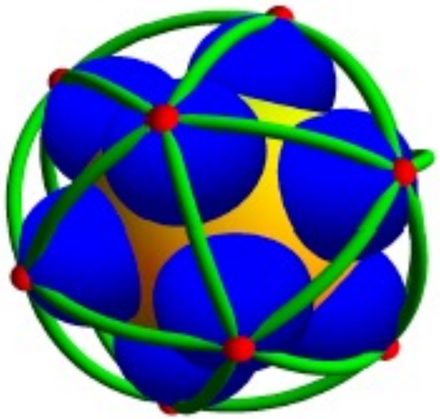
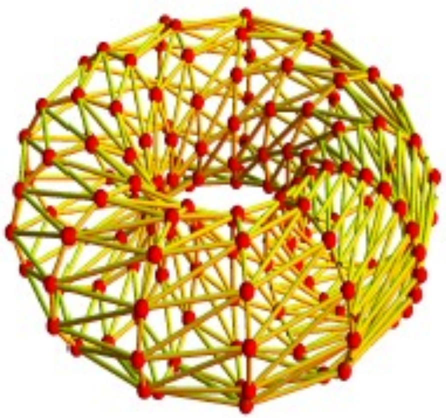
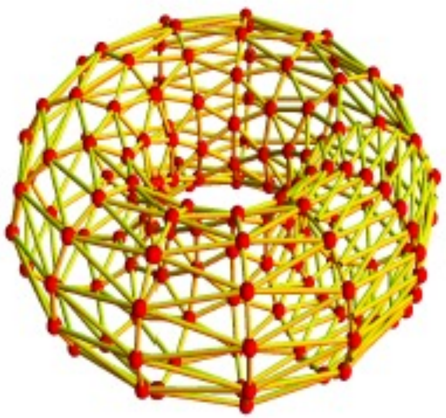
Mathematics Illustrations for 3D printing

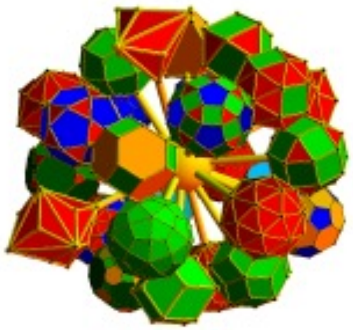


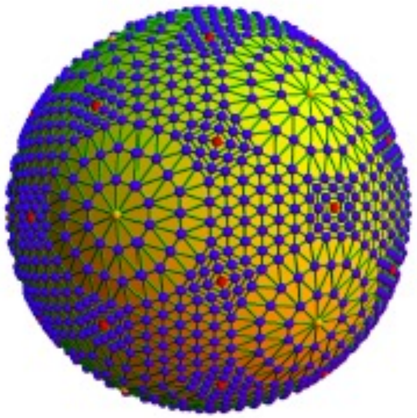
This figure aims to visualize that the kissing number of a sphere is ≥ 12. The Mathematica code producing this object is given in the text. It produces a file containing tens of thousands of triangles which the 3D printer knows to bring to live. The printed object visualizes that there is still quite a bit of space left on the sphere. Newton and his contemporary Gregory had a disagreement over whether this is enough to place a thirteenth sphere.



