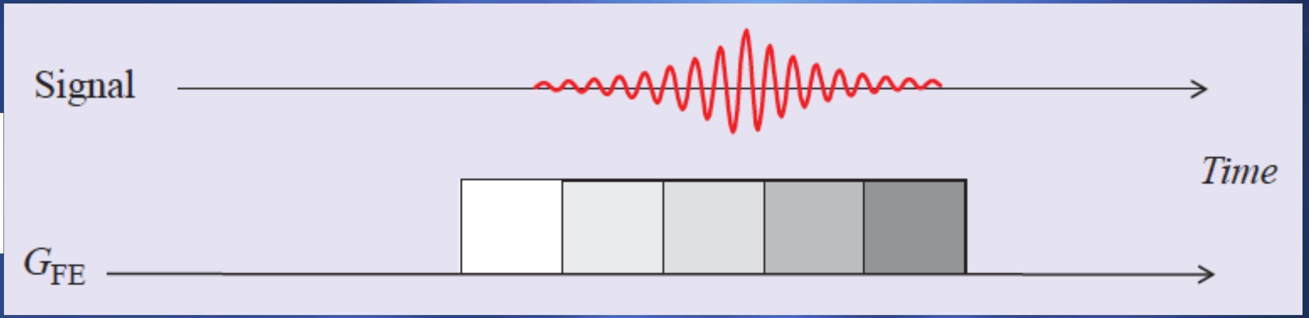
- ✓ Locked up inside each signal is a different phase distribution along the y-axis
- ✓ if we line up the signals next to each other and look across them there appear to be high-, medium- and low frequency components
  - horizontally in the diagram
- ✓ mathematically as well, there appears to be frequency information that we can extract using the Fourier transform



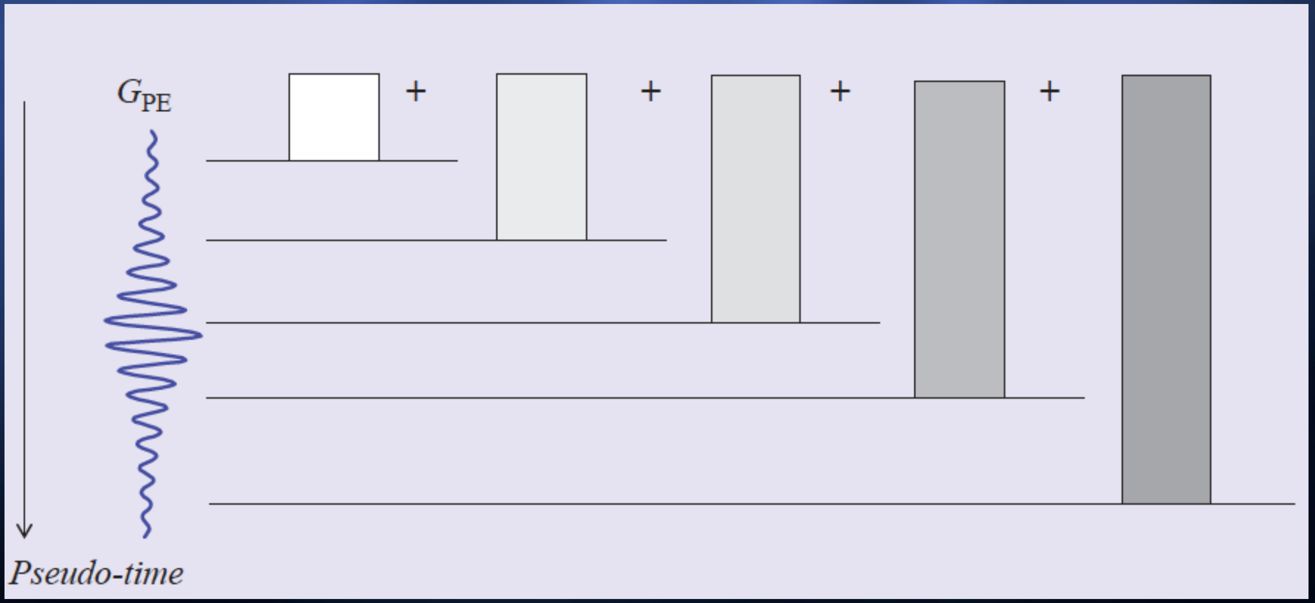
Phase Encoding

Equivalence of frequency encode acquired continuously in real time and phase encode acquired step-wise 'pseudo-time'

