$$\mathbb{R} = \bigcup_{n \geq 1} \underbrace{(-n, n)}_{\text{HA MISURA DI LEBESGUE } 2n}$$

SE $A \subset B$

$$P(B - A) = P(B) - P(A)$$

$$m(B - A) \cancel{= m(B)} - m(A)$$

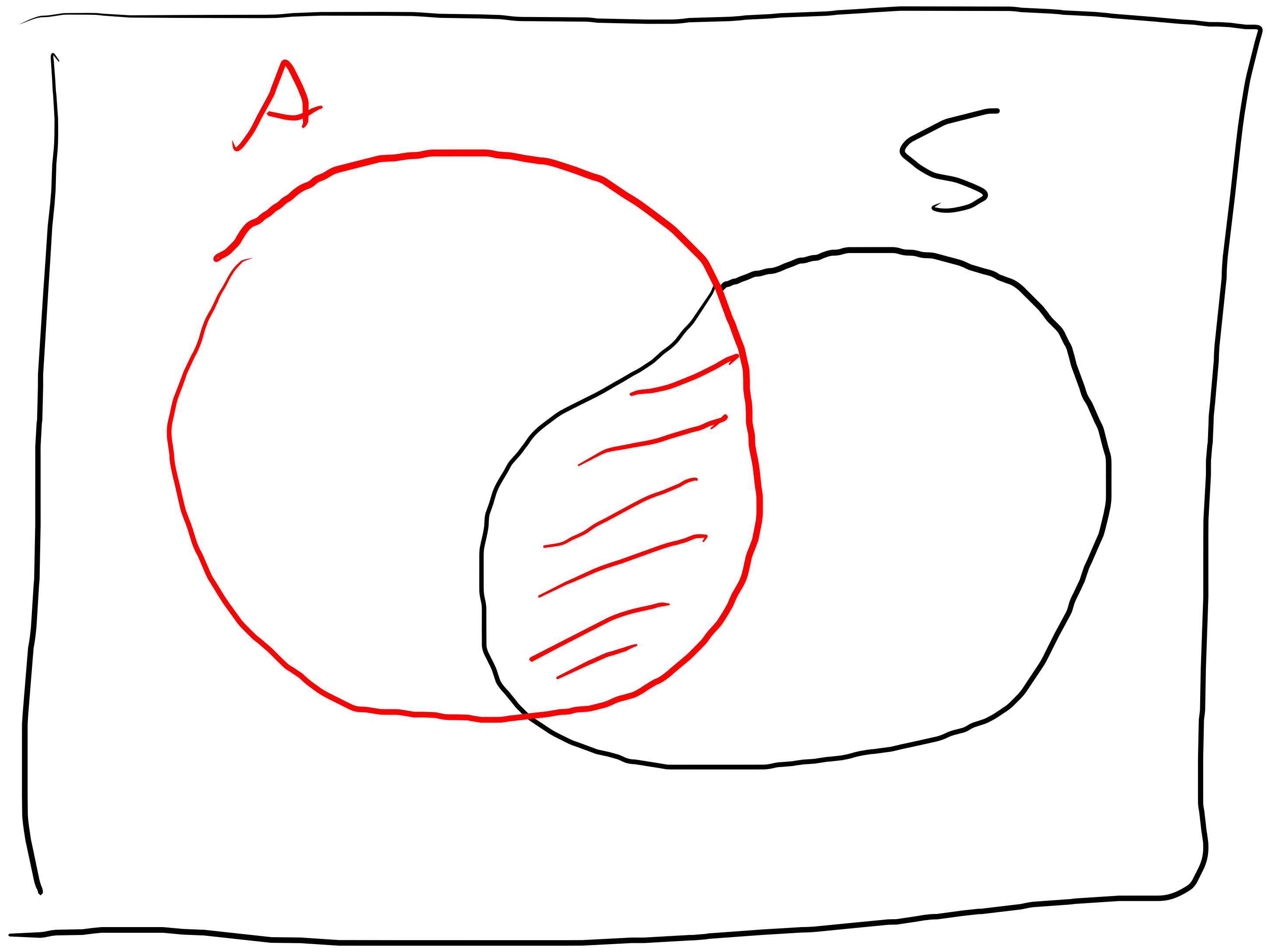
$$\underset{+\infty}{m(A)} + m(B - A) = \underset{+\infty}{m(B)}$$

$$A_n = [n, +\infty) \subset \mathbb{R} = \Omega$$

$$[1, +\infty) \supset [2, +\infty) \supset [3, +\infty) \supset \ldots$$

$$\lim_n A_n = \bigcap_{n \geq 1} A_n = \emptyset$$

$$\underbrace{m(\lim A_n)}_{=0} \neq \underbrace{\lim \underbrace{m(A_n)}_{+\infty}}_{+\infty}$$