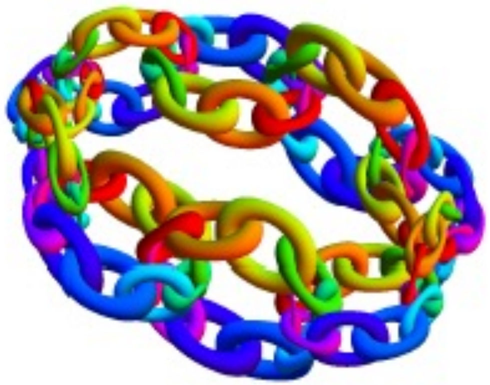


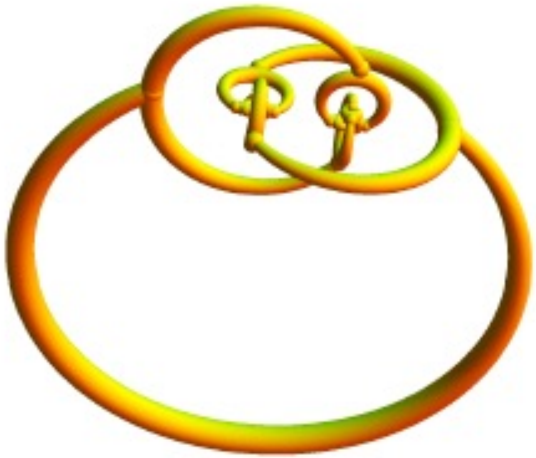
A Dehn twisted flat torus and an untwisted torus. The left and right picture show two non-isomorphic graphs, but they have the same topological properties and are isospectral for the Laplacian as well as for the Dirac operator. It is the easiest example of a pair of non-isometric but Dirac isospectral graphs.



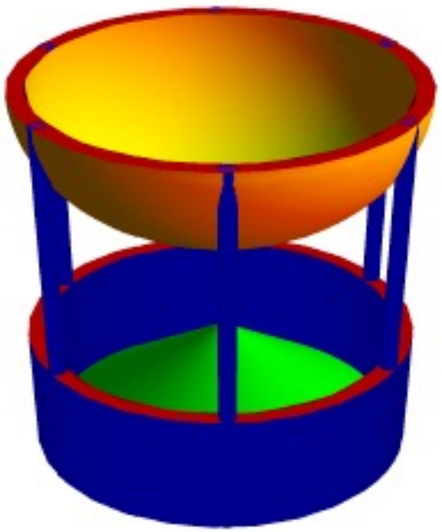


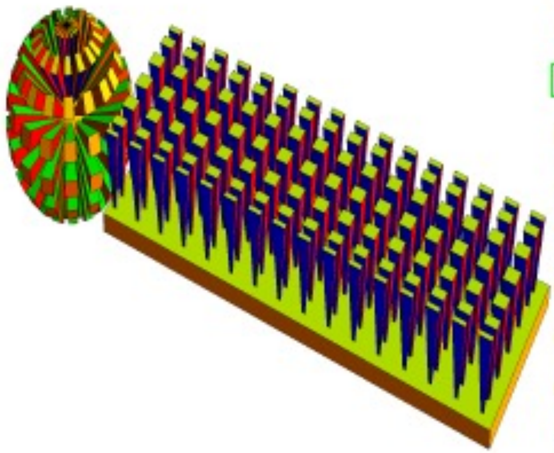
All 26 Archimedean and Catalan solids joined to a "gem" in the form of a DisdyakisDodecahedron. The gem has since been in the process of being printed (http://sdu.ictp.it/3d/gem/, http://www.3drucken.ch). The right figure shows a Great Rhombicosidodecahedron with 30 points of curvature 1/3 and 12 points of curvature $-2/3$. The total curvature is 2 and agrees with the Euler characteristic. This illustrates a discrete Gauss-Bonnet theorem



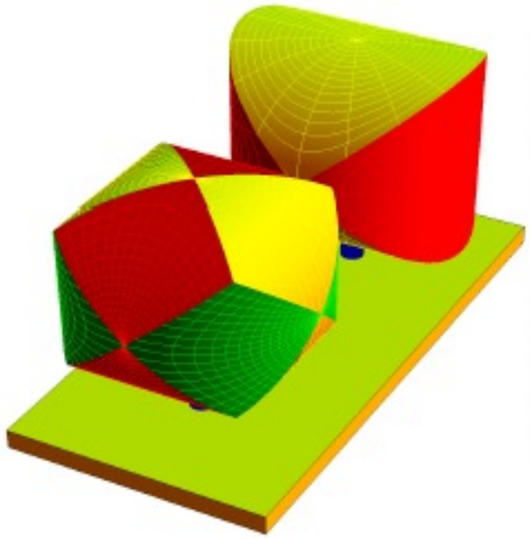


The Antoine necklace is a Cantor set in space whose complement is not simply connected. The Alexander sphere seen to the right is a topological 3 ball which is simply connected but which has an exterior which is not simply connected. Alexander spheres also make nice ear rings when printed.