$$\varphi(x,t_x) = \gamma G_x x t_x$$

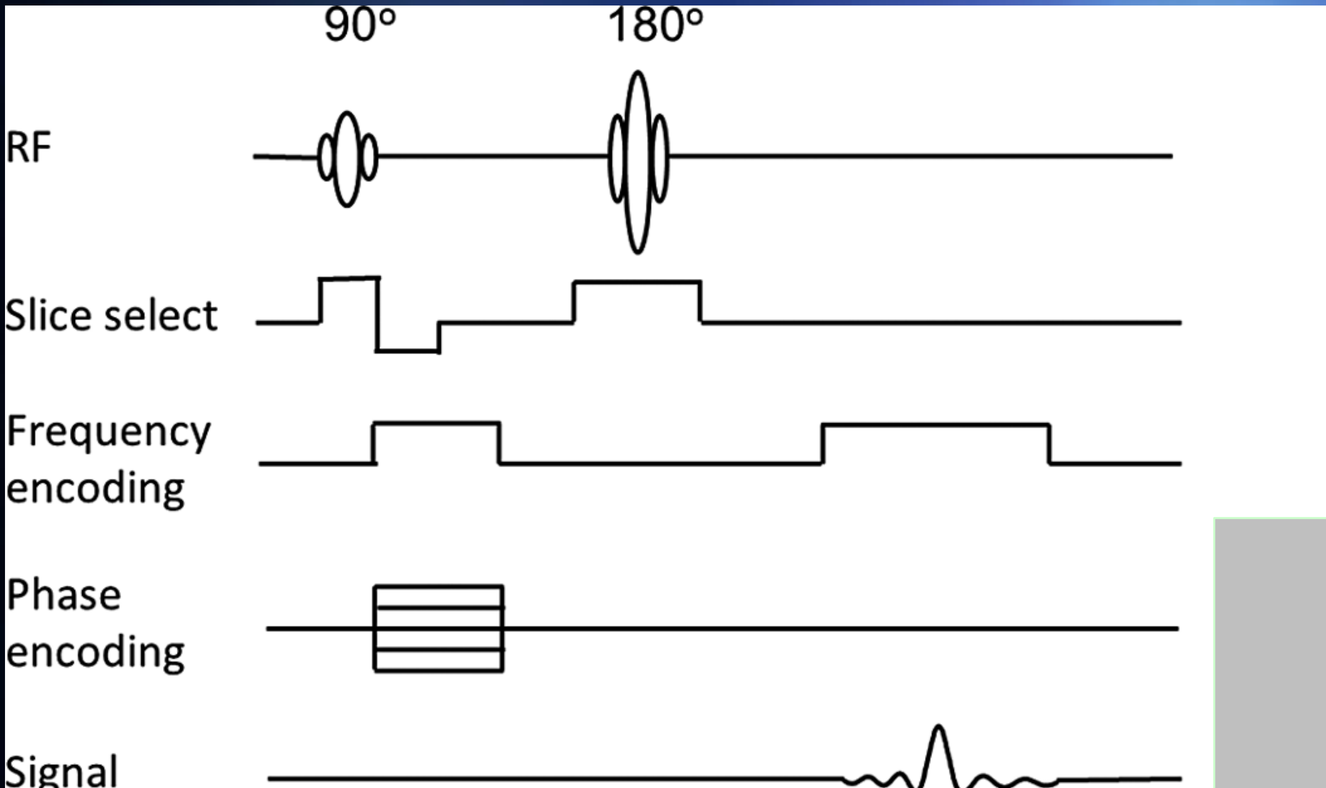


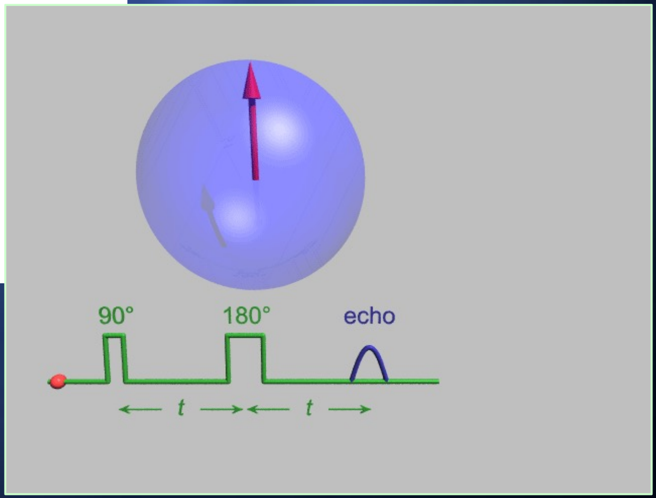
$$\varphi(y,t_y) = \gamma G_y y t_y$$



"MRI From Picture to Proton" D.W. McRobbie et al.
Ch 8 Spaced Out: Spatial Encoding