$\Omega \qquad \mathcal{F} = 2^{\Omega}$

$S \subset \Omega$ FISSATO

$$\gamma_S(A) = \begin{cases} \# S \cap A & \text{SE } S \cap A \text{ FINITO} \\ +\infty & \text{SE } S \cap A \text{ INFINITO} \end{cases}$$

$\gamma_S(A_1 \cup A_2) = \gamma_S(A_1) + \gamma_S(A_2)$

SE $A_1 \cap A_2 = \phi$

$\Omega = \mathbb{N}$

$S = \{$NUMERI PRIMI$\}$

$\gamma_S(A) =$ QUANTI N. PRIMI CI SONO IN A

$\gamma_S(\mathbb{N}) = +\infty$

$\mathbb{N} = \bigcup_{n \geq 1} \{0, 1, \dots, n\}$ $\quad \gamma_S(\{0,1,2, \dots n\}) < +\infty$

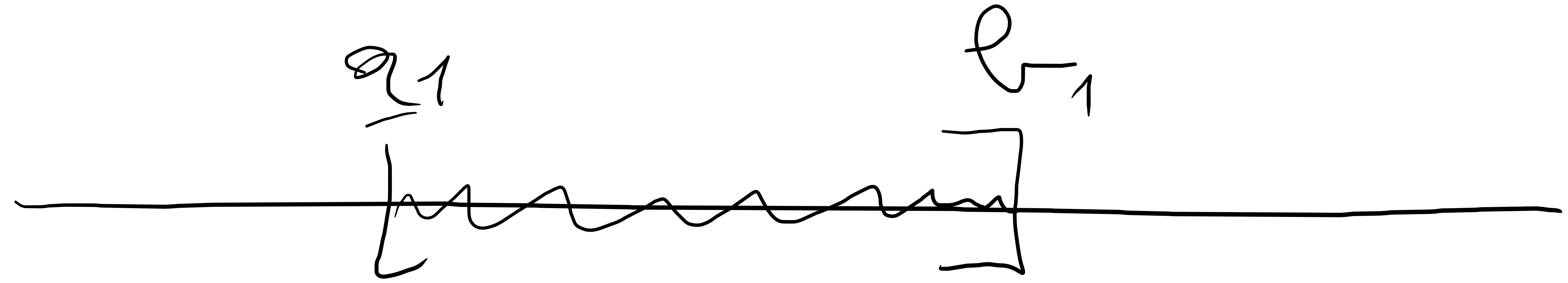
Ω QUALUNQUE

$S = \{\omega_0\}$

$$\delta_{\omega_0}(A) = \begin{cases} 0 & \text{SE } \omega_0 \notin A \\ 1 & \text{SE } \omega_0 \in A \end{cases}$$