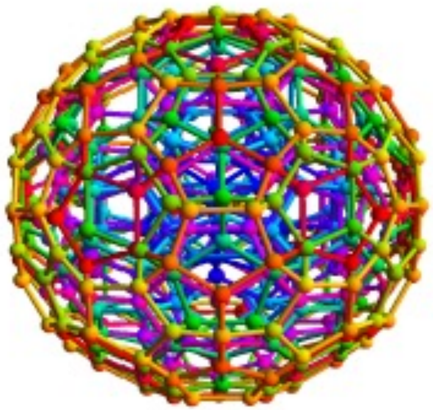


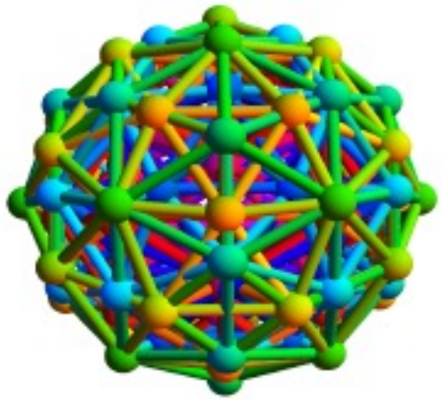


Two Archimedes type proofs that the volume of the sphere is $4\pi/3$. [44, 45, 98]. The first one assumes that the surface area A is known. The formula $V = Ar/3$ can be seen by cutting up the sphere into many small tetrahedra of volume $dAr/3$. When summing this over the sphere, we get $Ar/3$. The second proof compares the half sphere volume with the complement of a cone in a cylinder.

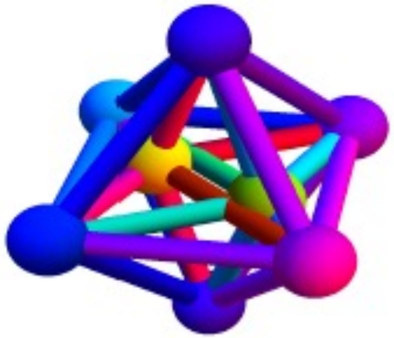


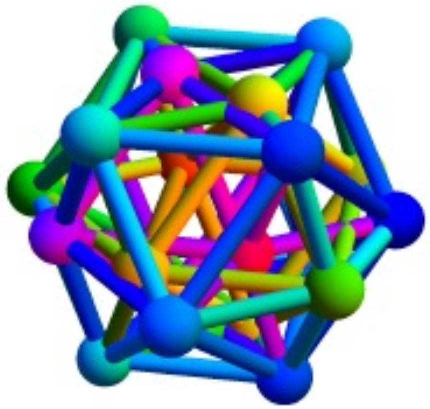
The hoof of Archimedes, the Archimedean dome, the intersection of cylinders are solids for which Archimedes could compute the volume with comparative integration methods [4]. The hoof is also an object where Archimedes had to use a limiting sum, probably the first in the history of humankind [76].



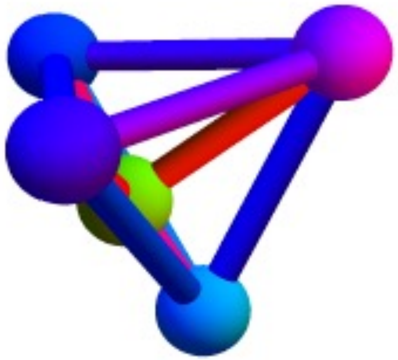


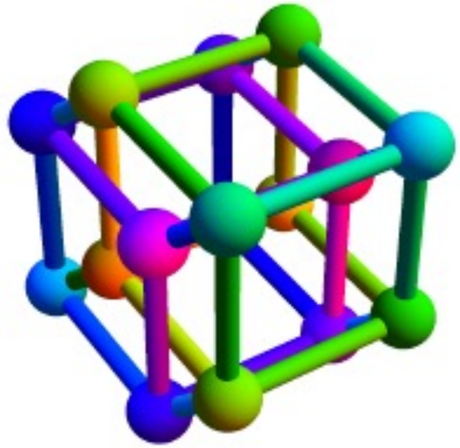
Two of the 6 regular convex polytopes in 4 dimensions. The color is the height in the four-dimensional space. We see the 120 cell and the 600 cell. The color of a node encodes its position in the forth dimension. We can not see the fourth dimension but only its shadow on the cave to speak with an allegory of Plato. The game of projecting higher dimensional objects into lower dimensions has been made a theme of [1].



