Y_K = N. DI INSUCCESSI PRIMA DI VEDERE IL K-ESIMO SUCCESSO

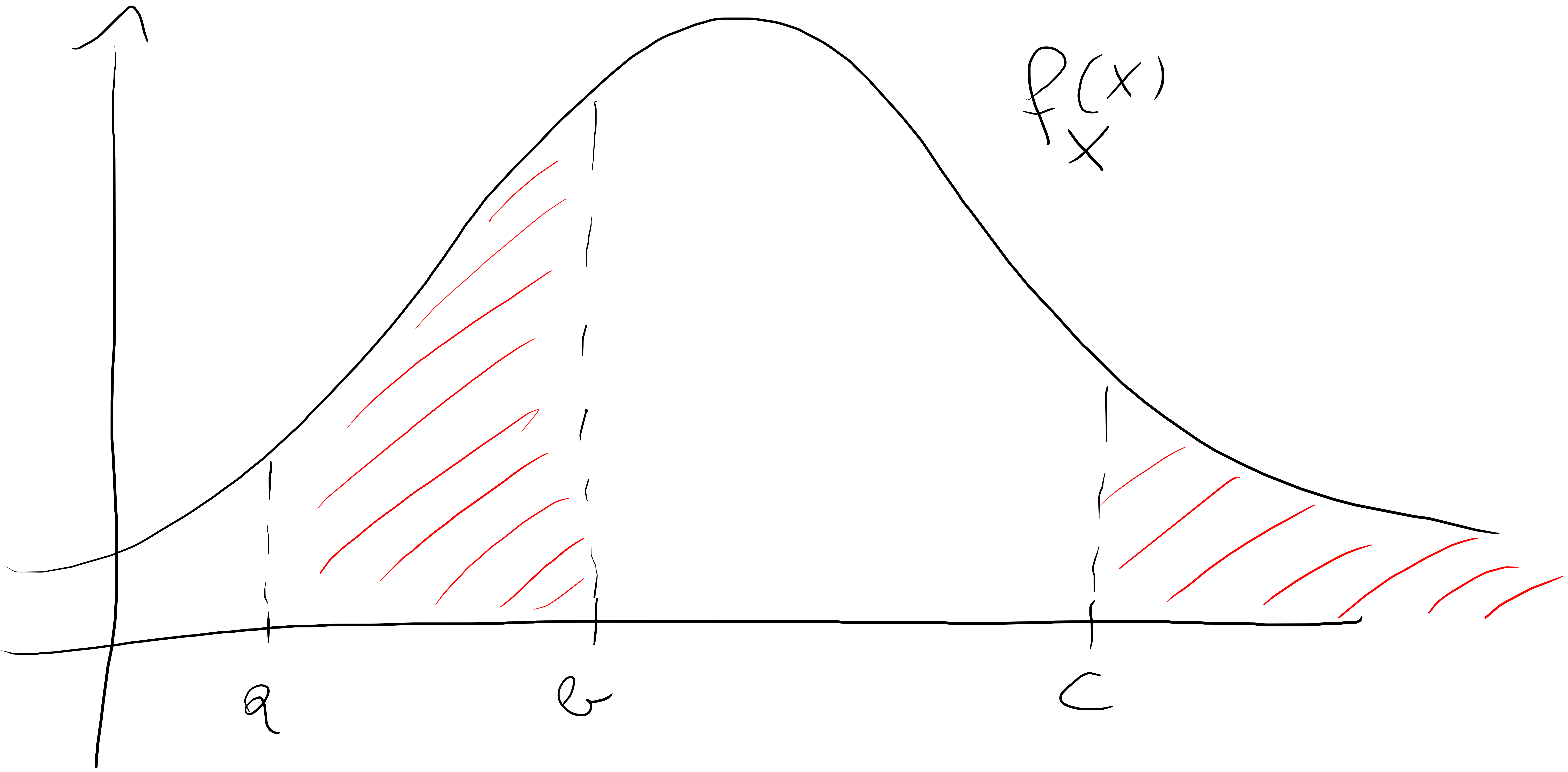
$Y_K = 0, 1, 2, 3, \dots$

$P(Y_K = h)$

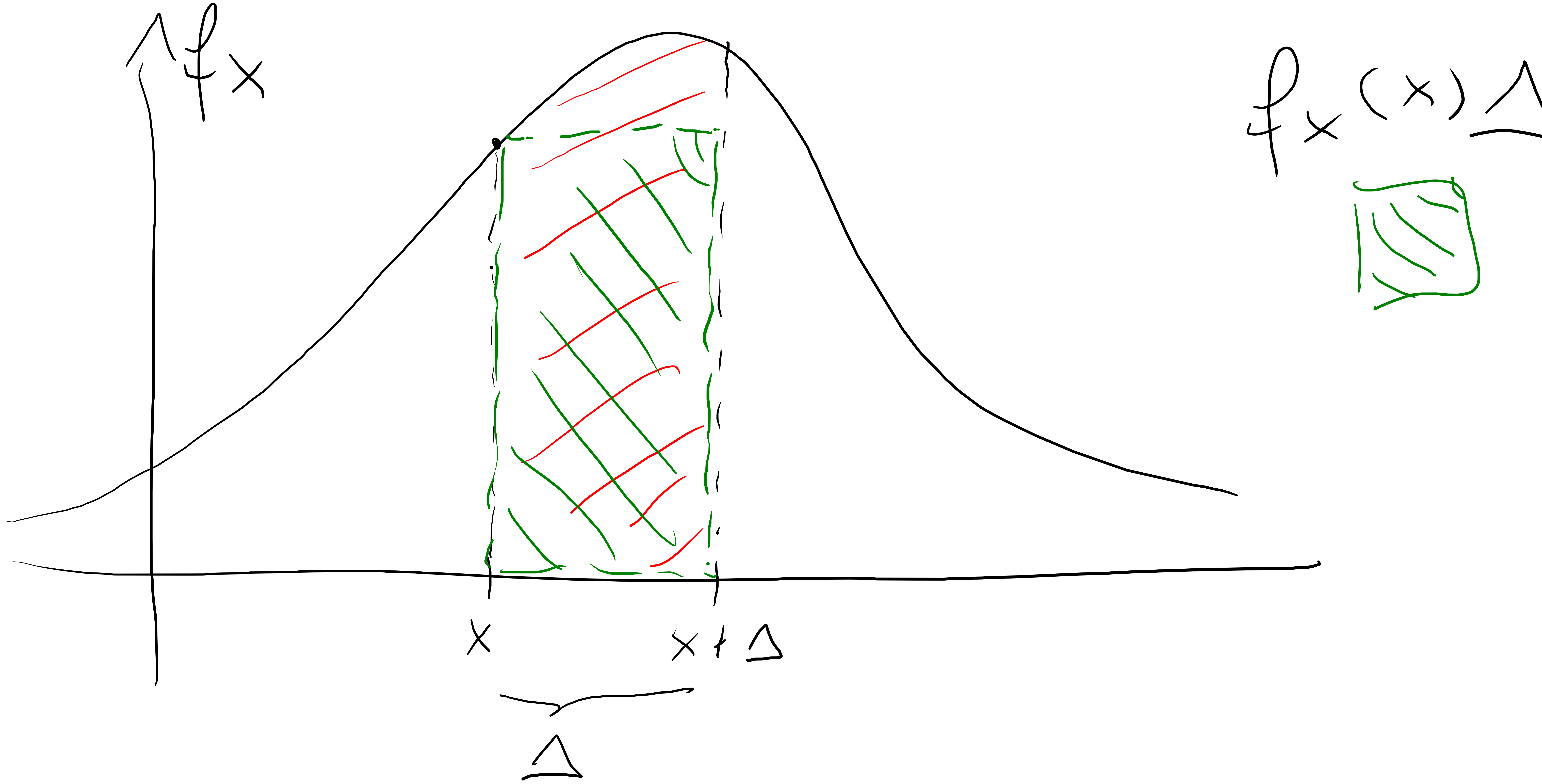
$$\overbrace{\underset{1}{\_\_}\ \ \underset{2}{\_\_}\ \ \underset{3}{\_\_}\ \ \_\_\ \ \_\_\ \ \cdots\ \ \_\_\ \ \underset{h+K-1}{\_\_}}^{\substack{h \text{ INSUCCESSI} \\ K-1 \text{ SUCCESSI}}}\ \ \overset{\substack{\text{K-ESIMO SUCCESSO} \\ \downarrow \\ E_{h+K}}}{\underset{h+K}{\_\_}}$$

