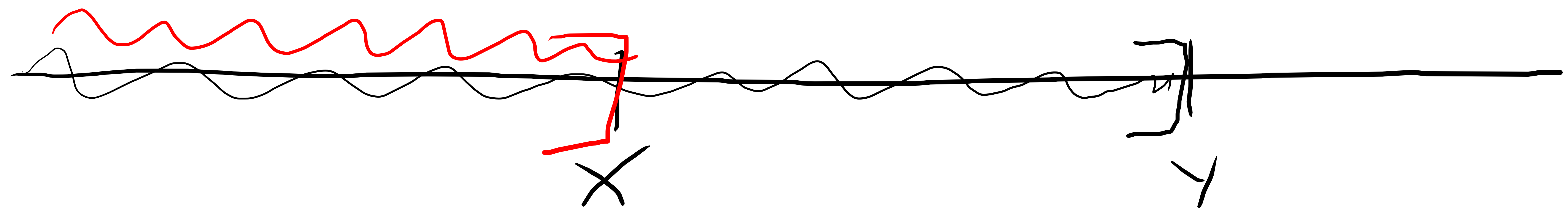
$$F_X(y) = P(X \le y) = P(X \le x) + P(x < X \le y)$$

ADDITIVITÀ

$F_X(x)$

$x < y$

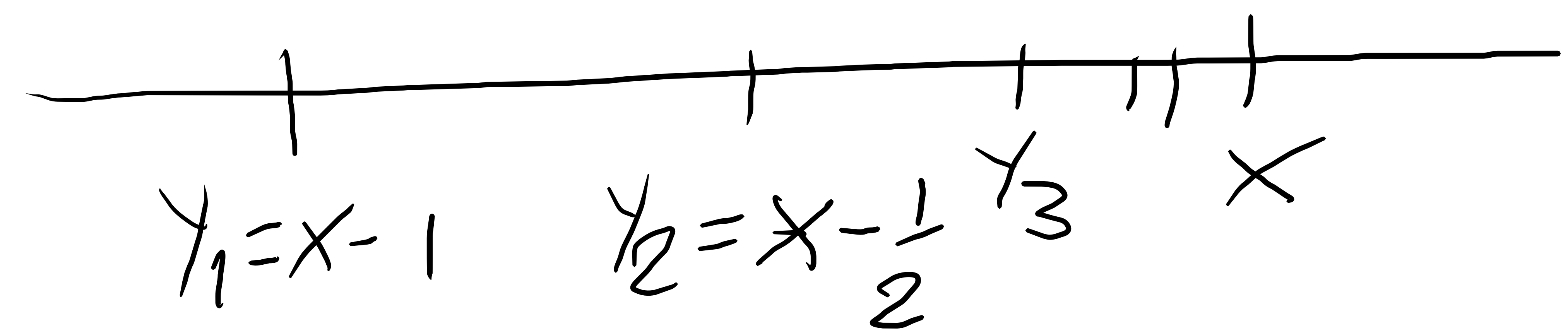


$$\overline{\{X \le x\}} = \{X > x\}$$

$$\lim_{y \uparrow x} F_X(y)$$

$$y_n = x - \frac{1}{n} \qquad y_n \uparrow x \quad (y_n < x)$$

$$F_X(y_n) = P(\underbrace{X \le y_n}_{A_n})$$



$A_n \subset A_{n+1}$ MONOTONA NON DECRESCENTE

$$\lim_n A_n = \bigcup_n A_n = \bigcup_n \{X \le y_n\} = \{X < x\}$$

SE ω : $\underline{\underline{X(\omega) < x}}$

PER ASSURDO $\omega \notin \bigcup_n \{X \leq y_n\}$

CIOÈ $\omega \notin \{X \leq y_n\}$ PER OGNI n

$\boxed{X(\omega) > y_n = x - \frac{1}{n}}$ PER OGNI n

$3 > 3 - \frac{1}{n} \quad \forall n \geq 1$

$\Rightarrow$ $\underline{\underline{X(\omega) \geq x}}$

$$P(\underbrace{\lim_n A_n}_{X<x}) = \lim_n P(\underbrace{A_n}_{X\le y_n})$$