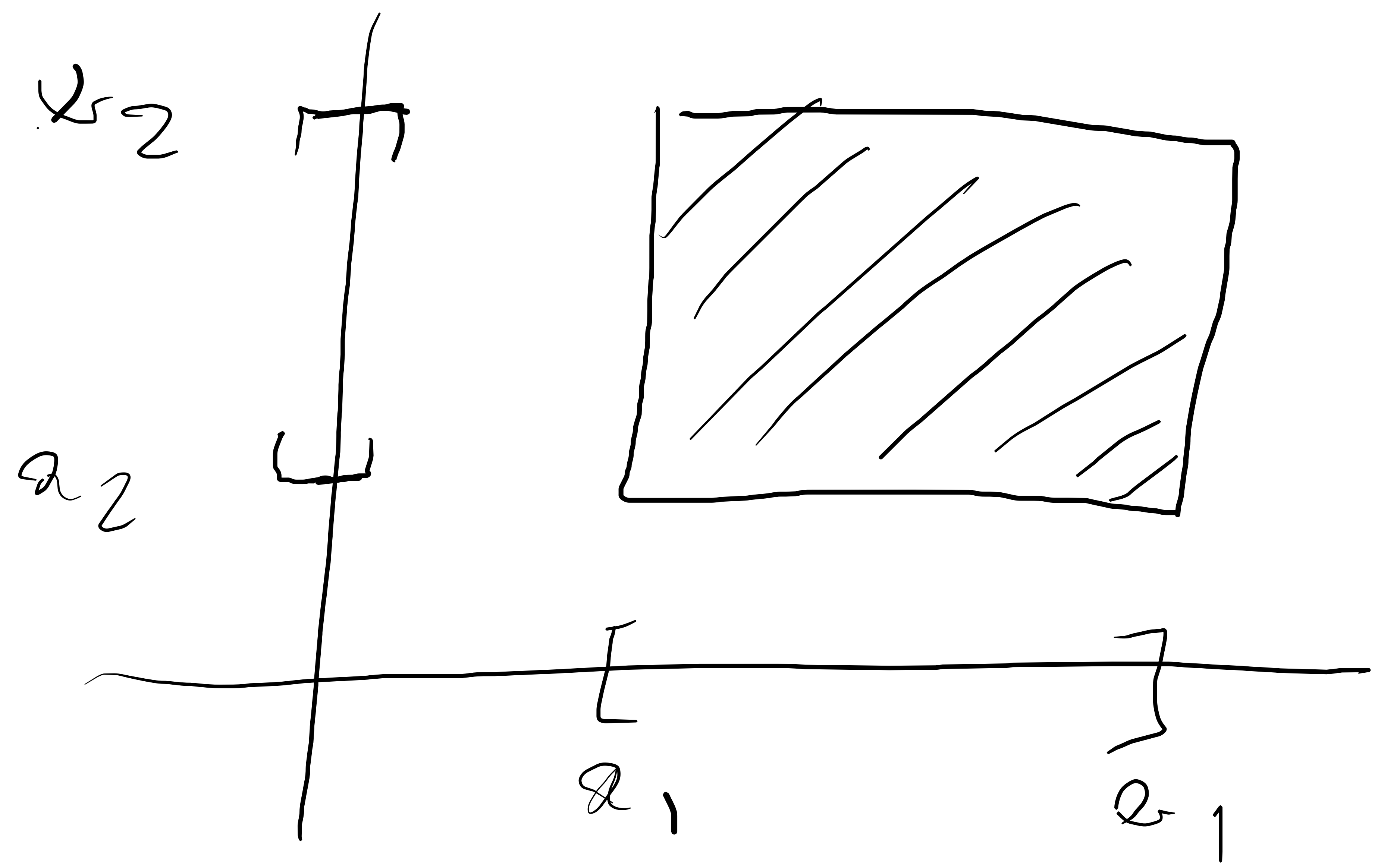
È UNA PROB.

IPER RETTANGOLO (CHIUSO LIMITATO)