$$= \binom{n+k-1}{k-1} p^k q^n \qquad q = 1-p$$

$$Y_1 = X$$

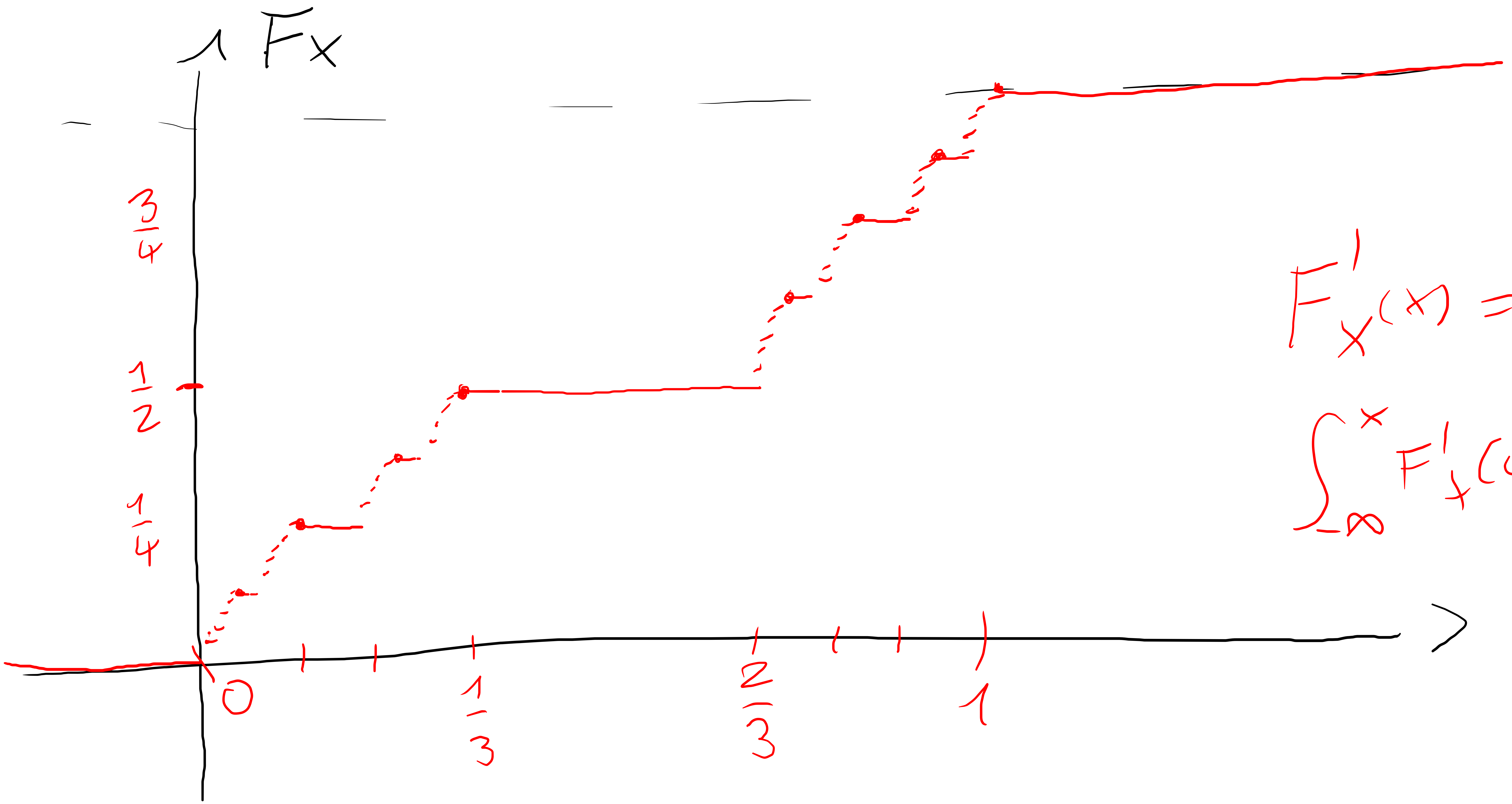


$$P(X \in (a,b) \text{ o } X > c) = \int_{(a,b) \cup (c,+\infty)} f_X(x)\,dx$$