$$= \lim_n F_X(y_n) = F_X(x-)$$

6) $P(X=x) = F_X(x) - F_X(x-)$

$$\{X \le x\} = \{X < x\} \cup \{X = x\}$$

ESEMPIO

$$P(X \geq x) = 1 - F_X(x-)$$

$$P(x < X < y) = F_X(y-) - F_X(x)$$

$x < y$

$$\{x < X \leq y\} = \{x < X < y\} \cup \{X = y\}$$

$F_X(y) - F_X(x)$ $\qquad$ $F_X(y) - F_X(y-)$

$$P(x \leq X < y) = ?$$

$$P(x \leq X \leq y) = ?$$

F FDF (SODDISFA (i), (ii), (iii))

$\exists$ P SU $(\mathbb{R}, \mathcal{B})$ TALE CHE

$$P((-\infty, x]) = F(x)$$

DEFINISCO

$$\Omega = \mathbb{R}$$
$$\mathcal{F} = \mathcal{B}$$
$$P = P$$

X APPLICAZIONE IDENTICA

$$X(x) = x$$

$$P_X(B) = P(\underbrace{X^{-1}(B)}_{= B}) = P(B)$$

$$F_X(x) = P(X \leq x) = P((-\infty, x]) = F(x)$$

$X = Y$ q.c. $\qquad P(X = Y) = 1$

$\Rightarrow F_X = F_Y \qquad (P_X = P_Y)$

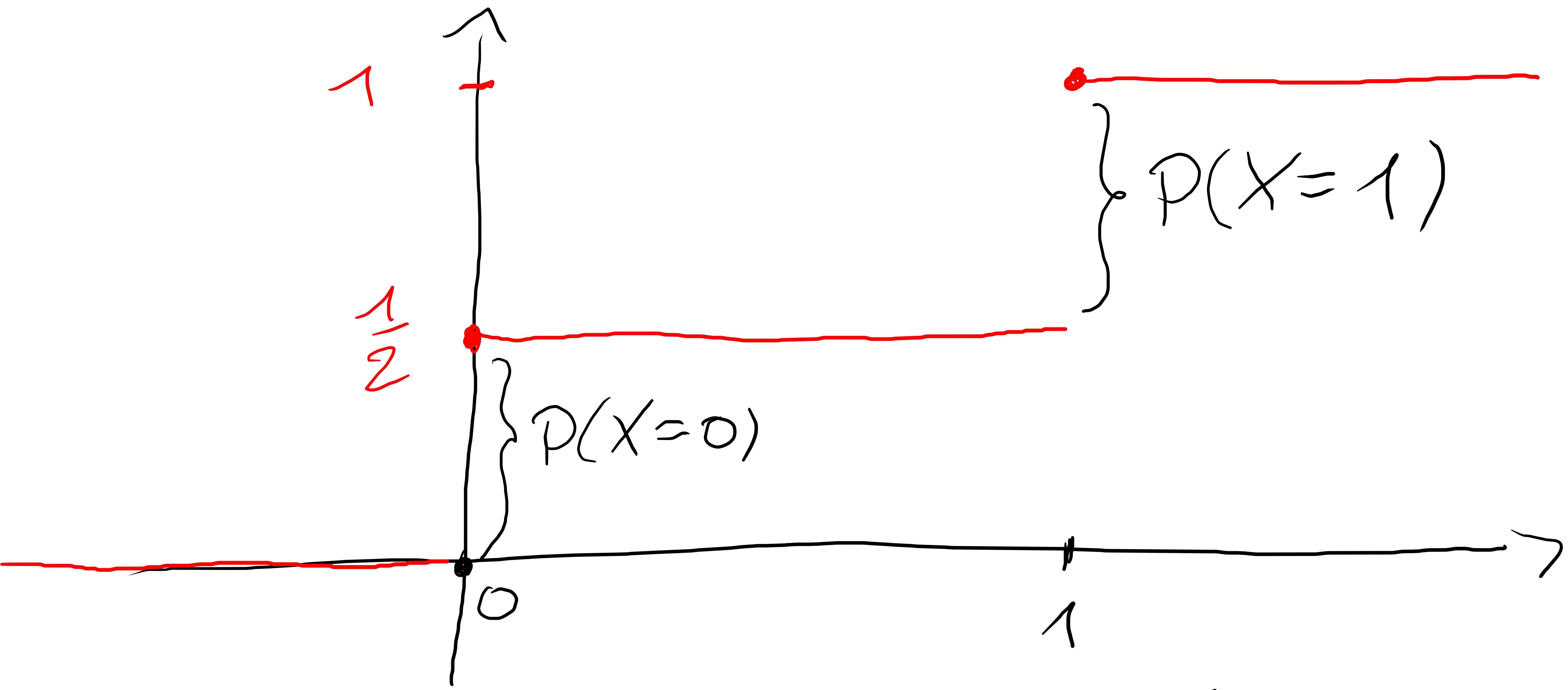
$$P(X \in B) = P(X \in B, X = Y) + \underbrace{P(X \in B, X \neq Y)}_{= 0}$$