$\underline{n=1}$

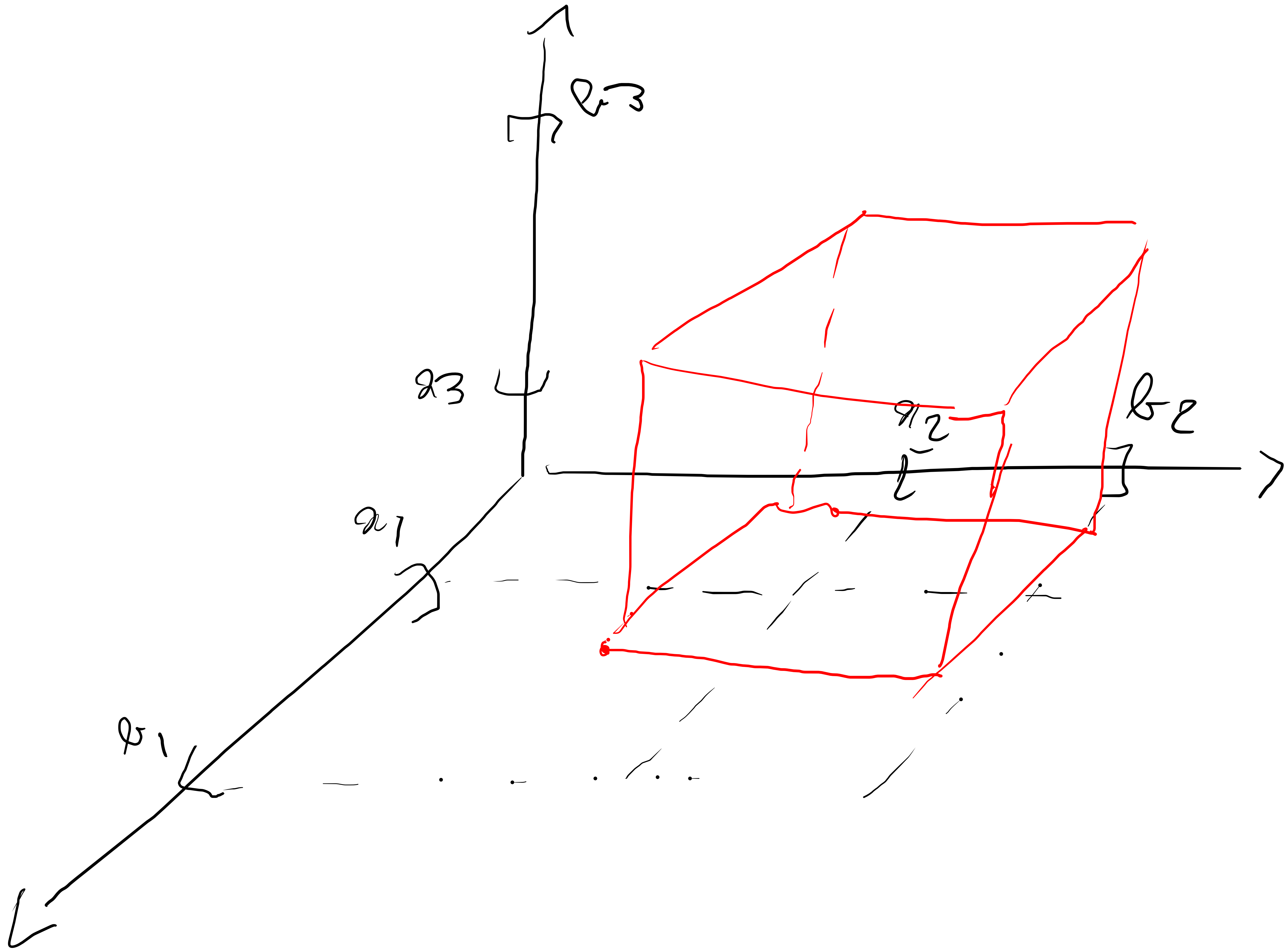
$b_1 - a_1$



$\underline{n=2}$

$(b_1 - a_1) \cdot$
$\cdot (b_2 - a_2)$

$n = 3$

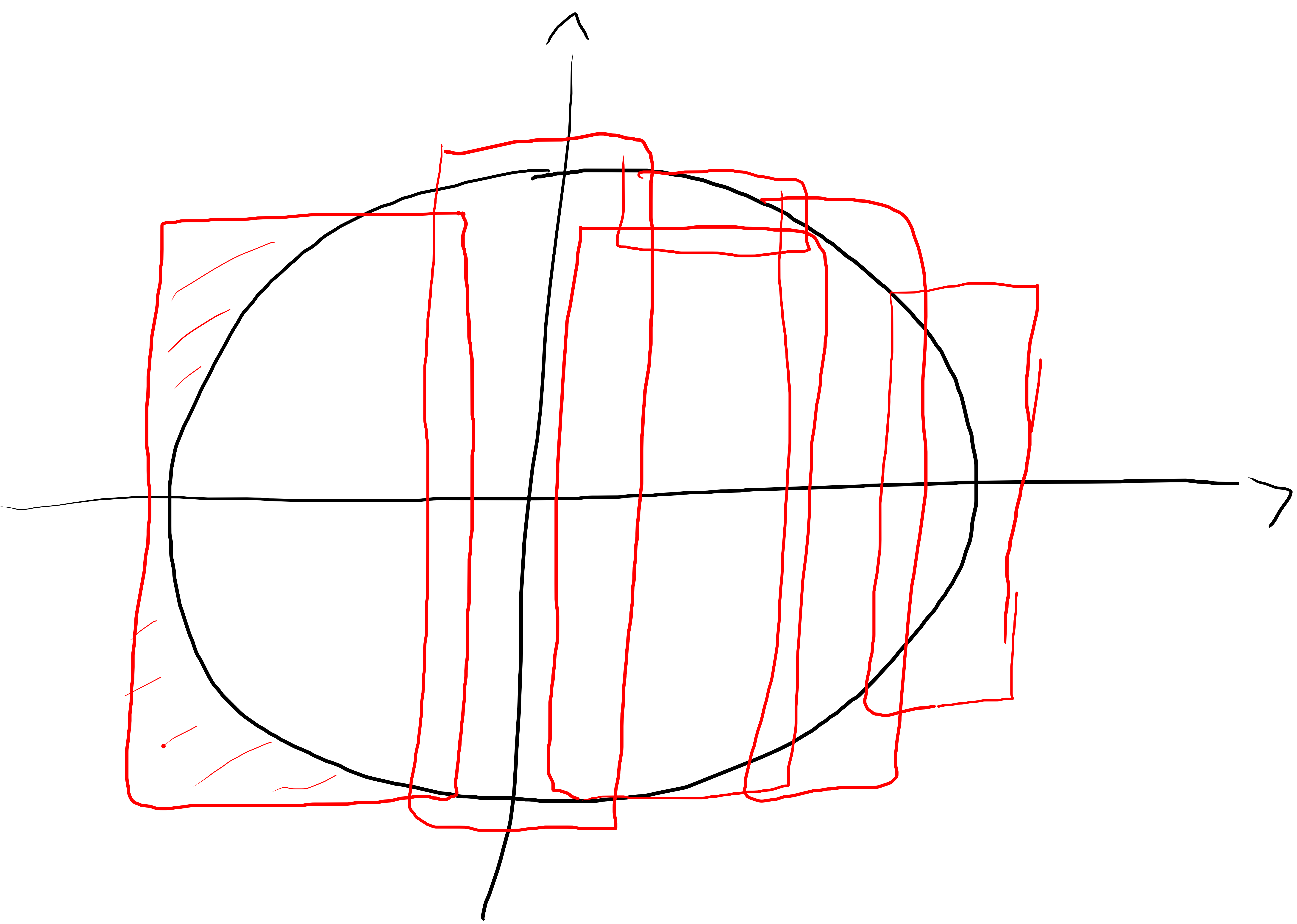


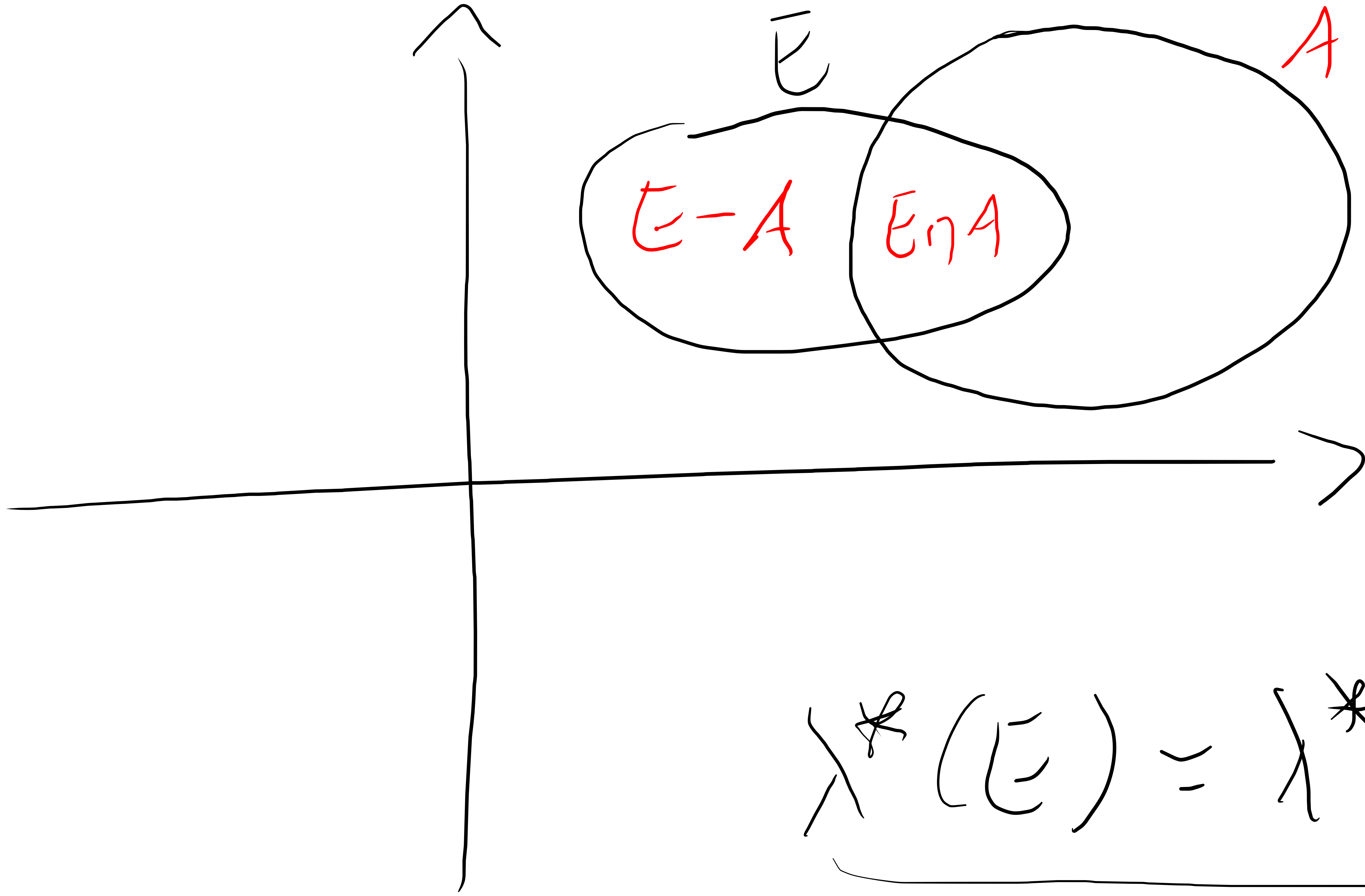
$(b_1 - a_1) \cdot$

$(b_2 - a_2) \cdot$

$(b_3 - a_3)$

$\Omega = \mathbb{R}^2$



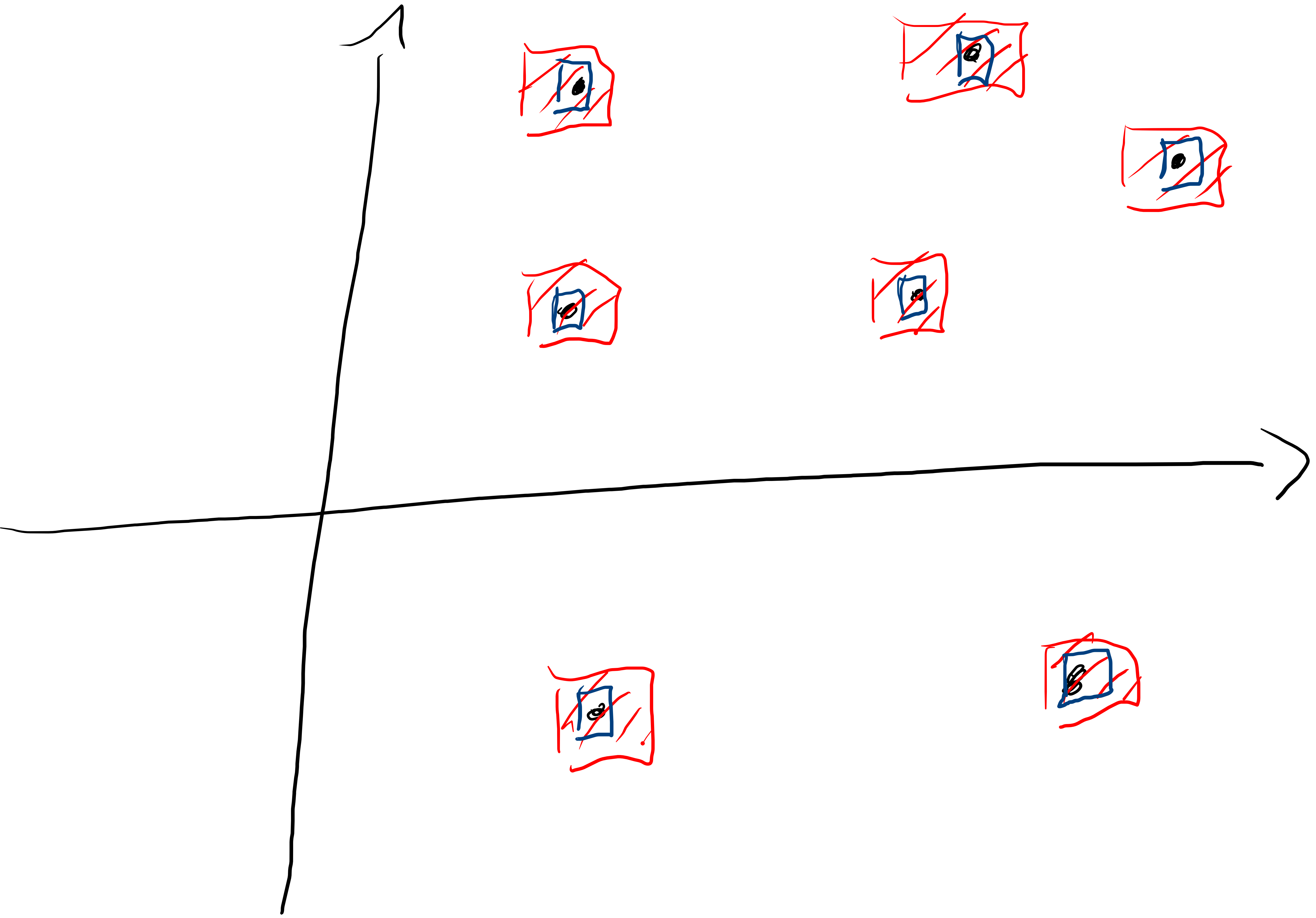