f_X
x
$x + \Delta$
Δ
$f_X(x)\Delta$

ESEMPIO DI CANTOR (CDF CONTINUA MA NON ASSOLUTAMENTE CONTINUA)



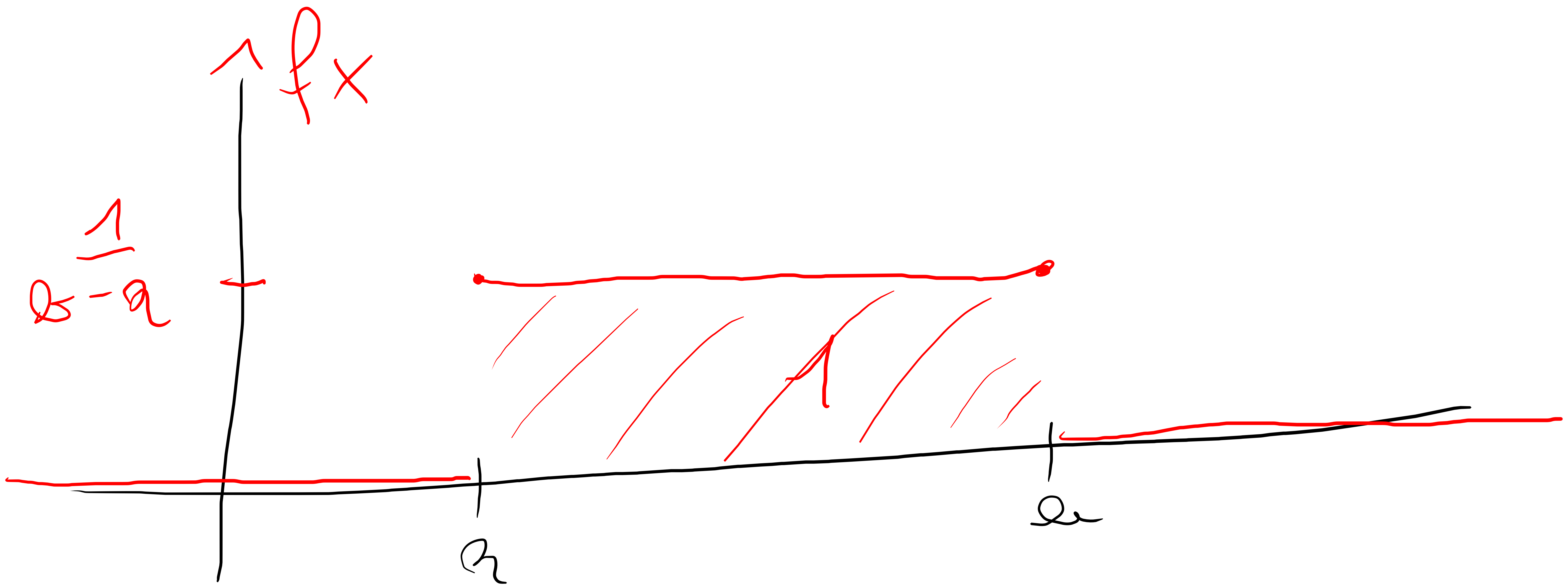
$F_X'(x) = 0$ Q.O.