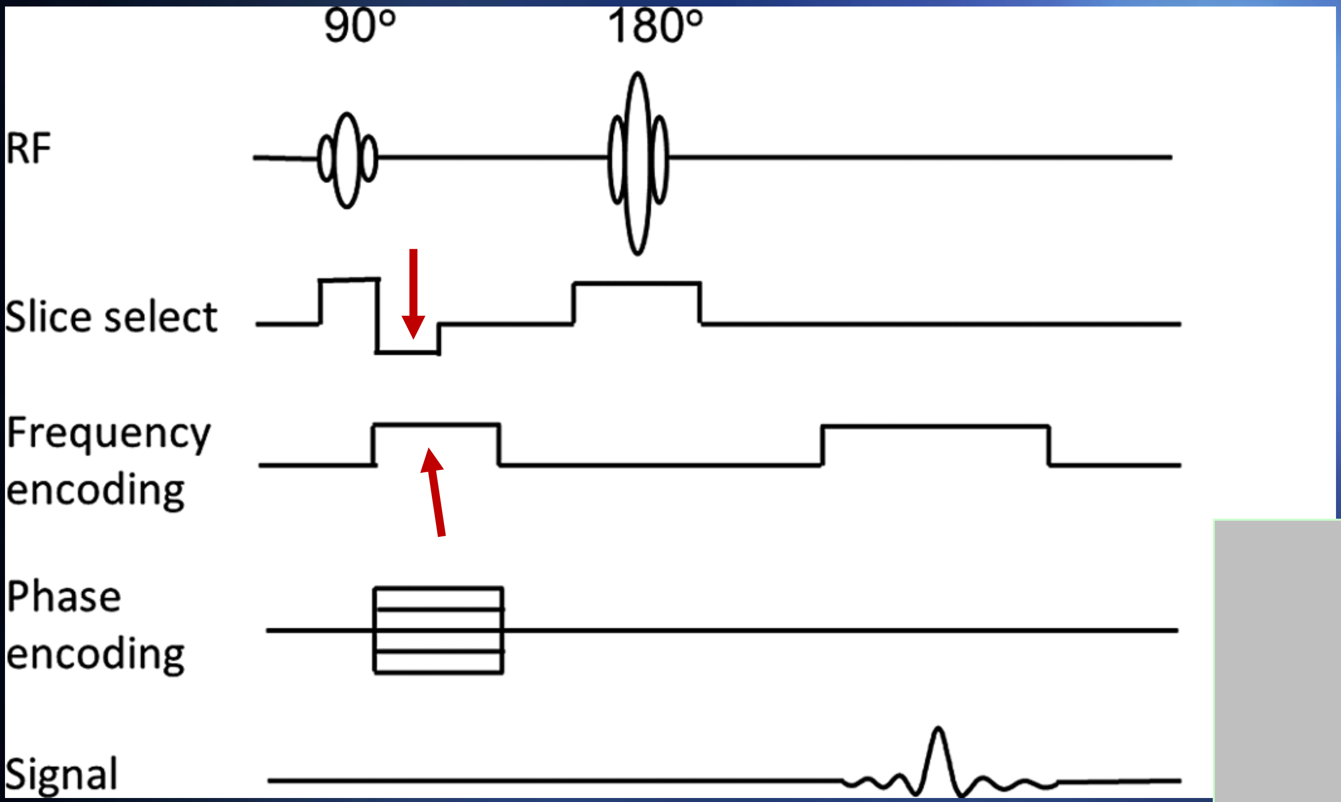
The Pulse Sequence Concept



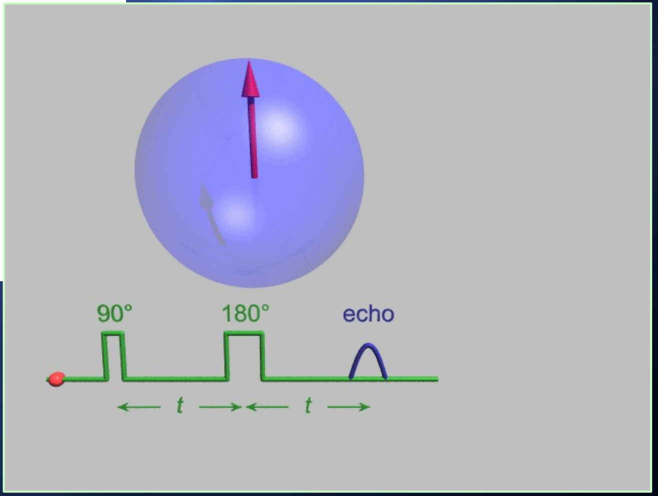


2-D spin-echo pulse sequence

The Pulse Sequence Concept

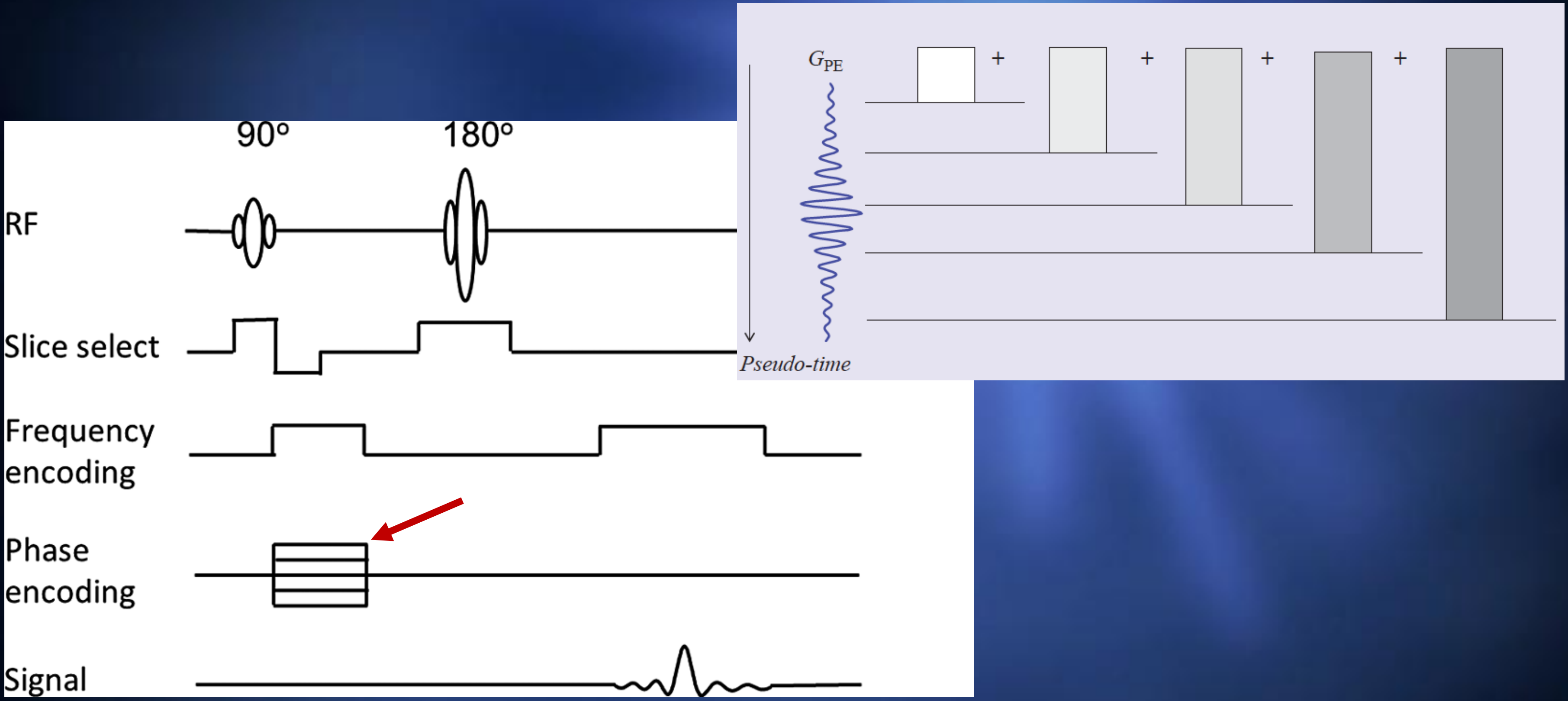


Rephasing gradients



2-D spin-echo pulse sequence

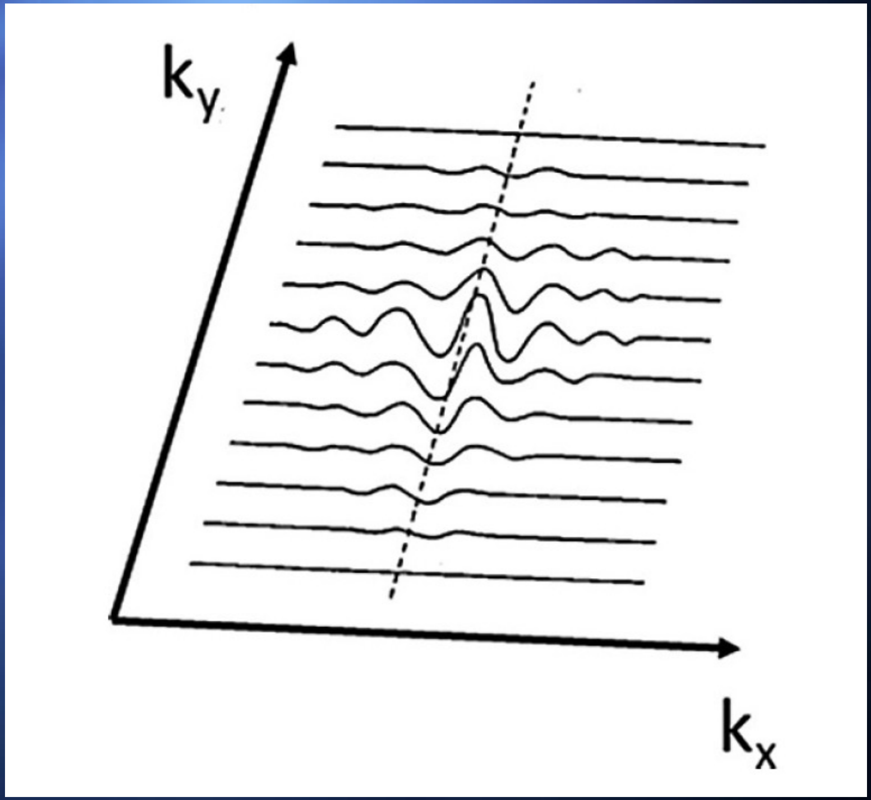
The Pulse Sequence Concept



- ✓ whole sequence is repeated multiple times
- ✓ each time phase encoding gradient has a different amplitude

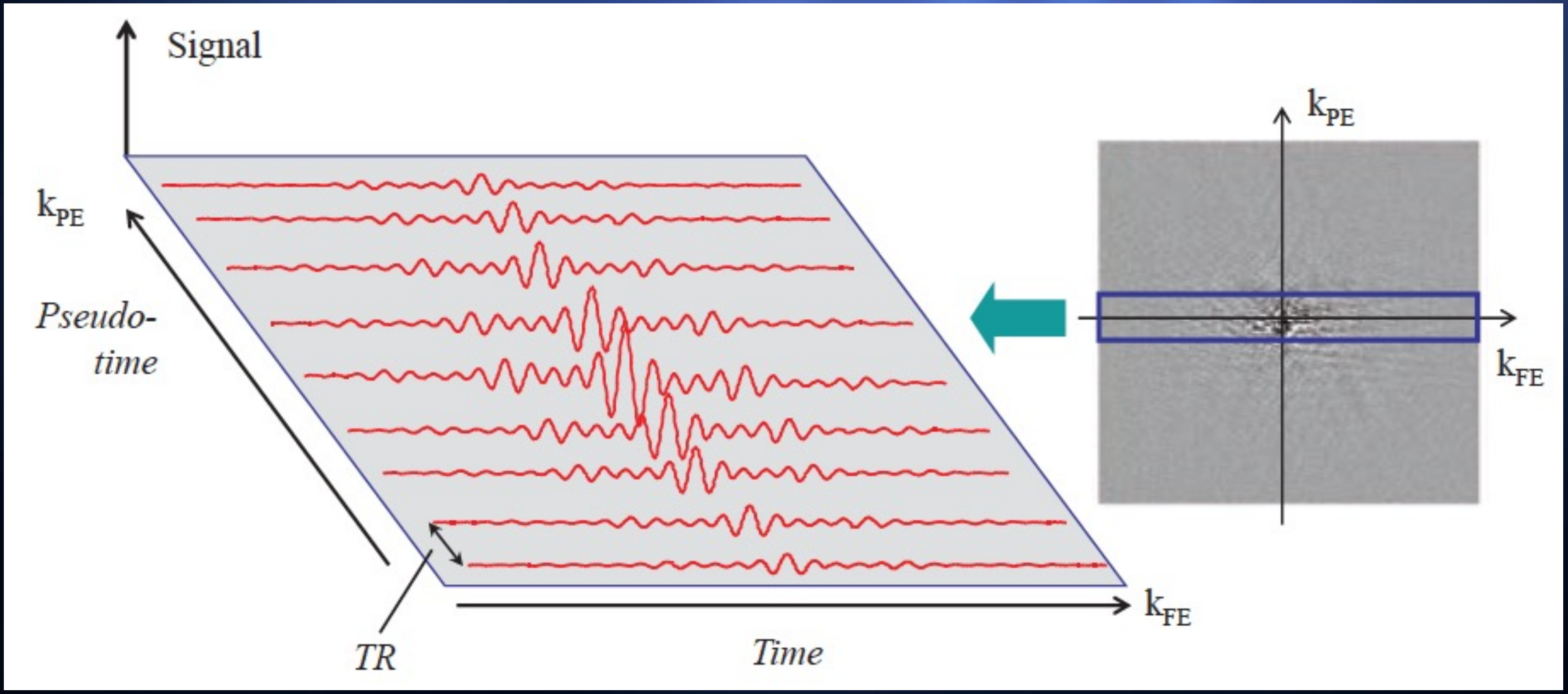
The k-Space Approach to Spatial Localisation