$$\underbrace{\lambda^*(E) = \lambda^*(E \cap A) + \lambda^*(E - A)}$$

$$\mathcal{L}^n = \{ A \subset \mathbb{R}^n \mid \cdot$$

PER OGNI $E \subset \mathbb{R}^n \}$

λ^*



$\mathbb{R}$ $\quad$ $\mathbb{Q}$ $\quad$ $\lambda(\mathbb{Q}) = 0$

$$X : \underset{\mathcal{F}}{\Omega} \to \underset{\mathcal{B}}{\mathbb{R}}$$

$$P_X(B) = P(X^{-1}(B)) = P(X \in B) \qquad B \in \mathcal{B}$$

$$P_X : \mathcal{B} \to [0,1]$$

P_X PROB SU: $\underbrace{(\mathbb{R}, \mathcal{B})}$

NUOVO SPAZIO
DI RIFERIMENTO

1) $P_X(\emptyset) = 0$, $P_X(\mathbb{R}) = 1$

$$P_X(\emptyset) = P(X^{-1}(\emptyset)) = P(\emptyset) = 0$$

$$P_X(\mathbb{R}) = P(X^{-1}(\mathbb{R})) = P(\Omega) = 1$$

2) σ-ADDITIVITÀ $(B_n)_{n\geq 1} \subset \mathcal{B}$ A DUE A DUE DISGIUNTI

$$P_X\left(\bigcup_{n\geq 1} B_n\right) = \sum_{n\geq 1} P_X(B_n)$$

$$P_X\left(\bigcup_{n\geq 1} B_n\right) = P\left(X^{-1}\left(\bigcup_{n\geq 1} B_n\right)\right)$$

$$= P\left(\bigcup_{n\geq 1} \underbrace{X^{-1}(B_n)}_{\in \mathcal{F}}\right)$$

A DUE A DUE DISGIUNTE

$n \neq m$

$$X^{-1}(B_n) \cap X^{-1}(B_m) = X^{-1}(\underbrace{B_n \cap B_m}_{=\emptyset}) = \emptyset$$