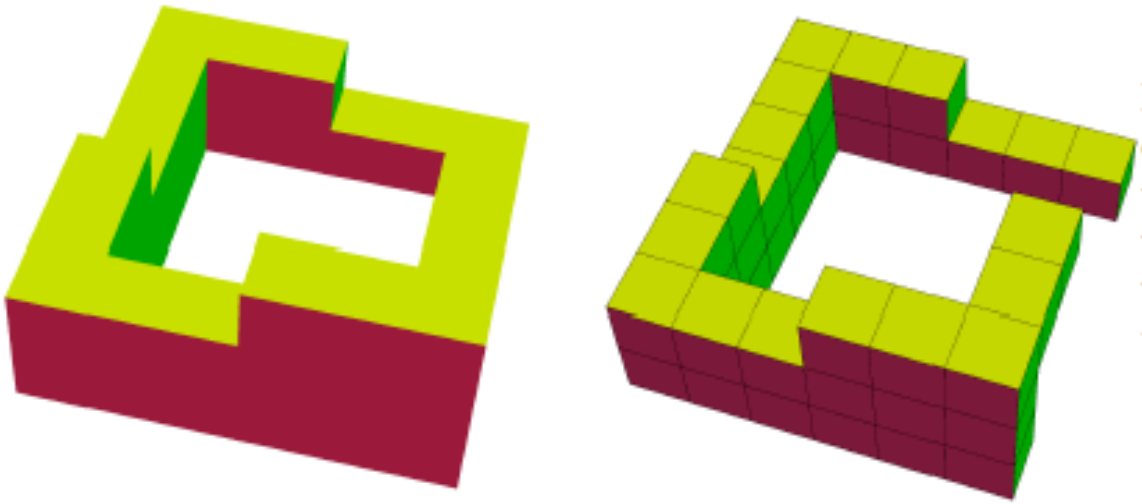


An other pair of the 6 regular convex polytopes in 4 dimensions. The color is the height in the four-dimensional space. We see the 16 cell (the analogue of the octahedron) and the 24 cell. The later allows to tessellate 4 dimensional Euclidean space similarly as the octahedron can tessellate 3 dimensional Euclidean space.

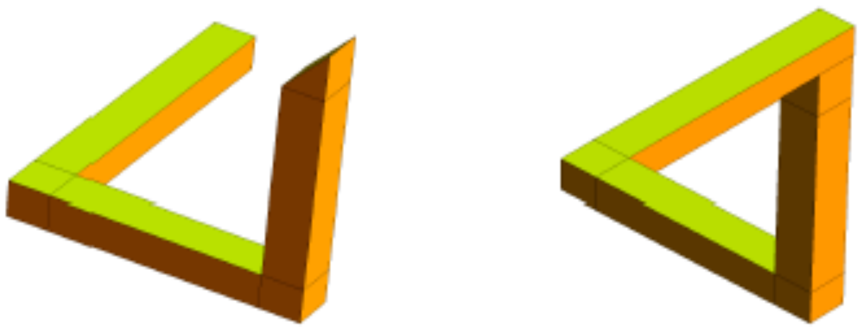




The 5 cell is the complete graph with 5 vertices and the simplest 4 dimensional polytop. The 8 cell to the right is also called the tesseract or hypercube. It is the 4 dimensional analogue of the cube and has reached stardom status among all mathematical objects.



Printing a simplified version of the Escher stairs. If the object is turned in the right angle, an impossible stair is visible. When printed, this object can visualize the geometry of impossible figures.



Printing the Penrose triangle. The solid was created by Oscar Reutersvard and popularized by Roger Penrose [35]. A Mathematica implementation has first appeared in [99]. The figure is featured on one of the early editions of [46] and of [8].