$$= P(Y \in B, X = Y) + \underbrace{P(Y \in B, X \neq Y)}_{= 0}$$

$$= P(Y \in B)$$

$X = 1_A \qquad P(A) = \frac{1}{2}$

$$1_A = \begin{cases} 0 & \bar{A} \\ 1 & A \end{cases}$$



$F_{1_A}(x) = P(1_A \le x)$

$F_{1_A}(0) = P(1_A \le 0)$

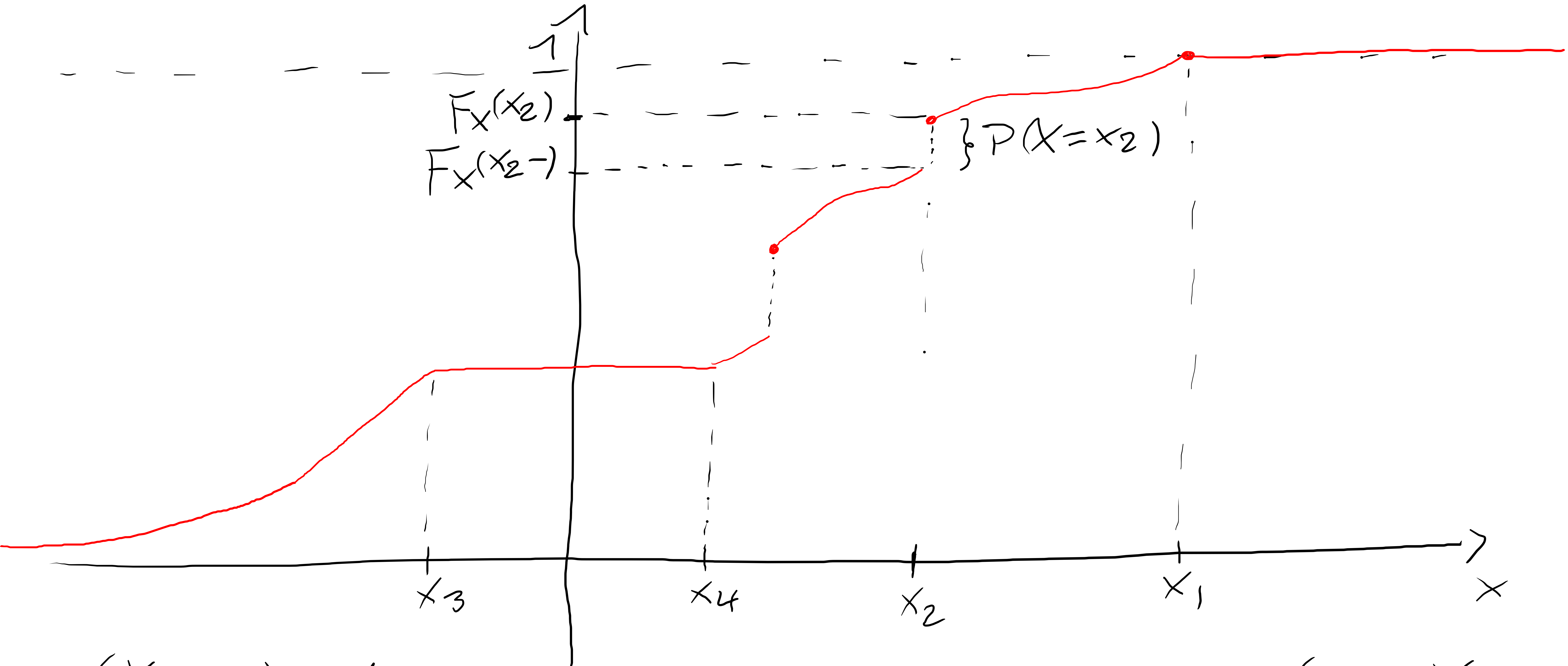
$= P(\bar{A}) = \frac{1}{2}$

$x \ge 1 \quad F_{1_A}(x) = P(1_A \le x)$

$= P(\Omega) = 1$

$Y = 1 - X = 1_{\bar{A}} \qquad F_X = F_Y$

MA $\quad P(X = Y) = 0$



$P(X \leq x_1) = 1$ $\quad$ $P(X = x_2) = F_X(x_2) - F_X(x_2-)$ $\quad$ $P(x_3 < X \leq x_4) = 0$

DATO IL LEMMA, PRENDO

$A_x = \{X = x\}, \quad x \in S$

$P(A_x) > 0$

A_x A DUE A DUE DISGIUNTI

$\Rightarrow$ S È DISCRETO (FINITO O NUMERABILE)

LEMMA $(\Omega, \mathcal{F}, P)$ $(A_\alpha)_{\alpha \in I} \subset \mathcal{F}$ A DUE A DUE DISGIUNTI

$P(A_\alpha) > 0$ PER OGNI $\alpha \in I$

$\Rightarrow I$ È DISCRETO

PROVA $I_n = \{\alpha \in I \mid P(A_\alpha) > \frac{1}{n}\}$ $n \geq 1$