- ✓ a 'stack' of echo signals, each collected following a separate repetition of the pulse sequence
- ✓ The central echo corresponds to the phase encoding gradient amplitude equal to zero
  - no variation in phase as a function of position along the y-axis



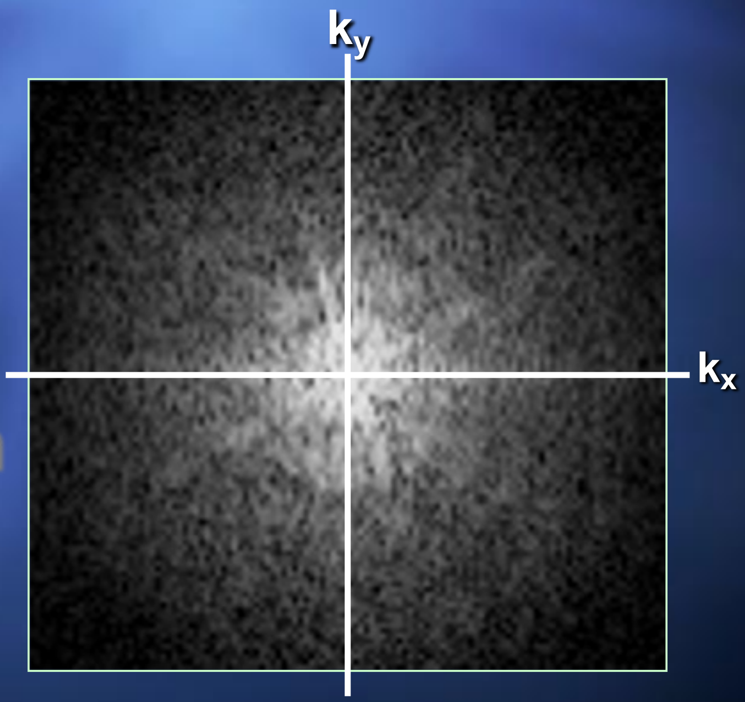
The k-Space Approach to Spatial Localisation

- ✓ **Echoes either side of the central echo are increasingly dephased**
  - **Due to phase encoding gradients of increasing amplitude**

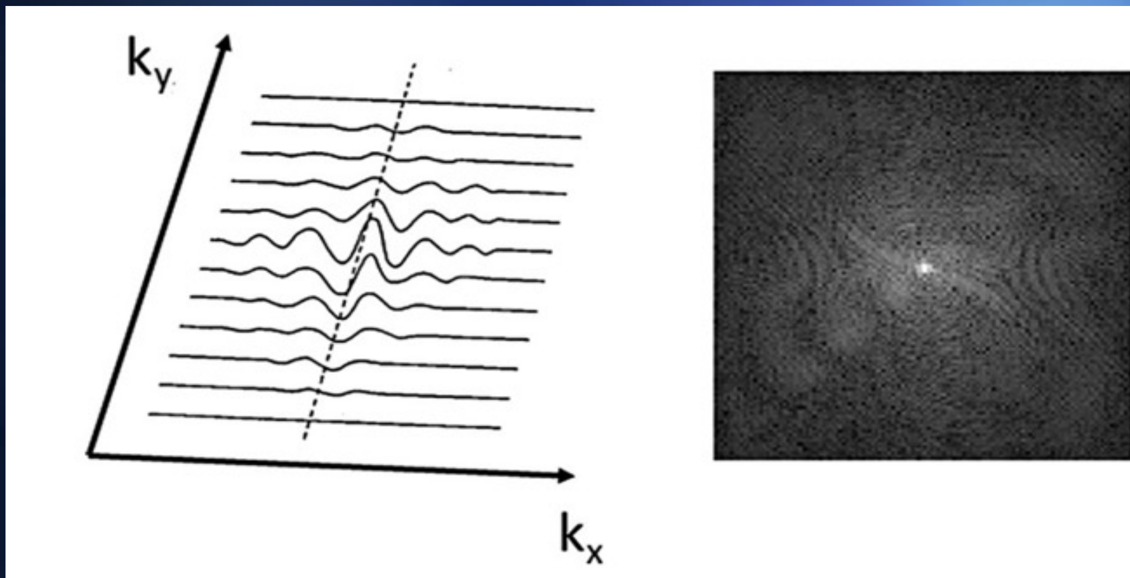


k-Space and Image Properties

- distribution of signal across k-space is sharply peaked at the centre
  - this is the point at which magnetisation throughout the imaged slice is in phase



The k-Space Approach to Spatial Localisation

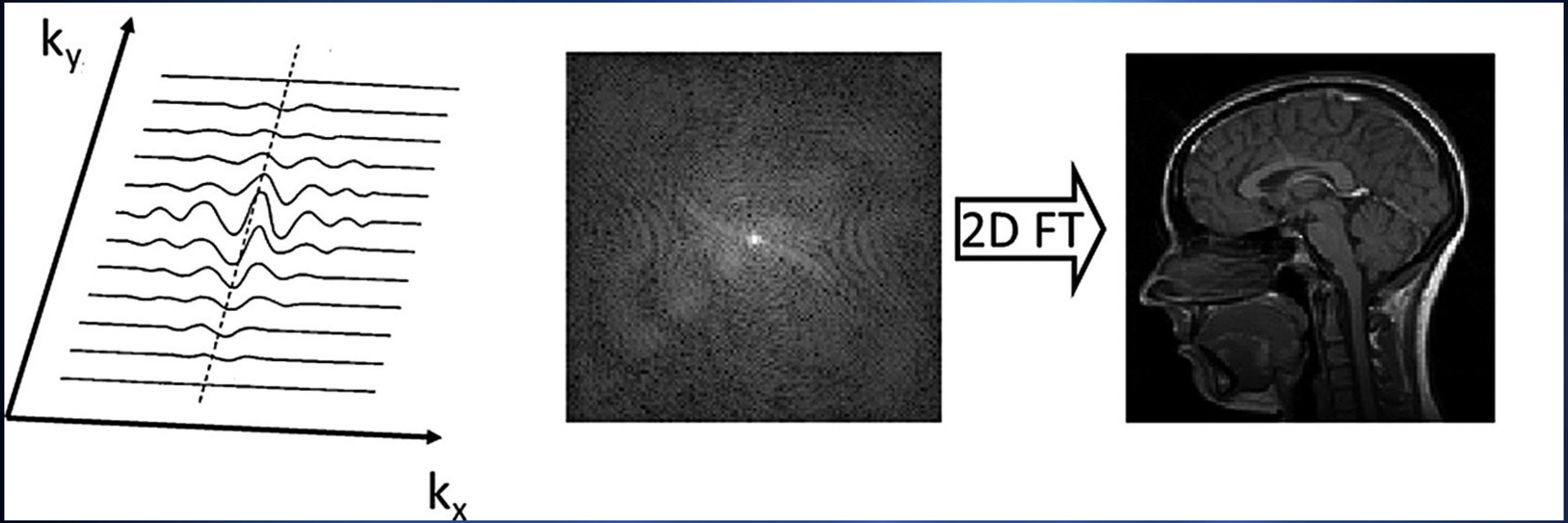


$$k_x = \gamma G_x t_x$$

$$k_y = \gamma G_y t_y$$

- ✓ the raw data array contains information from the entire selected slice
- ✓ The phase information within each data point reflects the ‘area’ of the gradients that M_{xy} has experienced on each axis at the point in time when the data point was collected

