

Cyber-Physical Systems

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Stochastic (Hybrid) Systems

Probabilistic Models

- Models for components that we studied so far were either deterministic or nondeterministic.
- The goal of models is to represent computation or time-evolution of a physical phenomenon.
- These models *do not* do a great job of capturing uncertainty.
- We can usually model uncertainty using probabilities, so probabilistic models allow us to account for likelihood of environment behaviors
- Machine learning/AI algorithms also require probabilistic modelling!

Stochastic Difference Equation Models

- ▶ We assume that the plant (whose state we are trying to estimate) is a stochastic discrete dynamical process with the following dynamics:

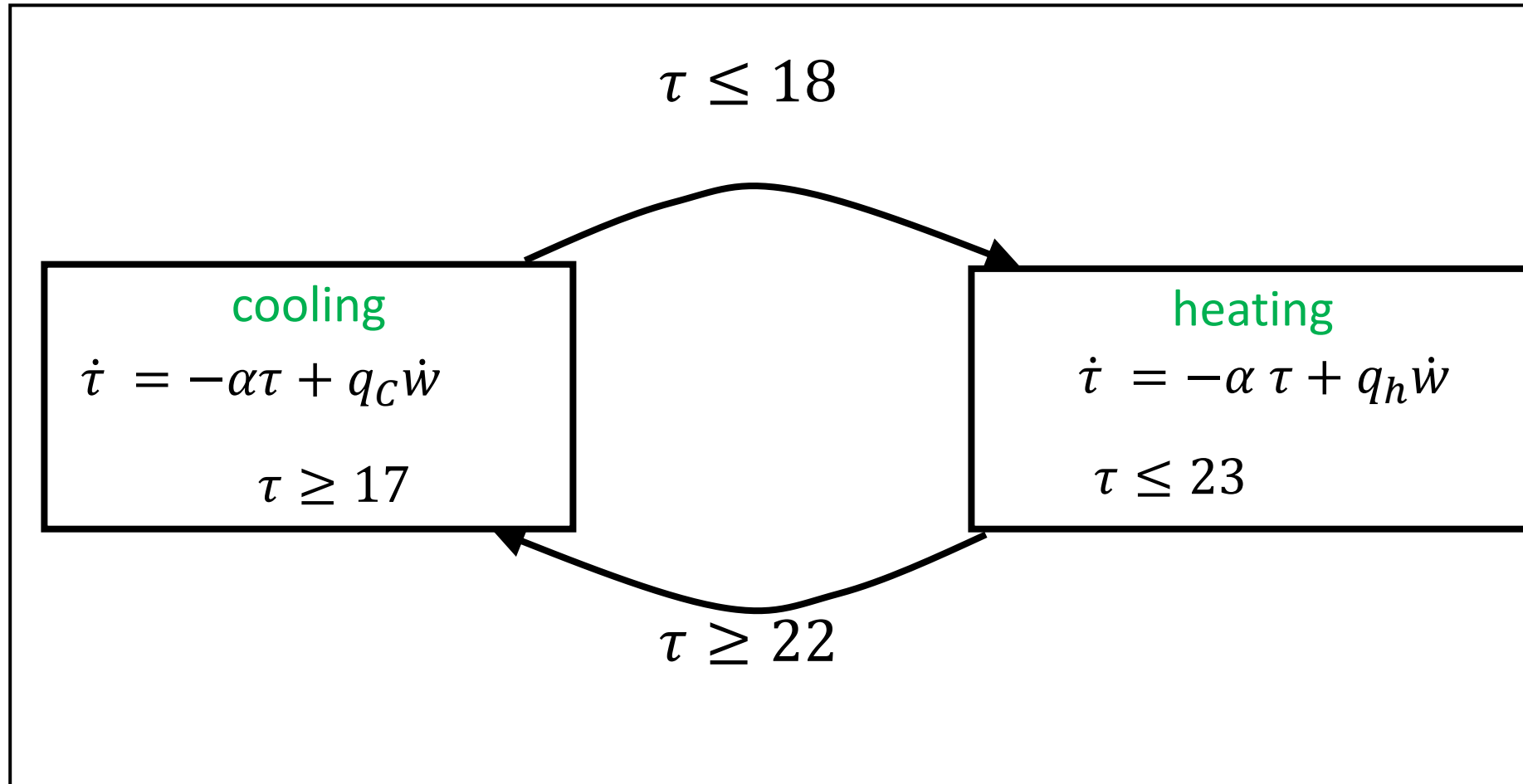
$$\mathbf{x}_k = A\mathbf{x}_{k-1} + B\mathbf{u}_k + \mathbf{w}_k \text{ (Process Model)}$$

$$\mathbf{y}_k = H\mathbf{x}_k + \mathbf{v}_k \text{ (Measurement Model)}$$

$\mathbf{x}_k, \mathbf{x}_{k-1}$	State at time $k, k - 1$
\mathbf{u}_k	Input at time k
\mathbf{w}_k	Random vector representing noise in the plant, $\mathbf{w} \sim N(\mathbf{0}, Q_k)$
\mathbf{v}_k	Random vector representing sensor noise, $\mathbf{v} \sim N(\mathbf{0}, R_k)$
\mathbf{y}_k	Output at time k

n	Number of states
m	Number of inputs
p	Number of outputs
A	$n \times n$ matrix
B	$n \times m$ matrix
H	$p \times n$ matrix

Example



The Family of Markov Models

<i>Markov Models</i>		<i>Do we have control over the state transitions?</i>	
		<i>No</i>	<i>Yes</i>
<i>Are the states completely observable?</i>	<i>Yes</i>	MC <i>Markov Chain</i>	MDP <i>Markov Decision Process</i>
	<i>No</i>	HMM <i>Hidden Markov Model</i>	POMDP <i>Partially Observable Markov Decision Process</i>

The Memoryless Property

$$p(s_{t+1}|s_{1:t}) = p(s_{t+1}|s_t).$$

The knowledge of the state s_t captures the complete information capturing all the relevant information about the present and the past of the system necessary for predicting its future evolution