$I_1 = \emptyset$

$\# I_2 \leq 1$

se $i_1, i_2 \in I_2$ $\quad P(A_{\alpha_{i_1}}) > \frac{1}{2}$

$P(A_{\alpha_{i_2}}) > \frac{1}{2}$

$$P(A_{\alpha_{i_1}} \cup A_{\alpha_{i_2}}) = P(A_{\alpha_{i_1}}) + P(A_{\alpha_{i_2}}) > 1$$

$$\# I_n \leq n-1$$

$\Rightarrow I_n$ FINITO PER OGNI $n \geq 1$

$$I = \bigcup_{n \geq 1} I_n$$ UNIONE DISCRETA DI INSIEMI DISCRETI $\Rightarrow$ È DISCRETO

$\supset$ OVVIA

$\subset$ SE $P(A_\alpha) > 0$, ESISTE $n \geq$ [illegible]
TALE CHE $P(A_\alpha) > \frac{1}{n}$
E QUINDI $\alpha \in I_n$

$$\underbrace{\lambda = \sum_{x \in S} P(X = x)}_{P(X \in S)} \in [0, 1]$$

$\lambda = 1$ VA DISCRETA

$\lambda = 0$ $(S = \emptyset)$ VA CONTINUA $P(X = x) = 0$ PER OGNI $x \in \mathbb{R}$

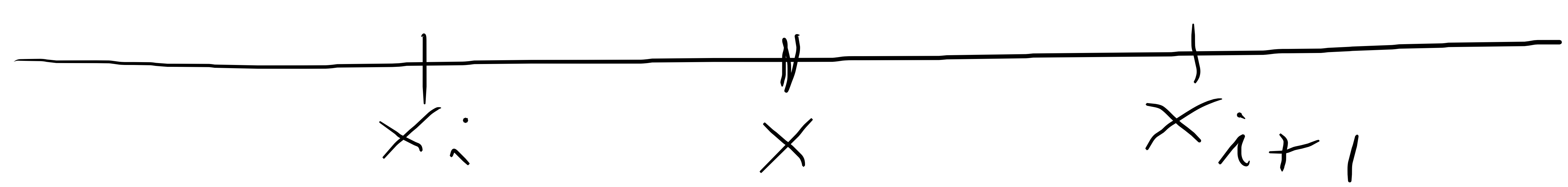
$0 < \lambda < 1$ VA MISTA

X VA DISCRETA

$x_i < x_{i+1}$ 2 VALORI DI X SUCCESSIVI (NON CI SONO ALTRI VALORI DI X TRA x_i E x_{i+1})