The k-Space Approach to Spatial Localisation

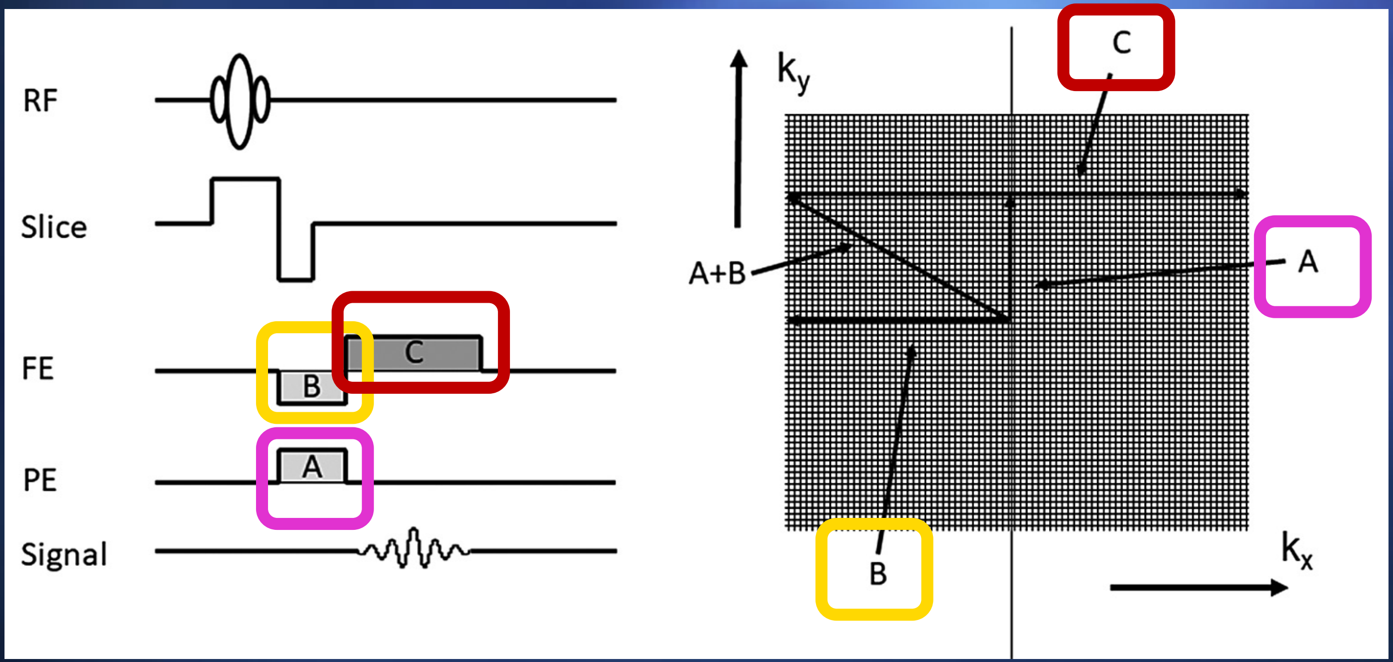


2D inverse Fourier transform extracts frequency content from the frequency-encoded echoes and from the echoes simulated using phase encoding

https://www.imaios.com/en/e-Courses/e-MRI/The-Physics-behind-it-all/Fourier-transform
http://www.imaios.com/en/e-Courses/e-MRI/The-Physics-behind-it-all/K-space

Navigating k-Space Using Gradients

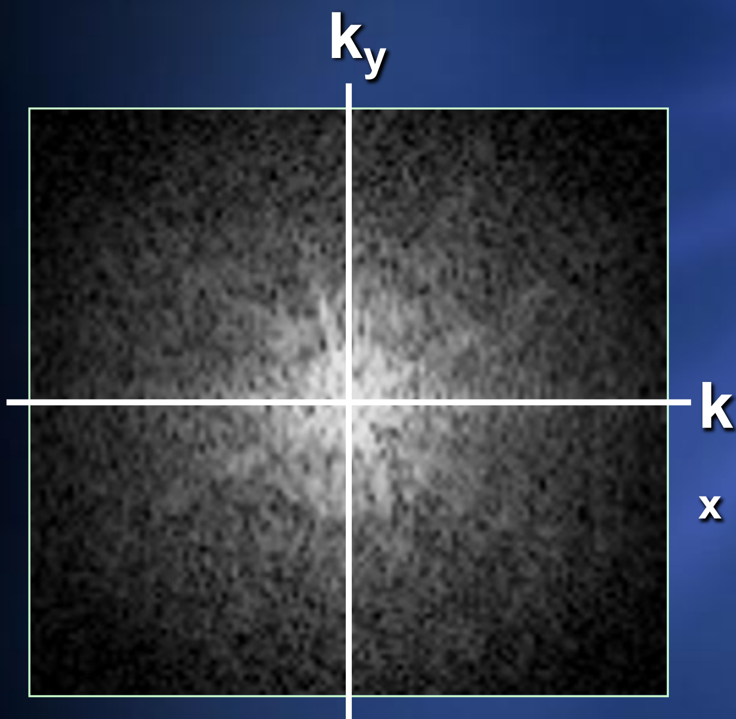
to reconstruct an MR image, we need to fill k-space and perform a 2D inverse Fourier transform



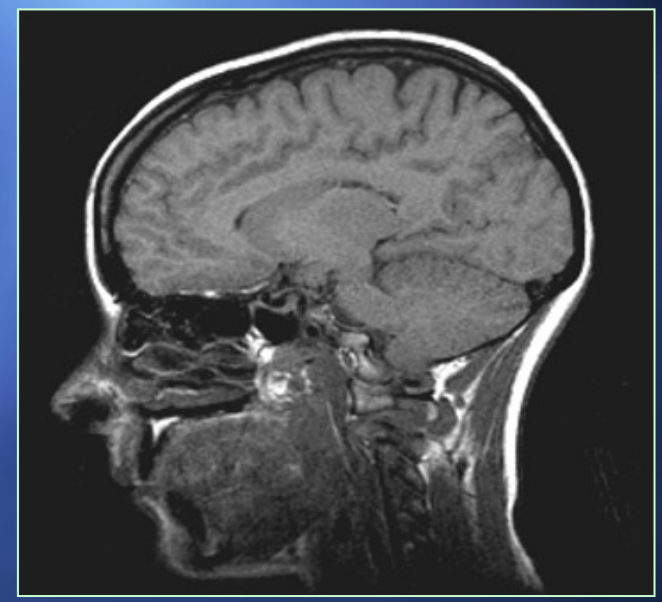
- ✓ **gradients in spatial localisation allow us to move around in k-space**
  - **filling it with data points as we go**

