31 ottobre

$$(e^{-x})' = -e^{-x}$$

$$sh(x) = \frac{e^x - e^{-x}}{2}$$

$$ch(x) = \frac{e^x + e^{-x}}{2}$$

$$\begin{cases} sh'(x) = ch(x) \\ ch'(x) = sh(x) \end{cases}$$

$$sh'(x) = \left(\frac{e^x - e^{-x}}{2}\right)' = \left(\frac{1}{2}(e^x - e^{-x})\right)' =$$

$$= \frac{1}{2}(e^x - e^{-x})' = \frac{1}{2}\left((e^x)' - (e^{-x})'\right)$$

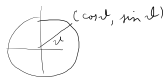
$$= \frac{1}{2}(e^x + e^{-x}) = ch(x)$$

$$ch'(x) = \left(\frac{e^x + e^{-x}}{2}\right)' = \frac{1}{2}(e^x + e^{-x})' = \frac{1}{2}\left((e^x)' + (e^{-x})'\right)$$

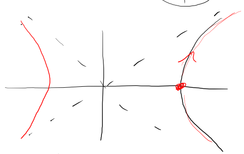
$$sh(x) = \frac{1}{2}(e^x - e^{-x})$$

$$1 = ch^2(x) - sh^2(x)$$

$$(coh(t), sh(t))$$



$$x^2 - y^2 = 1$$



$$ch^2(x) - sh^2(x) = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$$

$$= \frac{\cancel{e^{2x} + e^{-2x}} + 2}{4} - \frac{\cancel{e^{2x} + e^{-2x}} - 2}{4} = \frac{1}{2} - (-)\frac{1}{2} =$$

$$= 1$$

$$0 = \left(ch^2(x) - sh^2(x)\right)' = \left(ch^2(x)\right)' - \left(sh^2(x)\right)'$$

$$= 2\, ch(x)\, ch'(x) - 2\, sh(x)\, sh'(x)$$

$$= 2\, ch(x)\, sh(x) - 2\, sh(x)\, ch(x) = 0$$

$$(th(x))' = \left(\frac{sh(x)}{ch(x)}\right)' = \frac{sh'(x)\, ch(x) - sh(x)\, ch'(x)}{ch^2(x)}$$

$$= \frac{ch^2(x) - sh^2(x)}{ch^2(x)} = 1 - th^2(x)$$

$$= \frac{1}{ch^2(x)}$$

Teor (derivata funzione inversa)

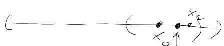
Sia $f: I \to \mathbb{R}$ continua in I e strettamente monotona. Qui I è un intervallo e sia $x_0 \in \mathring{I}$ e supponiamo che esista $f'(x_0) \neq 0$. Consideriamo l'intervallo $J = f(I)$ e funzione inversa $g: J \to I$ ed il punto $y_0 = f(x_0)$. Risulta $y_0 \in \mathring{J}$ e

$$g'(y_0) = \frac{1}{f'(x_0)}$$

Dim (Qui assumo f crescente.)

$y_0 \in \mathring{J}$. Se per assurdo avessi $y_0 = \max J$ dovrei avere che $f(x) \leq f(x_0) = y \quad \forall x \in I$ ma so che se $x_1 > x_0$ in I, allora $f(x_1) > f(x_0)$ perché per ipotesi f è strettamente crescente.

Siccome $x_0 \in \mathring{I}$



$\exists\, \varepsilon_0 > 0$ t.c. $(x_0 - \varepsilon_0, x_0 + \varepsilon_0) \subset I$

Dentro $(x_0 - \varepsilon_0, x_0 + \varepsilon_0)$ ci sono punti x_1 con $x_1 > x_0$

Ad esempio $x_1 = x_0 + \frac{\varepsilon_0}{2}$

$$\lim_{y \to y_0} \frac{g(y) - g(y_0)}{y - y_0} =$$

$x = g(y)$
$y = f(x)$
$y_0 = f(x_0)$
$x_0 = g(y_0)$

$$= \lim_{x \to x_0} \frac{x - x_0}{f(x) - f(x_0)} = \frac{1}{f'(x_0)}$$

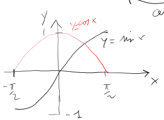
$$\Rightarrow \boxed{g'(y_0) = \frac{1}{f'(x_0)}}$$

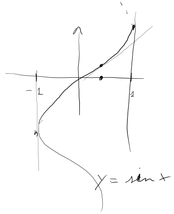
□

$\sin(x): \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$

$\arcsin(x)$

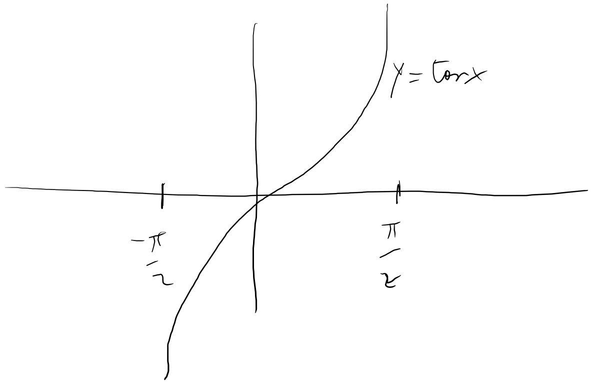
$\sin^2(x) + \cos^2(x) = 1$





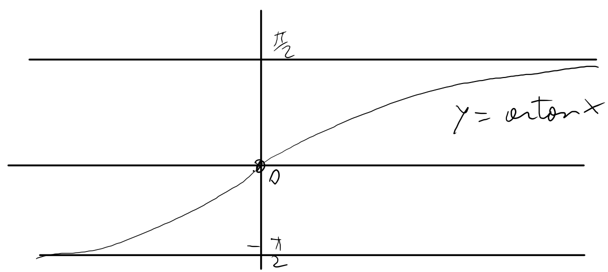
$$(\arcsin y)' = \frac{1}{\sqrt{1-y^2}}$$