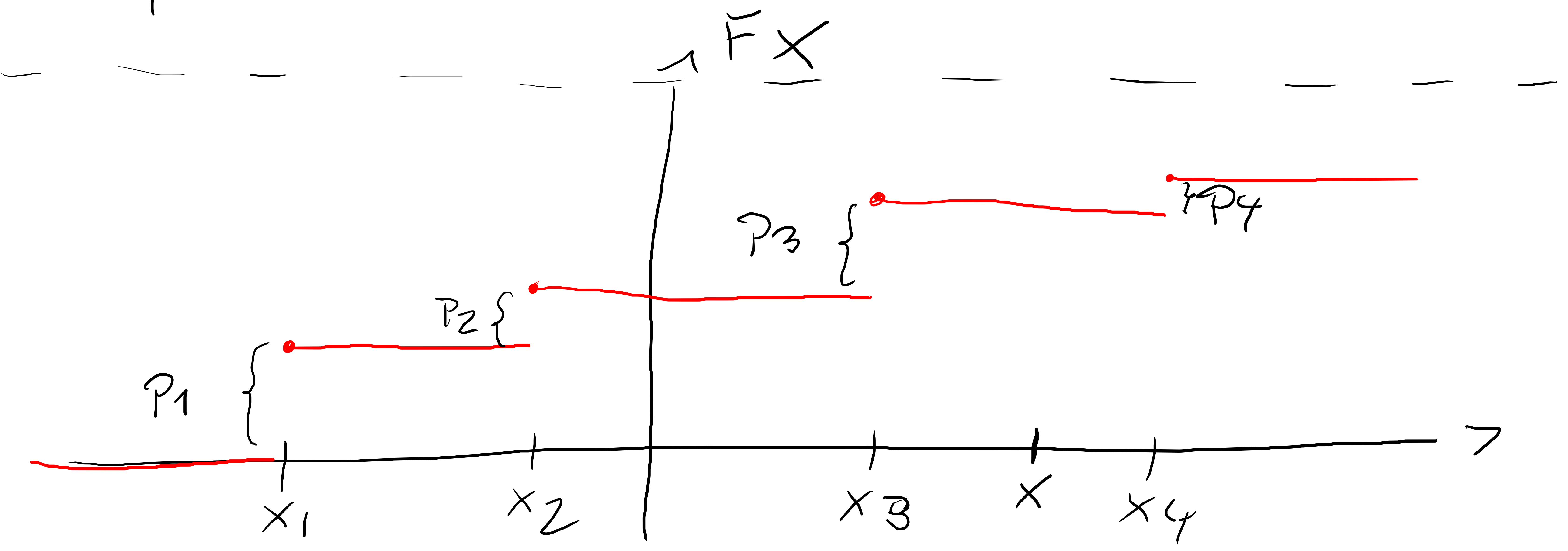
$x_i \le x < x_{i+1}$

$$F_X(x) = P(\underbrace{X \le x}_{=\{X \le x_i\}}) = P(X \le x_i) = F_X(x_i)$$



$x_1 < x_2 < x_3 < x_4 < \dots$

$p_1 \quad p_2 \quad p_3 \quad p_4 \quad \dots$

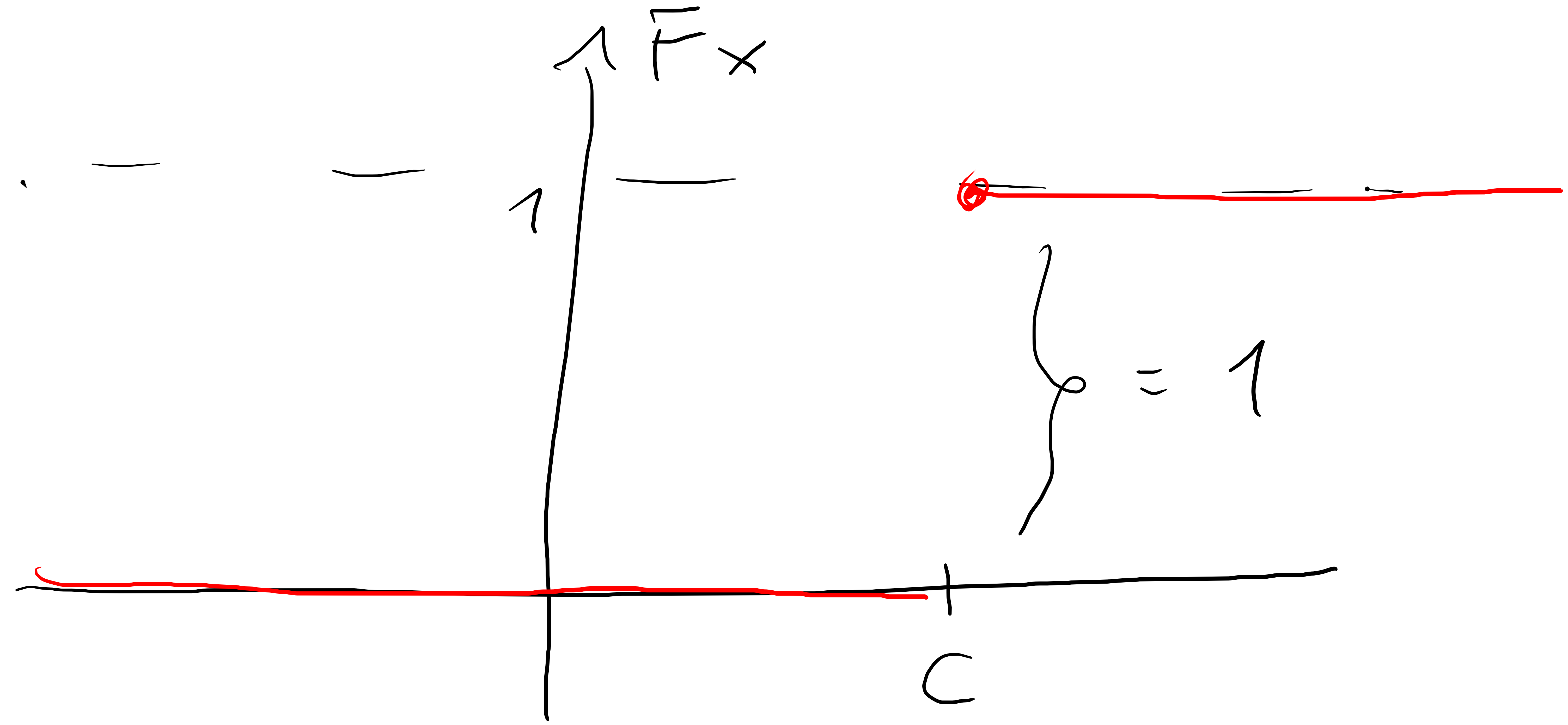


$F_X(x) = p_1 + p_2 + p_3$

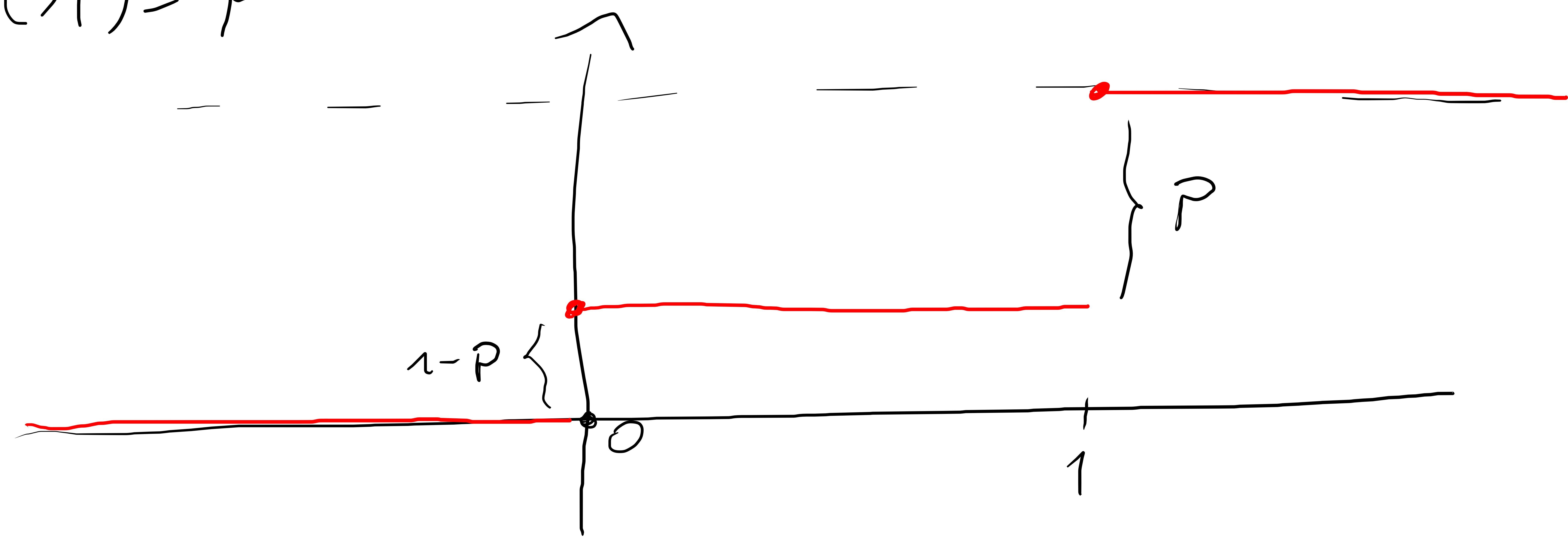
$$F_X(x) = \sum_{x_i \le x} p_i$$

SODDISFA (i), (ii) (iii)

$X = c$ q.c.



$X = 1_A \qquad P(A) = p$



$E_1, E_2, \ldots, E_n$ INDIPENDENTI, $P(E_i) = p$

$i = 1 \ldots n$

$X = \#\{i \mid E_i \text{ è } V\}$