http://www.imaios.com/en/e-Courses/e-MRI/The-Physics-behind-it-all/K-space

K space



FT



Frequency-space

K space

the acquired data

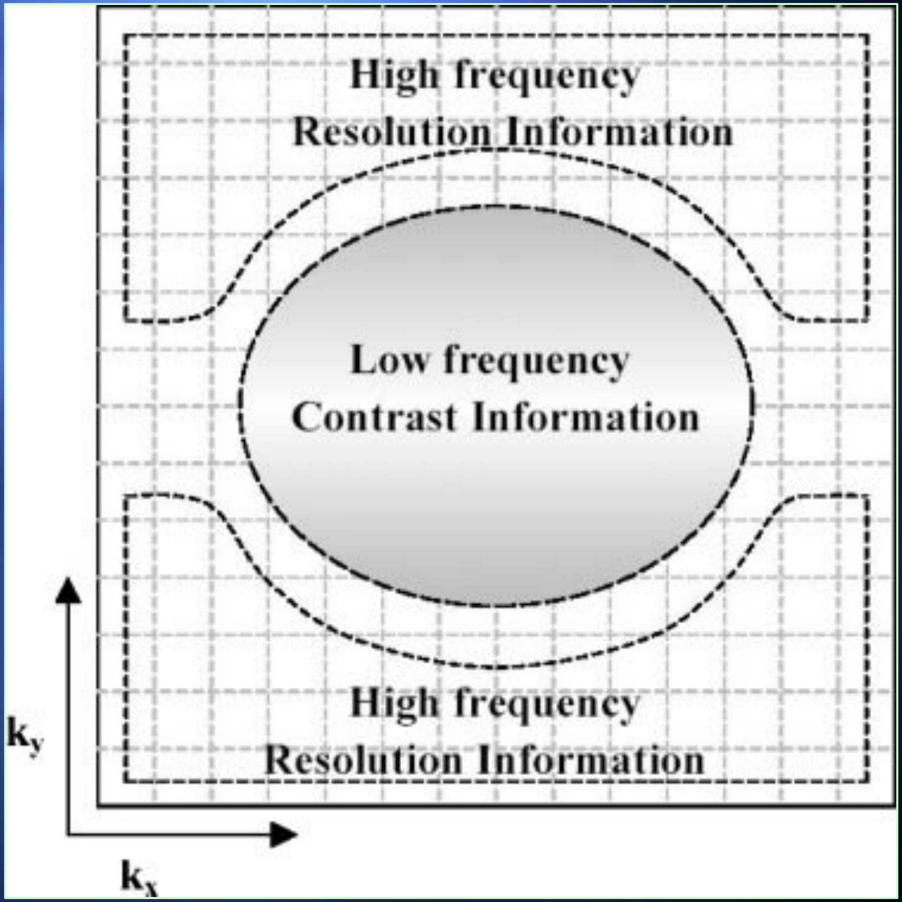
Image space

Filling the k-space

- linear phase encoding
  - starting at one extreme of the k_y-axis and work through to the other end by incrementing the amplitude of the phase encoding gradient on each repetition
- centric phase encoding
  - acquiring the central line of k-space first and working out towards the edges
  - alternating between positive and negative k_y values
- 'exotic' trajectories in the k-space
  - by switching the gradients appropriately
  - for example radially or in a spiral

k-Space and Image Properties

- ✓ low-frequency information is located at the center of k-space
  - about image contrast
- ✓ high-frequency information is at the edges of k-space
  - about spatial resolution and fine structure

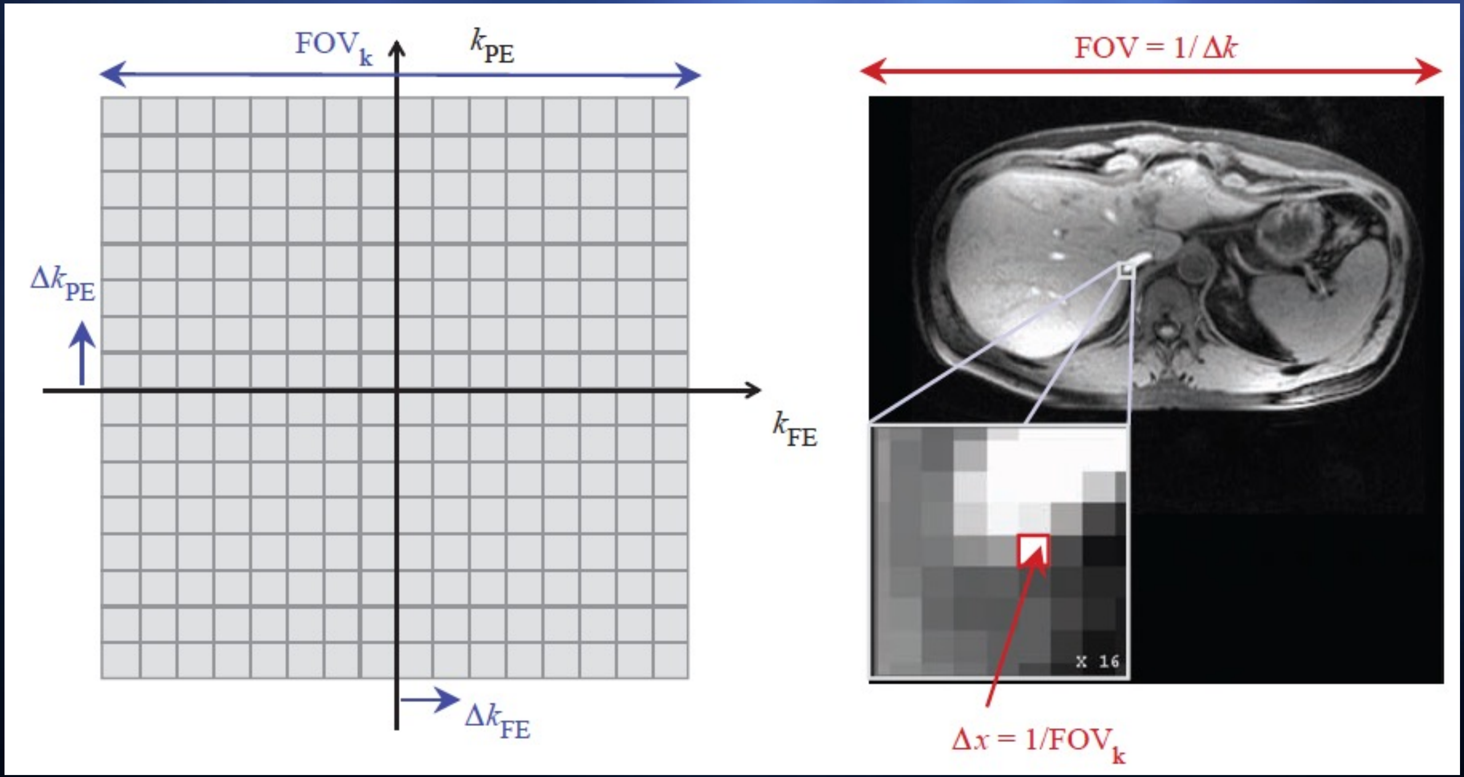


Gallagher AJR 2008; 190:1396–1405

http://www.imaios.com/en/e-Courses/e-MRI/The-Physics-behind-it-all/Spatial-frequency-image-contrast-and-resolution