$$(\arcsin y)' = \frac{1}{\sin'(x)} = \frac{1}{\cos(x)} = \frac{1}{\sqrt{1-\sin^2(x)}} = \frac{1}{\sqrt{1-y^2}}$$



$$\tan x : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$$

$$\arctan(y) : \mathbb{R} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$



$$(\arctan y)' = \frac{1}{1+y^2} \qquad y = \tan x$$

$$(\arctan y)' = \frac{1}{\tan'(x)} = \frac{1}{1+\tan^2(x)} = \frac{1}{1+y^2}$$

Def Siano f e $g : X (\subseteq \mathbb{R}) \longrightarrow \mathbb{R}$, sia $x_0 \in X'$,

allora diremo che $f(x) = o(g(x))$ se

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = 0$$

Osservazioni 1) Se $\lim_{x \to x_0} f(x) = 0$ allora

$f(x) = o(1)$

2) Se $\lim_{x \to x_0} f(x) = L \in \mathbb{R}$ allora possiamo

scrivere che $f(x) = L + o(1)$

Corollario Sia $f : I \to \mathbb{R}$, $x_0 \in \mathring{I}$, esiste $f'(x_0)$

Allora $f(x) = f(x_0) + f'(x_0)(x - x_0) + o(x - x_0)$

Dim Se $f'(x_0)$ esiste allora

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0)$$

$$\lim_{x \to x_0} \left[\frac{f(x) - f(x_0)}{x - x_0} - f'(x_0) \right] = 0$$

$$= \lim_{x \to x_0} \frac{f(x) - f(x_0) - f'(x_0)(x - x_0)}{x - x_0} = 0$$

$$\Rightarrow \quad f(x) - f(x_0) - f'(x_0)(x - x_0) = o(x - x_0)$$

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + o(x - x_0)$$

$f : I \to \mathbb{R}$, $x_0 \in \mathring{I}$

Corollario Se $\exists\ c_0 \in \mathbb{R}$ t.c.

$f(x) = f(x_0) + c_0 (x - x_0) + o(x - x_0)$

allora $c_0 = f'(x_0)$

Dim
$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{c_0 (x - x_0) + o(x - x_0)}{x - x_0}$$

$$= \lim_{x \to x_0} \left[\frac{c_0 \cancel{(x - x_0)}}{\cancel{x - x_0}} + \frac{o(x - x_0)}{x - x_0} \right]$$

$$= c_0 + \lim_{x \to x_0} \frac{o(x - x_0)}{x - x_0} = c_0 + 0 = c_0$$

Teor (sulla composizione) Sia $f: I \longrightarrow J \xrightarrow{g} \mathbb{R}$

$g: J \longrightarrow \mathbb{R}$

I e J due intervalli.

$x_0 \in I$, $y_0 \in J$ con $y_0 = f(x_0)$.

Supponiamo che esistano $f'(x_0)$ e $g'(y_0)$.

Allora

$$(g \circ f)'(x_0) = g'(y_0)\, f'(x_0) =$$
$$= g'(f(x_0))\, f'(x_0)$$

Dim

$$\boxed{f(x) = f(x_0) + f'(x_0)(x-x_0) + o(x-x_0)}$$

$$g(y) - g(y_0) = g'(y_0)(y-y_0) + o(y-y_0)$$

$$g(f(x)) - g(f(x_0)) \overset{\text{voglio}}{=} c_0 (x-x_0) + o(x-x_0)$$

dove $c_0 = g'(f(x_0))\, f'(x_0)$

$$g(f(x)) - g(f(x_0)) =$$

$$= g\big(\underbrace{f(x_0)}_{y_0} + \underbrace{f'(x_0)(x-x_0) + o(x-x_0)}_{h}\big) - g(\underbrace{f(x_0)}_{y_0})$$

$$= g(y_0 + h) - g(y_0) = g'(y_0)\, h + o(h)$$

$$= g'(y_0)\big(f'(x_0)(x-x_0) + o(x-x_0)\big) + o\big(f'(x_0)(x-x_0) + o(x-x_0)\big)$$

$$= g'(y_0) f'(x_0)(x-x_0) + \underbrace{g'(y_0)\, o(x-x_0)}_{o(x-x_0)} + \underbrace{o\big(f'(x_0)(x-x_0) + o(x-x_0)\big)}_{o(x-x_0)}$$

Qui $d_0\, o(x-x_0) = o(x-x_0)$ perché

$$\lim_{x \to x_0} \frac{d_0\, o(x-x_0)}{x-x_0} = \lim_{x \to x_0} d_0 \frac{o(x-x_0)}{x-x_0} = d_0\, 0$$
$$= 0$$

Se $f'(x_0) = 0$