$$P(X = i) = \binom{n}{i} \underbrace{p^i (1-p)^{n-i}}_{} \qquad i = 0, 1, \ldots, n$$

PROB DI UN GENERICO EVENTO

$E'_1 \cap E'_2 \cap \ldots \cap E'_n$

con i V, $n-i$ F

$E_1, E_2, \dots, E_n, \dots$ $\qquad P(E_i) = p$ $\quad$ INDIPENDENTI

$X \qquad 0, 1, 2, 3, \dots$

$$P(X = k) = P(\overline{E}_1 \cap \overline{E}_2 \cap \dots \cap \overline{E}_k \cap E_{k+1})$$

$$= (1-p)^k p = q^k p \qquad (q = 1-p)$$

X' = NUMERO DI TENTATIVI PER AVERE IL PRIMO SUCCESSO $= X + 1$

$$X' = 1, 2, 3, \ldots$$

$$P(X' = k) = P(X + 1 = k) = P(X = k - 1)$$

$$= q^{k-1} p$$

$x \in \mathbb{R}$

$\lfloor x \rfloor$ INTERO

$\lfloor x \rfloor \leq x$

$$\lfloor 3 \rfloor = 3 \qquad \lfloor 3.6 \rfloor = 3$$

$$\lfloor -2.3 \rfloor = -3$$

$\lfloor x \rfloor$ CARATTERIZZATO DA $\lfloor x \rfloor \leq x < \lfloor x \rfloor + 1$

PARTE INTERA

$$F_X(x) = P(X \leq x) = 0 \qquad x < 0$$

$$= \sum_{\substack{h=0,1,2\ldots \\ h \leq x}} q^h p \qquad x \geq 0$$

$$= p + qp + q^2 p + \ldots + q^{\lfloor x \rfloor} \cdot p$$

$$= p(1 + q + q^2 + \ldots + q^{\lfloor x \rfloor})$$

$$= \cancel{p} \frac{1-q^{\lfloor x \rfloor + 1}}{1-\cancel{q}} = 1 - q^{\lfloor x \rfloor + 1}$$