k-Space and Image Properties

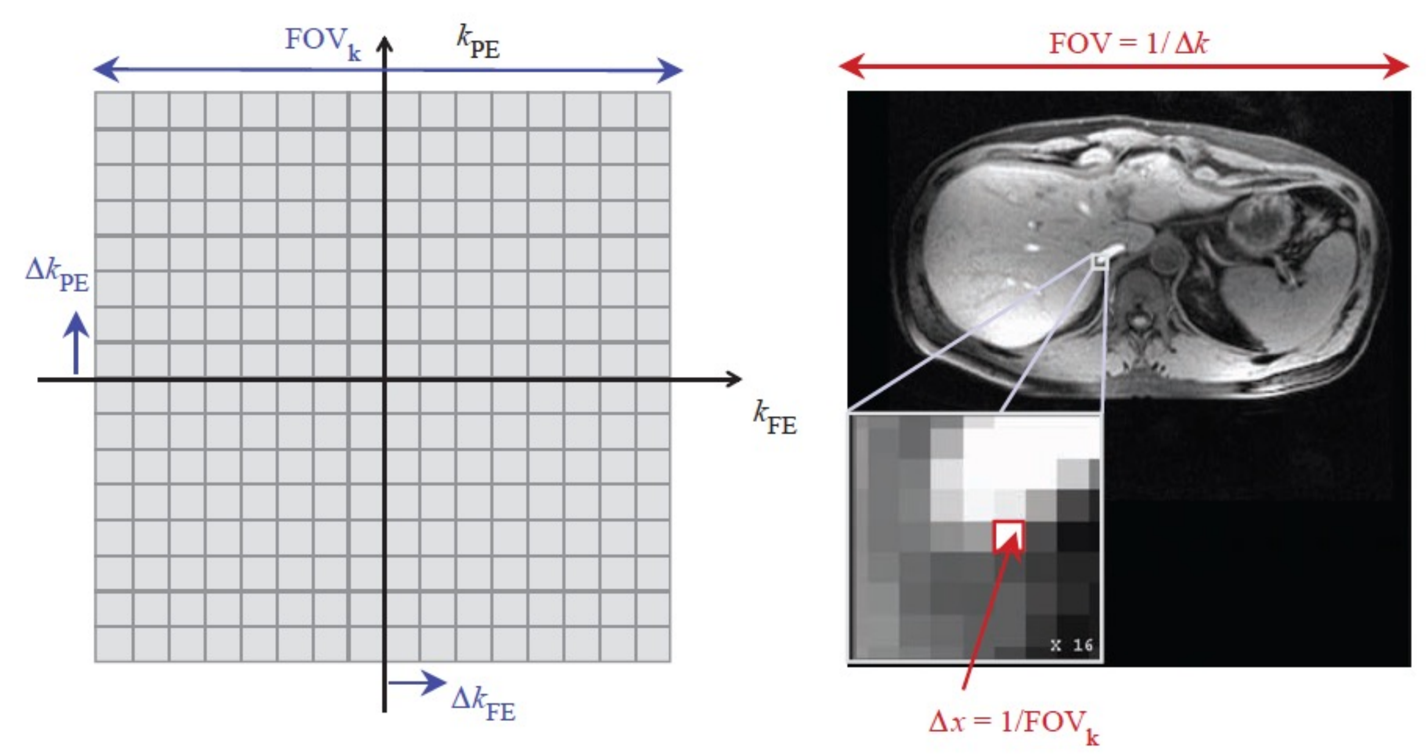
- acquiring more of the echo or more lines in the phase encoding direction brings in higher spatial frequency data and increases spatial resolution
  - More lines longer acquisition time !



k-Space and Image Properties

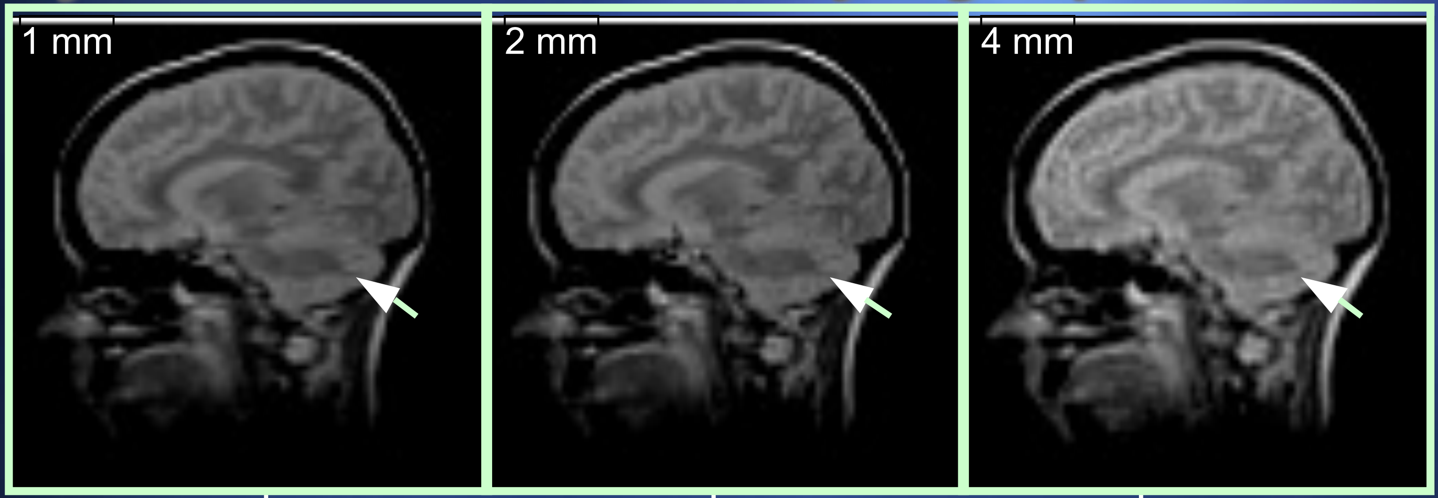
✓ **Increasing the density of data points in k-space increases the FOV**

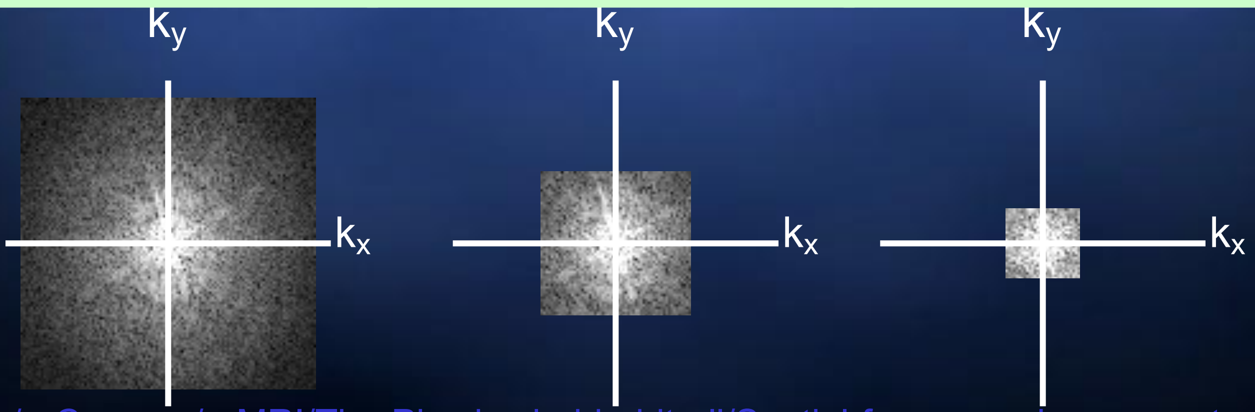
- increasing the rate at which the echo is digitally sampled
- adding phase encoding steps so as to decrease the interval between lines in the phase encoding direction
  - ❖ Acquisition time !!



k-Space and Image Properties

- ✓ **Sampling step defines FOV**
- ✓ **Sampling time defines pixel size**
  - ***Very short echo time and very high spatial resolution ?!***



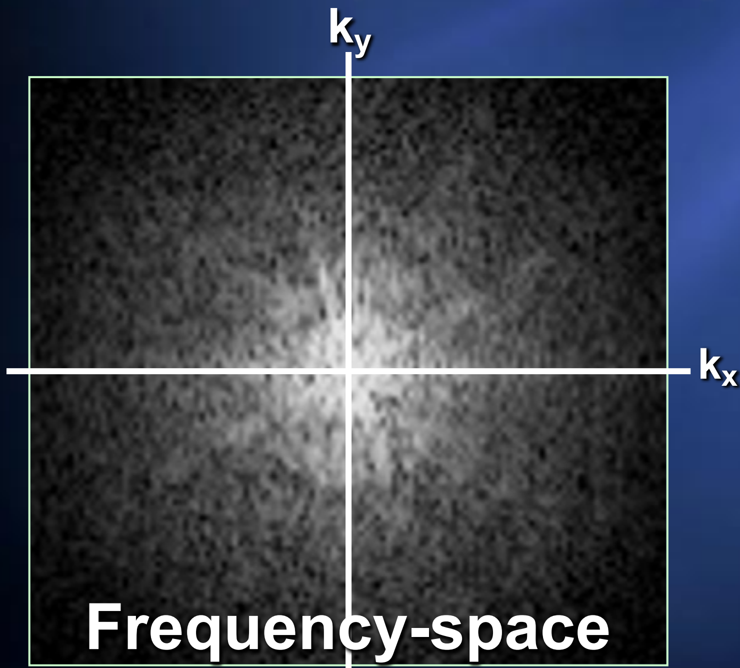


www.imaios.com/en/e-Courses/e-MRI/The-Physics-behind-it-all/Spatial-frequency-image-contrast-and-resolution

k-Space and Image Properties

✓ $S(k_x,k_y) = \iint dx\, dy\, M_t(x,y)\, e^{-ixk_x} e^{-iyk_y}$

- $k_x = \gamma G_x t$; $k_y = \gamma G_y T$
- $S(k_x,k_y)$ is in the complex plane
  - Magnitude and direction or real and imaginary part
  - Some symmetric images have real FT