$\int_{-\infty}^{x} F_X'(u)\,du = 0$ PER OGNI x

$f(x) \geq 0$ $\qquad \int_{-\infty}^{+\infty} f(x) = K > 0$

ALLORA È UNA DENSITÀ $\qquad \tilde{f}(x) = \frac{f(x)}{K}$

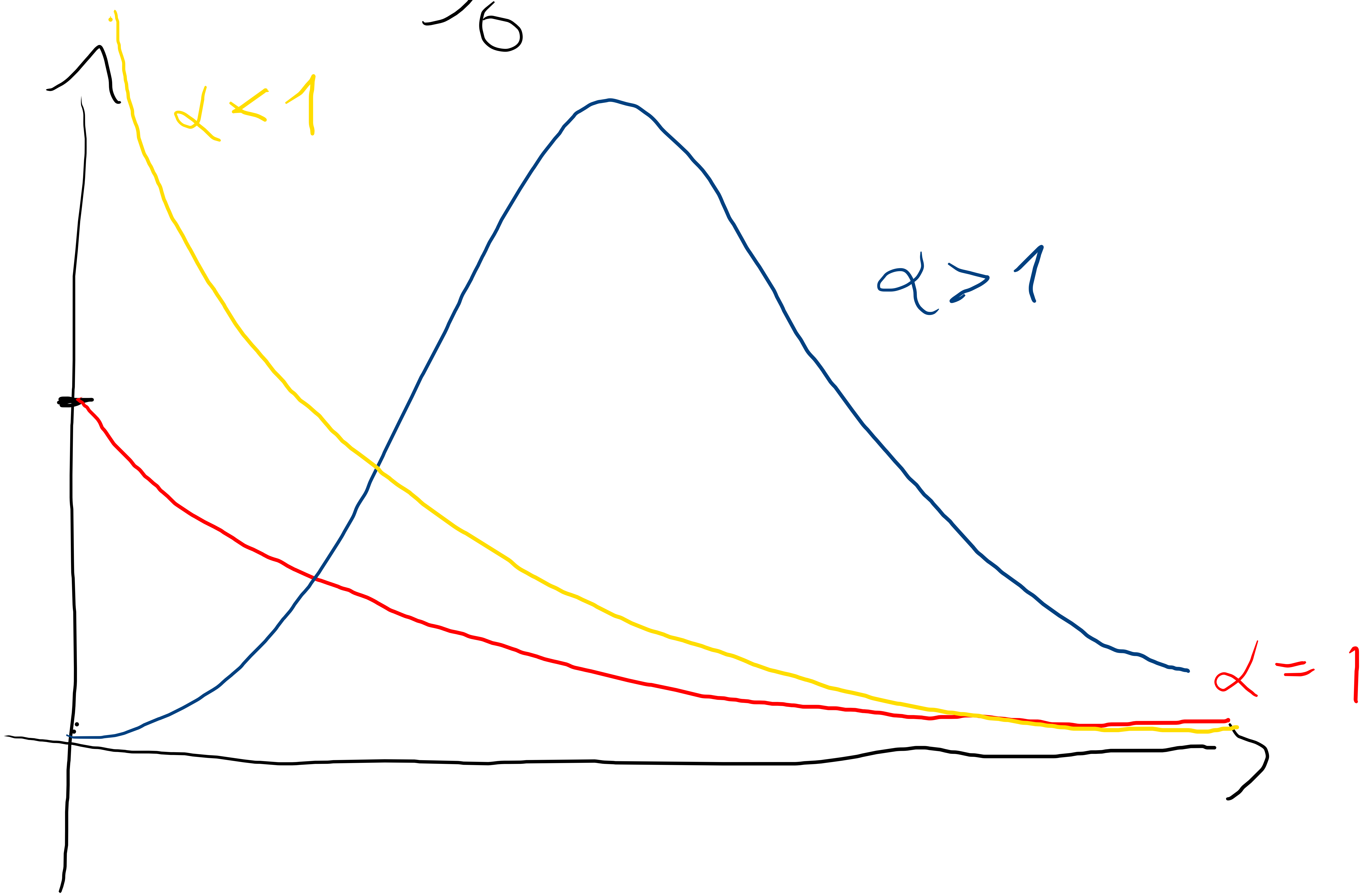
È UNA PDF



CDF of $U(a, b)$

$$F_X(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & x > b \end{cases}$$

$$\Gamma(\alpha) = \int_0^{+\infty} x^{\alpha-1} e^{-x} \, dx$$



$X \sim \exp(\lambda)$ $\qquad f_X(x) = \lambda e^{-\lambda x}$ $\qquad x > 0$

$$F_X(x) = \int_{-\infty}^{x} f_X(u)\,du = \underline{\underline{0}} \qquad \underline{\underline{x \leq 0}}$$

$$= \int_0^x \lambda e^{-\lambda u}\,du \qquad \underline{\underline{x > 0}}$$

$$= \left[-e^{-\lambda u}\right]_0^x = \underline{\underline{1 - e^{-\lambda x}}}$$

$$P(X > x) = e^{-\lambda x}$$

$$Y = F_X(X) = 1 - e^{-\lambda X} \in [0,1]$$

$$F_Y(y) = \begin{cases} 0 & y \leq 0 \\ 1 & y \geq 1 \end{cases}$$

$$= P(Y \leq y) \qquad 0 < y < 1$$

$$= P(1 - e^{-\lambda X} \leq y)$$

$$= P(-e^{-\lambda X} \leq y - 1) = P(e^{-X} \geq 1 - y)$$

log