σ-ADDITIVITÀ DI P

$$= \sum_{n\geq 1} \underbrace{P(X^{-1}(B_n))}_{P_X(B_n)}$$

$F_X(x) = P(X \leq x) \qquad F_X : \mathbb{R} \to [0,1]$

1) $x < y$ ALLORA $F_X(x) \leq F_X(y)$

$$\{X \leq x\} \subset \{X \leq y\}$$

2) $\lim_{y \downarrow x} F_X(y) = F_X(x)$

IL LIMITE ESISTE FINITO IN QUANTO F_X È MONOTONA (E LIMITATA)

È SUFFICIENTE CONSIDERARE IL LIMITE SU UNA SUCCESSIONE $y_n \downarrow x$ $\quad y_n > x$

$Y_n = X + \frac{1}{n}$ $\quad X < Y_{n+1} < Y_n \quad Y_n \to X$

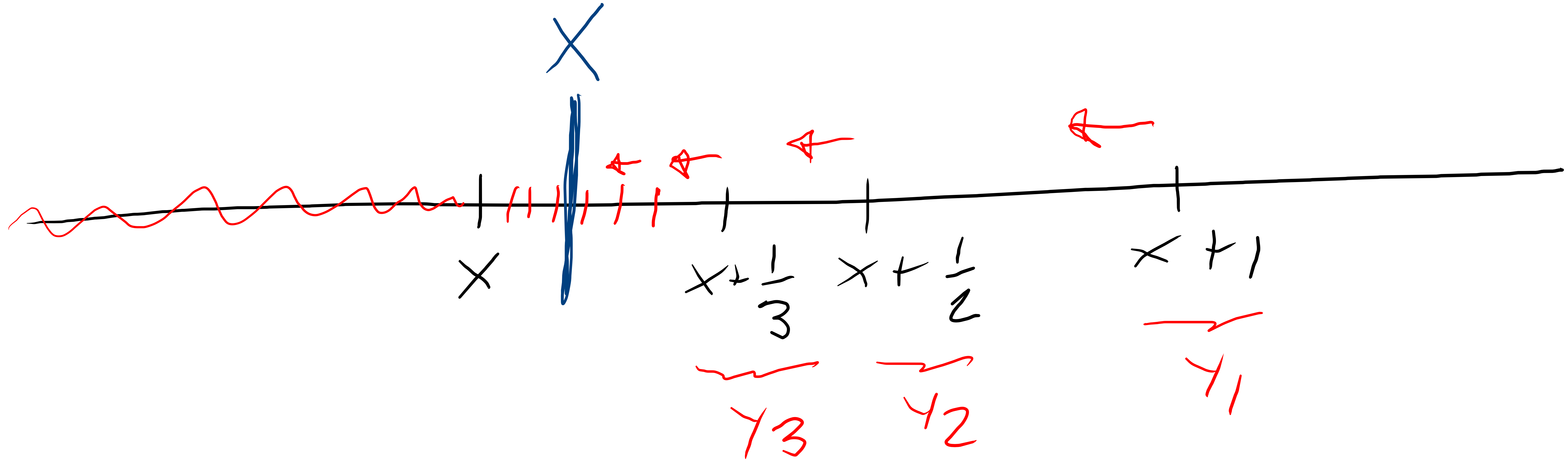
$\{X \le Y_n\} = A_n$

$\{X \le Y_{n+1}\} = A_{n+1}$

$A_{n+1} \subset A_n$

SUCCESSIONE MONOTONA NON CRESCENTE

$$\lim_{n \geq 1} A_n = \bigcap_{n \geq 1} A_n = \bigcap_{n \geq 1} \{X \le Y_n\} = \{X \le x\}$$



$\{X > x\}$

$$\underbrace{P(\underbrace{\lim A_n}_{\{X \le x\}})}_{F_X(x)} = \lim_n P(\overbrace{A_n}^{\{X \le y_n\}}) = \lim_n F_X(y_n)$$

3) $\lim_{x \to +\infty} F_X(x) = 1$

$y_n = n \qquad F_X(y_n) = P(\underbrace{X \leq n}_{A_n})$

$y_n \uparrow +\infty$

A_n MONOTONA CRESCENTE

$$\lim_n A_n = \bigcup_{n \geq 1} A_n = \bigcup_{n \geq 1} \{X \leq n\} = \Omega$$

$$1 = P(\underbrace{\lim A_n}_{\Omega}) = \lim P(A_n) = \lim_{n \geq 1} F_X(y_n)$$