Frequency-space
(k-space)

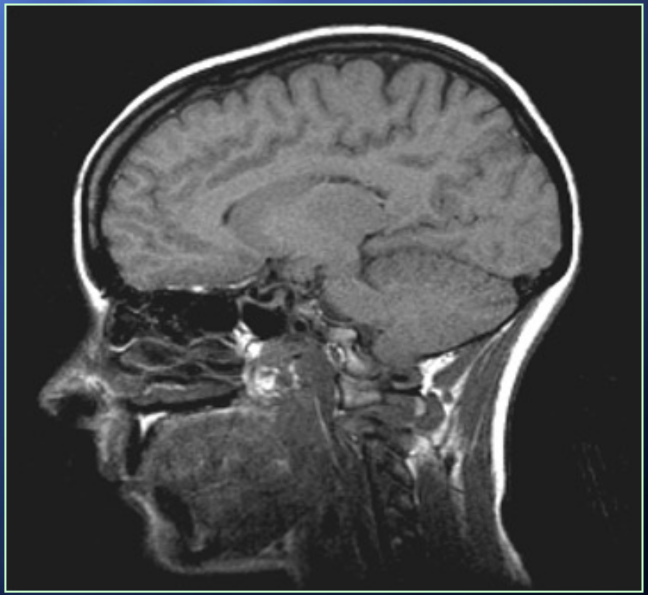


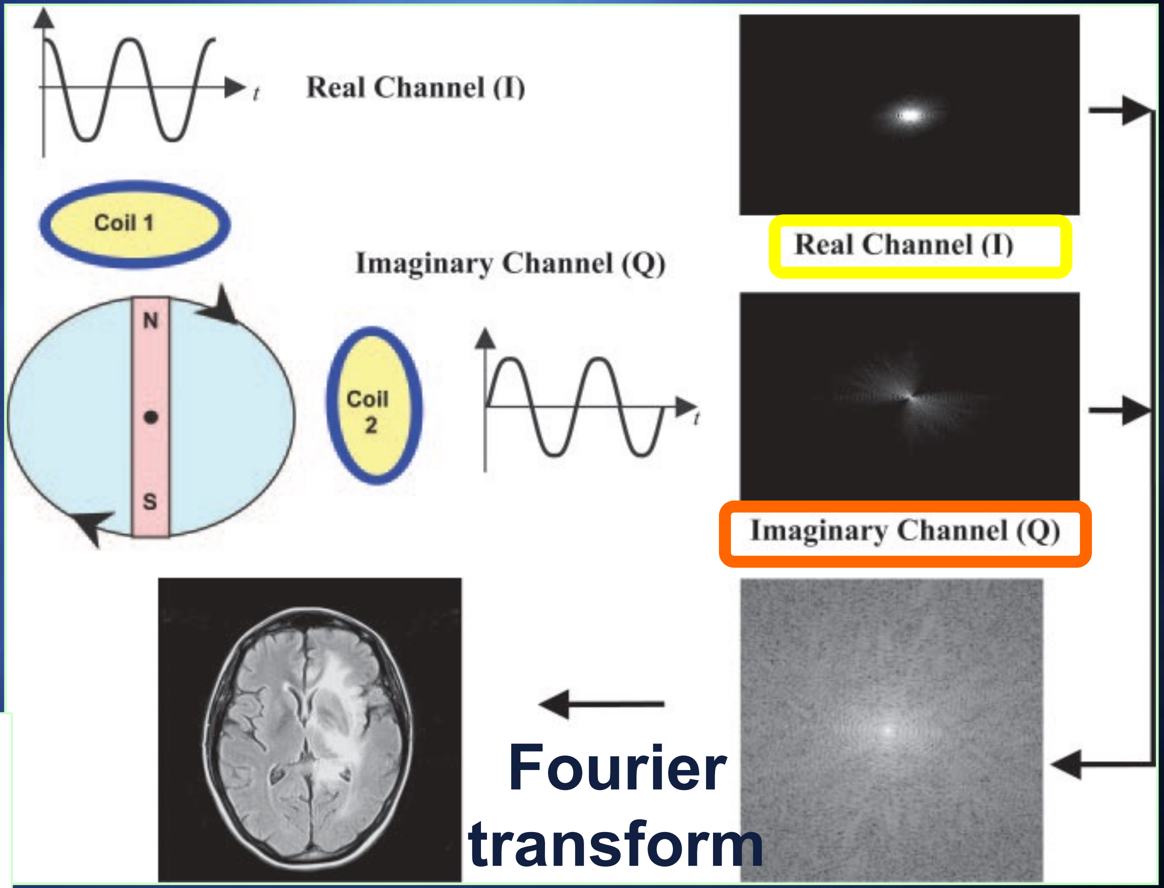
Image space

www.imaios.com/en/e-Courses/e-MRI/The-Physics-behind-it-all/K-space-linear-filling

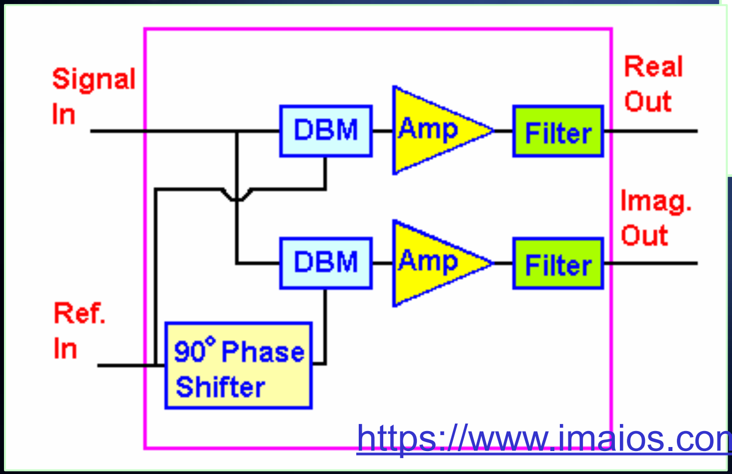
Image acquisition

$S(t) = Ne^{-i\omega_0 t}e^{-t/T2^*}$

- ✓ Quadrature coil detection shows the collection and combination of real and imaginary MR signal data to produce a complex map of k-space
  - Real and imaginary channel differs for 90º phase



Zhuo & Gullapalli, RadioGraphics 2006; 26:275–297



https://www.imaios.com/en/e-Courses/e-MRI/The-Physics-behind-it-all/2D-Fourier-transform

exercises

- A sample contains water at two locations, x = 0 cm and x = 2.0 cm. A one-dimensional magnetic field gradient of 1 G/cm is applied along the x-axis during the acquisition of an FID. What frequencies (relative to the isocenter frequency) are contained in the Fourier transformed spectrum?
- An NMR spectrum is recorded from a sample containing two water locations. The frequency encoding gradient is 1 G/cm along the y-axis. The spectrum contains frequencies of +1000 Hz and -500 Hz relative to the isocenter frequency. What are the locations of the water?