$$= P(-\lambda X \geq \log(1-y))$$

$$= P\left(X \leq -\frac{1}{\lambda}\log(1-y)\right)$$

$$= F_X\left(\underbrace{-\frac{1}{\lambda}\overbrace{\log(1-y)}^{<0}}_{>0}\right)$$

$$= 1 - e^{\cancel{-}\cancel{\lambda}\left(\cancel{-}\frac{1}{\cancel{\lambda}}\log(1-y)\right)}$$

$$= 1 - 1 + y = y$$

$$Y \sim U(0,1)$$

$$F_X^{-1}(p) = -\frac{1}{\lambda}\log(1-p) \qquad 0<p<1$$

$$F_X^{-1}(U) \sim \exp(\lambda)$$

$$U \sim U(0,1)$$

$$Z = X^{1/\gamma} \qquad \gamma > 0$$

$$Z \geq 0$$

$$F_Z(z) = 0 \qquad z \leq 0$$

$$F_Z(z) = P(Z \leq z) \qquad z > 0$$

$$= P(X^{1/\gamma} \leq z) = P(X \leq z^{\gamma})$$

$$= F_X(z^{\gamma})$$

$$= 1 - e^{-\lambda z^{\gamma}}$$

WEIBULL (λ, γ)

$$P(Z > z) = e^{-\lambda z^{\gamma}}$$

$X \sim N(\mu, \sigma^2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

$Y = e^X > 0$ PDF

$F_Y(y) = 0$ $y \leq 0$

$= P(e^X \leq y)$ $y > 0$

$= P(X \leq \log y) = F_X(\log y)$

$$F'_Y(y) = \begin{cases} 0 & y \leq 0 \\ \dfrac{1}{y\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(\log y - \mu)^2} & y > 0 \end{cases} = f_Y(y)$$